Day-Ahead Self Scheduling of a Virtual Power Plant in Energy and Reserve Electricity Markets under Uncertainty

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Abstract—This paper proposes a novel model for the day-ahead self-scheduling problem of a virtual power plant trading in both energy and reserve electricity markets. The virtual power plant comprises a conventional power plant, an energy storage facility, a wind power unit, and a flexible demand. This multi-component system participates in energy and reserve electricity markets as a single entity in order to optimize the use of energy resources. As a salient feature, the proposed model considers the uncertainty associated with the virtual power plant being called upon by the system operator to deploy reserves. Additionally, uncertainty in available wind power generation and requests for reserve deployment is modeled using confidence bounds and intervals, respectively, while uncertainty in market prices is modeled using scenarios. The resulting model is thus cast as a stochastic adaptive robust optimization problem which is solved using a column-and-constraint generation algorithm. Results from a case study illustrate the effectiveness of the proposed approach.

Index Terms—Robust optimization, self scheduling, stochastic programming, uncertainty, virtual power plant.

Notation

The main notation of this paper is stated below, where, for the sake of conciseness, subscripts \( t \) and \( \omega \) denoting time period and scenario indexes are dropped. Additional symbols with subscript \( \nu' \) are used to represent new variables corresponding to iteration \( \nu' \). Note that the value of a specific variable at iterations \( \nu \) and \( \nu' \) is represented by superscripts \( \nu ' \) and \( \nu ' ' \), respectively. In the out-of-sample assessment, subscript \( \nu \) and superscript \( \nu \) respectively denote new variables and parameters for sample \( \nu \).

A. Sets and Indexes

\( \mathcal{O} \) Set of sample indexes \( \nu \).
\( \mathcal{T} \) Set of time period indexes \( t \).
\( \Theta \) Feasibility set.
\( \Lambda \) Uncertainty set.
\( \nu, \nu' \) Indexes for the iterations of the column-and-constraint generation algorithm.
\( \Upsilon^{LL}, \Upsilon^{UL} \) Set of lower-level decision variables of the subproblem.

\( \Psi^D, \Psi^O \) Sets of decision variables of the deterministic problem and the problem solved in the out-of-sample assessment.
\( \Psi^{LS}, \Psi^M, \Psi^S \) Sets of decision variables of the linearized subproblem, the master problem, and the single-level equivalent of the subproblem.

\( \Omega \) Set of scenario indexes \( \omega \).

B. Parameters

\( C^{C,F}, C^{C,V} \) Fixed and variable cost coefficients of the conventional power plant (CPP) [\$, $/MWh].
\( D^D \) Minimum daily energy consumption of the flexible demand [MWh].
\( R^{R+}, R^{R-} \) Up- and down-reserve request coefficients [p.u.].
\( N \) Penalty cost coefficient for slack variables [$/MWh].
\( P^C, P^D \) Lower and upper bounds for the power generation of the CPP [MW].
\( P^E, P^F \) Minimum and maximum power consumption levels of the flexible demand [MW].
\( P^R^+, P^R^- \) Minimum and maximum power levels that can be traded in the energy market [MW].
\( P^{S,C}, P^{S,D} \) Maximum power that can be traded in the up- and down-reserve markets [MW].
\( P^{W,A} \) Charging and discharging capacities of the storage unit [MW].
\( \hat{P}^{W,A}, \hat{P}^{W,A} \) Available wind power generation [MW].
\( R^{C,D}, R^{C,U} \) Average and fluctuation levels of the available wind power generation [MW].
\( R^{C,SD}, R^{C,SU} \) Down and up ramping limits of the CPP [MW/h].
\( S^C, S^{SD}, S^{SU} \) Shut-down and start-up ramp rates of the CPP [MW/h].
\( S_0^C \) Initial stored energy of the storage unit [MWh].
\( S^S, S^S \) Lower and upper bounds for the stored energy of the storage unit [MWh].
\( S^{DC}, S^{UC} \) Shut-down and start-up cost coefficients of the CPP [\$/].
\( U_0^C \) Initial on/off scheduling status of the CPP.
\( \Gamma^R \) Reserve uncertainty budget.
\( \Gamma^W \) Wind uncertainty budget.
Δt
η^{S,C}, η^{S,D}
λ^E
λ^{R+, R^-}
μ^E
μ^{R+, R^-}
π_ω

C. Variables

$h^+, h^-$ Slack variables used in the subproblem and in the problem solved in the out-of-sample assessment [MW].

$k^{R+, R^-}$ Up- and down-reserve requests [p.u.].

$p^{C}$ Power generation of the CPP [MW].

$p^D$ Power consumption of the flexible demand [MW].

$p^{E}$ Power traded in the energy market [MW].

$p^{R+, R^-}$ Power capacity traded in the up- and down-reserve markets [MW].

$p^{S,C}, p^{S,D}$ Charging and discharging power levels of the storage unit [MW].

$p^W$ Production of the wind power unit [MW].

$p^{W,A}$ Available production of the wind power unit [MW].

$s^S$ Stored energy of the storage unit [MWh].

$u^C$ Variable that is equal to 1 if the CPP is generating electricity, being 0 otherwise.

$u^{W+, W-}$ Binary variables that are equal to 1 if the worst-case available wind power generation is equal to its upper and lower bounds, being 0 otherwise.

$v^{C,SD}, v^{C, SU}$ Variables representing the shut-down and start-up statuses of the CPP.

$\vartheta$ Auxiliary variable of the master problem.

I. INTRODUCTION

NOWADAYS, environmental concerns have boosted the production of electricity from renewable units. Nevertheless, despite the significant technological improvements made over the last decade, several operational issues hinder the large-scale penetration of renewable-based power production in existing generation portfolios. One of the handicaps of renewable units is their uncertain and variable production, which is an obstacle for their participation in electricity markets. Combining renewable units with other technologies such as conventional generating units, storage facilities, or flexible demands might be a solution to either supply energy when the renewable production is low or store energy if the renewable production is high. Such a combination yields the notion of virtual power plant (VPP), which can be defined as an aggregation of different distributed energy resources that operate as a single entity [1]. Relevant examples of real-life VPPs can be found in [2]–[4]. In addition, the interested reader is referred to [5] for a recent literature review on approaches for VPP operation.

Within the context of the electricity markets implemented across Europe and the U.S., this paper addresses the self-scheduling problem of a VPP trading energy and reserves in a day-ahead setting. The VPP under consideration comprises a conventional power plant (CPP), a storage facility, a wind power unit, and a flexible demand. Hence, this problem consists in deciding one day ahead the levels of power and reserves that maximize the overall profit of the VPP under uncertainty. Uncertainty sources of this problem are market prices, available wind power generation, and the requests by the system operator to deploy reserves.

Some relevant closely related works are [6]–[11]. Reference [6] uses a deterministic price-based unit commitment model to address the bidding strategy of a VPP. Zhao et al. [7] solve a stochastic bilevel optimization problem in order to devise the least-cost bidding strategy of a VPP in both day-ahead and balancing energy markets. In [8], robust optimization without recourse is implemented to address the bidding strategy of a VPP in the day-ahead and real-time markets under uncertain market prices and wind power production. The optimal risk-averse offering strategy for a VPP trading in a joint market for energy and spinning reserve is modeled in [9]. Uncertainties in renewable-based generation, calls for the reserve service, and prices in electricity markets are characterized through a two-stage stochastic programming approach. The application of stochastic programming for the medium-term self-scheduling of a VPP that cooperates with neighbour VPPs and trades energy in an electricity market is proposed in [10]. In the recent work [11], a new approach is presented for the offering strategy of a VPP in the day-ahead energy market wherein uncertainties in wind power production and market prices are modeled with confidence bounds and scenarios, respectively.

In spite of their relevance, some important practical aspects are disregarded by existing approaches. Note that reserve market trading is ignored in [7], [8], [10], and [11]. Furthermore, uncertainty is neglected in [6], while the approaches described in [7]–[10] rely on a far unrealistic assumption, namely, the availability of an accurate characterization of all uncertainty sources. Such an assumption is particularly questionable for wind power generation and for the requests by the system operator to deploy reserves due to the intermittency of the former and the lack of publicly available historical data for the latter. Therefore, new approaches for the self-
scheduling problem of a VPP participating in energy and reserve electricity markets are yet to be explored.

Built on our previous work [11], this paper describes the combination of scenario-based stochastic programming [12] and adaptive robust optimization (ARO) [13] to consider reserve trading in the self-scheduling of a VPP. The incorporation of this practical aspect constitutes a relevant extension of [11] due to the increased complexity of the optimization process, wherein uncertain reserve deployments become part of the constraint set. Note that self-scheduling decisions driven by the imprecise characterization of such uncertain parameters may lead to not only suboptimal but also infeasible VPP operation. In order to address this issue, the resulting stochastic ARO formulation features a significant modeling novelty with respect to [6]–[11], namely the characterization of the requests for reserve deployment using intervals. Moreover, uncertain market prices are modeled using scenarios, as is usual in previous works dealing with VPPs [9]–[11], while the uncertainty in available wind power generation levels is represented through confidence bounds, as is customary in related works for generation scheduling under uncertainty [14]–[20]. Consequently, different from the relevant literature contributions [6]–[11], the proposed approach allows effectively incorporating reserve trading in the self-scheduling problem while properly accounting for all uncertainty sources involved.

The proposed stochastic ARO model is formulated as a trilevel problem that involves three nested optimization levels. This problem can be addressed by replacing the two lowermost optimization levels with a set of constraints modeling the reaction of the VPP against every possible realization of reserve deployment requests and available wind power generation [21]. In order to overcome the intractability issue of the realization-dependent approach, suitable decomposition-based methods for structurally similar instances of trilevel programming are available such as Benders decomposition [15] and the column-and-constraint generation algorithm (CCGA) [22]. Both techniques involve the iterative solution of a master problem representing an approximation of the original trilevel program, and a subproblem associated with the two lowermost optimization levels. The only distinctive aspect featured by both methods arises in the master problem. Under a Benders decomposition framework, the master problem relies on the iterative addition of cutting planes with dual information from the subproblem. In contrast, the CCGA gives rise to a master problem wherein decision variables (columns) and primal lower-level constraints are iteratively added based on the solution of the preceding subproblem.

Motivated by the superior performance over Benders decomposition [22] and recent examples of successful application for power system operation [18]–[20] and planning [23] problems, the proposed solution approach relies on the CCGA. It is worth emphasizing that the proposed methodology guarantees finite convergence to the optimal solution. As a distinctive methodological feature in comparison with our previous work [11], we use an equivalent binary-variable-based representation of the uncertainty set. This equivalent uncertainty characterization enables the application of the duality theory of linear programming, which substantially improves the computational performance of the proposed decomposition-based method.

The contributions of this paper are threefold:

1) A novel stochastic ARO approach is provided for the self-scheduling problem of a VPP trading in energy and reserve electricity markets.

2) For the first time in the literature, a model relying on intervals is provided to characterize the uncertainty in the requests for reserve deployment.

3) The resulting trilevel optimization problem is effectively solved using an enhanced CCGA, as compared with our previous closely related work [11]. The use of the Karush-Kuhn-Tucker (KKT) optimality conditions reported in [11] to tackle the subproblem is here replaced with the application of the duality theory of linear programming. As a consequence, the dimension of the single-level equivalent for the subproblem is significantly reduced, which is beneficial from a computational perspective.

The rest of this paper is organized as follows. Section II presents the problem formulation. Section III is devoted to the solution methodology. Results from a case study are provided and analyzed in Section IV. Section V closes the paper with some remarkable conclusions and future avenues of research. Finally, the linearization of the subproblem is described in the Appendix.

II. FORMULATION

This section provides the formulation of the self-scheduling problem of a VPP trading in energy and reserve electricity markets. In order to facilitate the understanding of the proposed model, we first present a deterministic instance followed by the description of the uncertainty characterization.

A. Deterministic Model

In this simpler formulation, available wind power generation, requests for reserve deployment, and market prices are considered known parameters. Based on [9], the deterministic model is formulated as the following mixed-integer linear program:

\[
\begin{align*}
\max_{\phi, d} & \sum_{t \in T} \left[ \mu_t^E p_t^E \Delta t + \left( \mu_t^R - K_t^R \mu_t^R + R_t^R \Delta t \right) p_t^R \right. \\
& \left. + \left( \mu_t^R - R_t^R \mu_t^R - \Delta t \right) p_t^R - \left( C_{C,t}^C u_t^C + C_{C,t}^V p_t^C \Delta t \right) \right] \\
\text{subject to:} & \quad p_t^E + R_t^R p_t^R - K_t^R p_t^R = p_t^W - p_t^D + p_t^{S,C} \left( S_{S,C}^C, \forall t \in T \right) \\
& \quad P_t^E \leq p_t^E \leq T_t^E, \forall t \in T \\
& \quad 0 \leq p_t^R + R_t^R, \forall t \in T \\
& \quad 0 \leq p_t^R, \forall t \in T \\
& \quad u_t^C, u_t^{S,C} \in \{0, 1\}, \forall t \in T \\
& \quad v_t^C, v_t^{C,SD} \geq 0, \forall t \in T 
\end{align*}
\]

(1a)
\[ u^C_t - u^C_{t-1} = v^C_{t, \text{SU}} - v^C_{t, \text{SD}}, \forall t \in \mathcal{T} \quad (1h) \]
\[ P^D_t \leq P^C_t \leq P^D_t, \forall t \in \mathcal{T} \quad (1i) \]
\[ p^C_t - p^C_{t-1} \leq \left( R^D_t u^C_t + R^C_{t, \text{SU}} v^C_{t, \text{SU}} \right) \Delta t, \forall t \in \mathcal{T} \quad (1j) \]
\[ p^C_t - p^C_{t-1} \leq \left( R^D_t u^C_t + R^C_{t, \text{SD}} v^C_{t, \text{SD}} \right) \Delta t, \forall t \in \mathcal{T} \quad (1k) \]
\[ \sum_{t \in \mathcal{T}} p^D_t \Delta t \geq D^D \quad (1m) \]
\[ 0 \leq p^C_t \leq P^C, \forall t \in \mathcal{T} \quad (1n) \]
\[ 0 \leq p^D_t \leq P^D, \forall t \in \mathcal{T} \quad (1o) \]
\[ s^S_t = s^S_{t-1} + \left( \eta^S_t p^C_t - \frac{1}{\eta^S_t D^D} p^D_t \right) \Delta t, \forall t \in \mathcal{T} \quad (1p) \]
\[ s^S_t \leq s^S_t \leq s^S_t, \forall t \in \mathcal{T} \quad (1q) \]
\[ 0 \leq p^W_t \leq P^W, \forall t \in \mathcal{T} \quad (1r) \]

where the set \( \mathcal{S} = (p^C_t, p^D_t, p^R_+, p^R_-, p^W_t, s^S_t, u^C_t, v^C_{t, \text{SU}}, v^C_{t, \text{SD}}) \) \( \forall t \in \mathcal{T} \) includes the optimization variables of problem (1).

Problem (1) is driven by the maximization of the total profit of the VPP (1a), which is expressed in terms of the revenues and costs associated with the participation in the energy and reserve electricity markets, as well as the operating costs of the CPP. Note that, in the up- and down-reserve markets, we consider revenues from both capacity \( \left( \bar{\mu}^R_+ p^R_+ + \bar{\mu}^R_- p^R_- \right) \) and energy \( \left( K^R_+ p^R_+ \Delta t - K^R_- p^R_- \Delta t \right) \). Variables \( p^R_+ \) and \( p^R_- \) are the power capacities traded in the up- and down-reserve markets, respectively. Analogously, \( K^R_+ p^R_+ \Delta t \) and \( K^R_- p^R_- \Delta t \) represent the respective reserve deployments requested by the system operator, which depend on the actual system imbalance and the activation process for corrective actions [24]. Parameters \( K^R_+ \) and \( K^R_- \) lie in the range \([0,1]\) and thus represent the fractions of the corresponding capacities that are dispatched by the system operator in the up- and down-reserve markets, respectively. In other words, \( p^R_+ \) and \( p^R_- \) denote the maximum VPP production increase and decrease that can be requested by the system operator.

Constraints (1b) model the power balance of the VPP. Constraints (1c)–(1e) set bounds for the power traded in the energy, up-reserve, and down-reserve markets, respectively. The operation of the CPP is characterized in (1f)–(1k). Binary variables are used for on/off statuses and shut-downs (1f), whereas start-ups are represented by nonnegative continuous variables (1g). Constraints (1h) establish the relationship between these variables and guarantee that start-up variables, albeit continuous, are binary valued. Constraints (1i) bound the power production of the CPP. Constraints (1j)–(1k) impose the up- and down-ramp capability of the CPP.

Based on the demand-side models presented in [8], [11], and [25], constraints (1f) and (1m) define the demand requirements of the VPP. Constraints (1i) impose consumption limits at every time period, whereas constraint (1m) sets the minimum energy consumption along the scheduling horizon.

Constraints (1n)–(1q) model the operation of the storage unit. Constraints (1n) and (1o) bound the charging and discharging power levels, respectively. Constraints (1p) establish the energy storage level at every time period. Constraints (1q) set the limits on the energy stored at every time period.

Finally, constraints (1r) limit the wind power production.

B. Uncertainty Characterization

In the deterministic problem (1), market prices, available wind power generation, and requests for reserve deployment are assumed to be known by the VPP. In a practical setting [21], [26], the self-scheduling problem under consideration is solved one day in advance and, hence, these pieces of information are uncertain and unknown.

Decision making under uncertain parameters can be handled by scenario-based stochastic programming [12]. Within such a framework, uncertainty is characterized by a prespecified discrete set of realizations or scenarios and their corresponding probabilities of occurrence.

The accuracy of the uncertainty representation strongly depends on the knowledge of the actual probability distribution and on the size of the scenario-based discretization. Moreover, the accuracy of the uncertainty representation may have a significant impact on the solution quality. For uncertain parameters in the objective function, solution optimality is solely involved. In contrast, for uncertain parameters in the constraint set, solution quality may be affected in terms of not only optimality but also feasibility. Unfortunately, the larger the scenario set, and hence the more accurate uncertainty representation, the larger the computational burden. Therefore, a tradeoff between accuracy and tractability is required. Such a tradeoff may be hard to implement when probabilistic information is lacking and/or when solution feasibility is involved.

For the self-scheduling problem under consideration, uncertainty sources are categorized in two groups according to their presence in the problem formulation, namely 1) market-clearing prices for energy and reserves, which appear in the objective function, and 2) available wind power generation levels and reserve deployment requests, which are part of the constraint set.

The abundance of publicly available information and their lack of impact on solution feasibility render market-clearing prices suitable for a scenario-based stochastic programming framework [12], as customarily adopted in the closely related literature [9]–[11]. Scenarios for market prices can be generated using either historical data, as done in this paper, or effective forecasting tools [27] such as the ARIMA model applied in [11]. Market prices, which are denoted by \( \lambda^E_t, \lambda^R_+ t, \lambda^R_- t, \lambda^W_t \), are represented using a finite set of scenarios indexed by \( \omega \). Each scenario \( \omega \) has a probability of occurrence \( \pi_\omega \), such that the sum of the probabilities over all scenarios is equal to 1, i.e., \( \sum_{\omega \in \Omega} \pi_\omega = 1 \).

As for available wind power generation levels and reserve deployment requests, we argue that the assumption made in [7], [9], and [10] that their probability distributions are known is non-trivial. Despite the wide availability of data, the short-term uncertainty characterization of wind power is still challenging [28]. Moreover, to the best of our knowledge,
no information is publicly available for reserve deployment requests. Thus, the use of a computationally tractable set of scenarios is prone to yield infeasibility when uncertainty unfolds. This is a highly undesirable aspect in the context of power system operation, wherein the penalty associated with infeasible solutions is very high.

Hence, based on the growing body of literature on generation scheduling under uncertainty [14]–[20], [29], we propose dealing with uncertain wind power generation and reserve deployment requests by adjustable robust optimization [13] as an alternative to scenario-based stochastic programming. As a result, solution feasibility is ensured for all possible realizations of available wind power generation and reserve deployment requests within a pre-specified deterministic uncertainty set relying on confidence bounds and intervals. Since such a solution immunization comes at the expense of a reduction in the expected profit, the conservativeness of the solution is controlled by imposing user-defined uncertainty budgets.

For the problem at hand, wind-related uncertainty is modeled by variables:

\[ p_t^W, A, \tilde{p}_t^W A, \tilde{p}_t^W W, \delta_t^W, k_t^R, k_t^L \]

and the uncertainty budget \( \Gamma^W \) representing the maximum number of periods at which the available wind power generation experiences fluctuations with respect to the average. Similarly, uncertain requests for the fractional deployment of the scheduled reserve capacities are modeled by variables \( k_t^R \in [0, 1] \) and \( k_t^L \in [0, 1] \), and the uncertainty budget \( \Gamma^R \), which denotes the maximum number of periods at which the VPP is called upon to provide reserves. Both sources of uncertainty are thus characterized by a cardinality-constrained uncertainty set [30], [31]. For polyhedral uncertainty sets such as the cardinality-constrained uncertainty set under consideration, the worst-case uncertainty realization corresponds to an extreme or vertex of the polyhedron representing the uncertainty set [17], [21]. Thus, polyhedral uncertainty sets can be equivalently characterized by solely modeling the finite set of extremes or vertices of the polyhedron. As done in [14], [16], [17], [19], [20], and [23], binary variables are used to model the extreme-based equivalent for the original cardinality-constrained uncertainty set. The resulting binary-variable-based equivalent set \( \Lambda \) is formulated as follows:

\[
\Lambda = \left\{ \pi_{\text{ML}} : u_t^W, u_t^W, k_t^R, k_t^L \in \{0, 1\}, \forall t \in T \right\}
\]

\[
\begin{align*}
p_t^W, A &= \tilde{p}_t^W A - u_t^W - \tilde{p}_t^W A + u_t^W + \tilde{p}_t^W A, \forall t \in T \quad (2a) \\
u_t^W &+ u_t^W \leq 1, \forall t \in T \\
\sum_{t \in T} (u_t^W &+ u_t^W) \leq \Gamma^W \quad (2c) \\
k_t^R &+ k_t^R \leq 1, \forall t \in T \\
\sum_{t \in T} (k_t^R &+ k_t^L) \leq \Gamma^R \quad (2f)
\end{align*}
\]

where \( \pi_{\text{ML}} \) is the set of variables characterizing the uncertainty set. Binary variables \( u_t^W \) and \( u_t^W \) are associated with the worst-case available wind power generation, whereas \( k_t^R \) and \( k_t^L \) can be equivalently characterized as binary variables to model the worst-case reserve deployment requests. Constraints (2a) model the integrality of such binary variables. The use of the equivalent binary-variable-based representation for wind power generation uncertainty constitutes an additional major modeling difference with respect to our previous work disregarding reserves [11].

In (2b), the available wind power generation is expressed in terms of the average and fluctuation levels associated with the corresponding confidence bounds. Constraints (2c) guarantee that the available wind power generation cannot be simultaneously equal to the minimum and the maximum values at every time period. Constraint (2d) controls the conservativeness of the model through the wind uncertainty budget, \( \Gamma^W \), with values ranging between 0 and \( |T| \). For \( \Gamma^W = 0 \), the available wind power generation at each time period is equal to the corresponding average value, thereby disregarding its uncertainty. On the other hand, the maximum level of uncertainty corresponds to \( \Gamma^W = |T| \) because variables \( p_t^W, A \) are allowed to take values different from the average at all time periods.

Regarding the uncertainty in reserve deployment requests, constraints (2e) guarantee that the VPP cannot be called upon to provide both upward and downward reserves at the same time period. Constraint (2f) sets the reserve uncertainty budget, \( \Gamma^R \), that controls the conservativeness of the model associated with the requests for reserve deployment. For \( \Gamma^R = 0 \), the VPP is never called upon by the system operator to deploy reserves, i.e., \( k_t^R = k_t^L = 0, \forall t \in T \). In contrast, for \( \Gamma^R = |T| \), the VPP can be requested to provide upward or downward reserves at all time periods.

Conservativeness parameters \( \Gamma^R \) and \( \Gamma^W \) thus model the tradeoff between expected profit and uncertainty immunization and so depend on the preferences of the operator of the VPP. A detailed discussion on how to select appropriate values for such parameters is beyond the scope of this paper.

### C. Stochastic Adaptive Robust Optimization Model

The proposed stochastic ARO model is formulated as an instance of trilevel programming that involves three nested optimization levels. The upper level determines the energy and reserve market decisions, as well as the schedule of the CPP, maximizing the worst-case expected profit of the VPP. Given the upper-level decision vector, the middle level identifies the worst-case realization of available wind power generation levels and reserve deployment requests, i.e., that minimizing the expected profit of the VPP. Finally, the lower level models the VPP operation maximizing the expected profit for given upper- and middle-level decisions. The proposed trilevel counterpart is formulated as follows:

\[
\begin{align*}
\text{max}_{\pi} & \sum_{t \in T} \left[ \sum_{i \in \Omega} \left[ \lambda_{\text{E}}^t p_t^E \Delta t + \lambda_{\text{R}}^t p_t^R + \lambda_{\text{L}}^t p_t^L - (C_{\text{C,F}}^i u_t^i + S_{\text{C,SU}}^i u_t^i + S_{\text{D,C,SD}}^i u_t^i) \right] \\
& - \left( C_{\text{C,F}}^i u_t^i + S_{\text{C,SU}}^i u_t^i + S_{\text{D,C,SD}}^i u_t^i \right) \right] \\
& + \min_{\gamma_{\text{ML}}} \sum_{t \in T} \left[ \sum_{i \in \Omega} \left[ \lambda_{\text{E}}^t p_t^E \Delta t + \lambda_{\text{R}}^t p_t^R + \lambda_{\text{L}}^t p_t^L - (C_{\text{C,F}}^i u_t^i + S_{\text{C,SU}}^i u_t^i + S_{\text{D,C,SD}}^i u_t^i) \right] \\
& - \left( C_{\text{C,F}}^i u_t^i + S_{\text{C,SU}}^i u_t^i + S_{\text{D,C,SD}}^i u_t^i \right) \right]
\end{align*}
\]

(3a)
subject to: Constraints (1c) – (1h). \hspace{1cm} (3b)

Problem (3) involves three optimization levels:

1) The upper level related to the decisions for the energy and reserve markets, as well as to the schedule of the CPP, i.e., $\mathcal{Y}^{UL} = \left\{ p_t^E, p_t^{R+}, p_t^{R-}, \mu_t^C, \nu_t^C, v^{SU}_t, v^{CSU}_t \right\}_{t \in T}$.

2) The middle level associated with the worst-case realization of reserve deployment requests and available wind power generation levels, i.e., $\mathcal{Y}^{ML}$.

3) The lower level modeling the reaction against upper- and middle-level decisions, i.e., $\mathcal{Y}^{LL} = \left\{ p_t^C, p_t^D, \eta_t^C, p_t^W, s_t^S, S_t^S \right\}_{t \in T}$.

Problem (3) is driven by the maximization of the worst-case expected profit (3a) subject to constraints (3b) modeling the feasibility of upper-level decisions as described in Section II-A. Note that $\Lambda$ and $\Theta$ are the uncertainty and feasibility sets that are, respectively, related to the middle- and lower-level optimization variables. Set $\Lambda$ is explained in Section II-B whereas set $\Theta$ models the feasible space of lower-level optimization variables as follows:

$$\Theta = \left\{ \mathcal{T}^{UL}: p_t^E + k_t^{R+} - k_t^{R-} = p_t^W + p_t^D + p_t^C + p_t^{S\prime} - p_t^{S\prime}, \forall t \in T \right\} \hspace{1cm} (4a)$$

$$p_t^{C} \leq p_t^{C}, \forall t \in T \hspace{1cm} (4b)$$

$$p_t^{C} - p_t^{C-1} \leq \left( R_t \mu_t^{C} + R_t^{C,SD} v^{CSU}_t \right) \Delta t, \forall t \in T \hspace{1cm} (4c)$$

$$p_t^{D} \leq p_t^{D}, \forall t \in T \hspace{1cm} (4d)$$

$$\sum_{t \in T} p_t^{D} \Delta t \geq D^{D} \hspace{1cm} (4e)$$

$$0 \leq s_t^{S}, \forall t \in T \hspace{1cm} (4g)$$

$$0 \leq s_t^{S} \leq S_t^{S}, \forall t \in T \hspace{1cm} (4h)$$

$$s_t^{S} = s_t^{S-1} + \left( \eta_t^{S} p_t^{C} - \frac{1}{\eta_t^{S,D}} p_t^{S\prime} \right) \Delta t, \forall t \in T \hspace{1cm} (4i)$$

$$S_t^{S} \leq S_t^{S}, \forall t \in T \hspace{1cm} (4j)$$

subject to:

$$0 \leq p_t^{W} \leq p_t^{W,A}, \forall t \in T \hspace{1cm} (4k)$$

The feasibility set $\Theta$ is parameterized in terms of upper-level decisions $p_t^{E}, p_t^{R+}, p_t^{R-}, \mu_t^C, \nu_t^C$ and $v_t^{SU}$ and middle-level optimization variables $k_t^{R+}, k_t^{R-},$ and $p_t^{W,A}$.

Constraints (4) are identical to constraints (1b) and (1i)–(1r) in the deterministic problem (1).

Fig. 1 shows the nested structure of the proposed trilevel problem.

III. SOLUTION METHODOLOGY

The stochastic ARO problem (3) is solved using a CCGA [22], which involves the iterative solution of a master problem and a subproblem. This method guarantees finite convergence to optimality and provides a measure of the distance to the optimal solution along the iterative process.

A. Master Problem

The master problem at iteration $\nu$ is formulated below:

$$\max_{\omega \in \Omega} \sum_{\omega \in \Omega} \pi_{\omega} \left[ \sum_{t \in T} \left[ \mu_t^E p_t^E \Delta t + \lambda_t^{R+} p_t^{R+} + \lambda_t^{R-} p_t^{R-} - \left( C_t^{C,F} u_t^C + SUC_t^{C,SU} + SDC_t^{C,SD} v_t^{CSU} \right) \right] \right] + \theta$$ \hspace{1cm} (5a)

subject to:

Constraints (3b) \hspace{1cm} (5b)

$$\nu \leq \sum_{\omega \in \Omega} \pi_{\omega} \left[ \sum_{t \in T} \left[ \lambda_t^{R+} p_t^{R+} - \lambda_t^{R-} p_t^{R-} - C_t^{C,V} p_t^{W,\nu} \right] \right], \nu = 1, \ldots, \nu$$ \hspace{1cm} (5c)

$$p_t^{E} + k_t^{R+} p_t^{R+} - k_t^{R-} p_t^{R-} = p_t^{W} + p_t^{D} + p_t^{C} + p_t^{S\prime} - p_t^{S\prime}, \forall t \in T \hspace{1cm} (5d)$$

$$p_t^{C} \leq p_t^{C}, \forall t \in T \hspace{1cm} (5e)$$

$$p_t^{C} - p_t^{C-1} \leq \left( R_t \mu_t^{C} + R_t^{C,SD} v^{CSU}_t \right) \Delta t, \forall t \in T \hspace{1cm} (5f)$$

$$\forall t \in T, \nu = 1, \ldots, \nu \hspace{1cm} (5g)$$

$$\forall \nu = 1, \ldots, \nu$$

$$\forall \nu = 1, \ldots, \nu$$
\[
p_t^{C} - p_{t+1}^{C} \leq \left( R^{C,D} u_t^{C} + R^{C,SD} v_t^{C,SD} \right) \Delta t, \\
\forall t \in T, \nu = 1, \ldots, \nu \\
\Delta D \leq p_{t}^{D} \leq \Delta D, \forall t \in T, \nu = 1, \ldots, \nu \\
\sum_{t \in T} p_t^{S} \Delta t \geq \Delta D, \nu = 1, \ldots, \nu \\
0 \leq p_t^{S} \leq \Phi_{t}^{S}, \forall t \in T, \nu = 1, \ldots, \nu \\
0 \leq p_t^{W} \leq \Phi_{t}^{W}, \forall t \in T, \nu = 1, \ldots, \nu \\
s_{t+1}^{S} - s_{t}^{S} = \left( \eta_{t}^{S,C} p_t^{C} - \frac{1}{\eta_{t}^{S,D}} p_t^{S,D} \right) \Delta t: \phi_t, \forall t \in T (6a) \\
\Delta s_{t}^{S} \leq \Phi_{t}^{S}, \forall t \in T (6b) \\
0 \leq p_t^{W} \leq \Phi_{t}^{W}, \forall t \in T (6c) \\
h_{t}^{+}, h_{t}^{-} \geq 0, \forall t \in T (6d)
\]

where set \(Y_{\mathcal{LL}} = \{ Y_{\mathcal{UL}}, \{ h_{t}^{+}, h_{t}^{-} \} \}_{t \in T} \) whereas lower-level dual variables are presented after a colon. The subproblem (6) corresponds to the two lowermost optimization levels of the original trilevel problem (3) parameterized in terms of \( p_t^{E(\nu)} \), \( p_t^{R(\nu)} \), \( p_t^{R(-\nu)} \), \( C(\nu) \), \( v_t^{C,SD(\nu)} \), and \( v_t^{C,SU(\nu)} \), which are provided by the preceding master problem (5). In order to guarantee the feasibility of the subproblem along the iterative process, power balance constraints (6b) are relaxed with nonnegative slack variables \( h_{t}^{+} \) and \( h_{t}^{-} \) (6m), which are penalized in the objective function (6a) using a sufficiently large positive cost coefficient \( N \).

The subproblem (6) is a min-max bilevel model whose lower level is continuous and linear in its decision variables. Such a property allows using the duality theory of linear programming to convert the min-max subproblem (6) into a single-level equivalent. This conversion consists in replacing 1) the inner optimization in (6) with its dual constraints, and 2) the objective function in the outer optimization in (6) with the dual lower-level objective function [32]. Note that this transformation is customary in the related literature [14–20], [29]. Therefore, problem (6) is transformed into the single-level equivalent below:

\[
\min_{p_t^{E}, p_t^{W}, p_t^{D}, p_t^{S}} \sum_{t \in T} \left( \lambda_{t}^{R} p_t^{R} - R^{C,V} p_t^{C} - N (h_{t}^{+} + h_{t}^{-}) \right) \Delta t \\
- \sum_{t \in T} \left( R^{C,D} u_t^{C} + R^{C,SD} v_t^{C,SD} \right) \Delta t: \phi_t, \forall t \in T (7a) \\
\alpha_{t} + p_t^{D} - p_t^{S} - \sigma_t - \sigma_{t+1} - \sigma_{t+2} = C^{G,V}, \\
\forall t = 1, \ldots, |T|-1 (7b) \\
\alpha_{t} + p_t^{D} - p_t^{S} - \sigma_t - \sigma_{t+1} = C^{G,V}, \\
\forall t = 1, \ldots, |T|-1 (7c) \\
\alpha_{t} \leq N, \forall t \in T (7d) \\
\alpha_{t} \leq N, \forall t \in T (7e) \\
\alpha_{t} \leq N, \forall t \in T (7f) \\
\alpha_{t} \leq N, \forall t \in T (7g) \\
\alpha_{t} \leq N, \forall t \in T (7h) \\
\alpha_{t} - \sigma_t \leq 0, \forall t \in T (7i)
\]

subject to:

Constraints (2)

\[
\alpha_{t} + p_t^{D} - p_t^{S} - \sigma_t - \sigma_{t+1} - \sigma_{t+2} = C^{G,V}, \\
\forall t = 1, \ldots, |T|-1 (7c)
\]

Subject to:

Constraints (2)

\[
\alpha_{t} + p_t^{D} - p_t^{S} - \sigma_t - \sigma_{t+1} - \sigma_{t+2} = C^{G,V}, \\
\forall t = 1, \ldots, |T|-1 (7c)
\]
\[ - \alpha_t - \delta_t - \eta^{S,C}_{t} \phi_t \Delta t \leq 0, \forall t \in \mathcal{T} \]  
\[ \phi_t + \psi_t - \bar{\psi}_t - \phi_{t+1} = 0, t = 1, \ldots, |\mathcal{T}| - 1 \]  
\[ \phi_{|\mathcal{T}|} + \psi_{|\mathcal{T}|} - \bar{\psi}_{|\mathcal{T}|} = 0 \]  
\[ \gamma_t, \delta_t, \kappa_t, \bar{\rho}_t, \bar{\sigma}_t, \tau_t, \xi \geq 0, \forall t \in \mathcal{T} \]  
where set \( \Psi^S = \{ k_t^{R+}, k_t^{R-}, p_t^{W,A}, u_t^{W+}, u_t^{W-}, \alpha_t, \gamma_t, \delta_t, \kappa_t, \bar{\rho}_t, \bar{\sigma}_t, \tau_t, \xi \}_{\forall t \in \mathcal{T}} \).

The dual lower-level objective function is minimized in (7a). Constraints (7b) are associated with the uncertainty set, while constraints (7c)–(7n) are the lower-level dual feasibility constraints. In (7a), \( \alpha_t k_t^{R+} + p_t^{A}, \alpha_t k_t^{R-} - p_t^{A} \) are nonlinear terms in the form of products of a lower-level dual (continuous) variable and a middle-level variable. Nevertheless, such bilinear terms can be replaced with equivalent mixed-integer linear expressions using the exact linearization scheme explained in [33] and in the Appendix. The use of mixed-integer linear programming is advantageous due to the guaranteed finite convergence to optimality and the availability of efficient off-the-shelf software based on the branch-and-cut algorithm.

### C. Algorithm

The iterative master-subproblem algorithm comprises the following steps:

1. Set the lower and upper bounds, LB and UB, for the optimal worst-case expected profit to \( -\infty \) and \( \infty \), respectively, and select the convergence tolerance, \( \epsilon \).

2. Initialize the iteration counter \( \nu \leftarrow 0 \).

3. Solve the master problem (5). Note that, if \( \nu = 0 \), then constraints (5c)–(5n) are not included.

4. Update the upper bound, UB, using the optimal value of the objective function (5a) of the master problem (5), \( z^{M*} \), i.e., \( UB = z^{M*} \).

5. Using the results obtained in Step 3, set \( p_t^{E(t)} = p_t^{E*} \), \( p_t^{R+} = p_t^{R+*} \), \( p_t^{R-} = p_t^{R-*} \), \( u_t^{W+} = u_t^{W+*} \), \( u_t^{W-} = u_t^{W-*} \), \( v_t^{C,SD} = v_t^{C,SD*} \), and \( v_t^{C,SU} = v_t^{C,SU*} \), and solve the linear equivalent of the subproblem (7).

6. Update the lower bound, LB, using equation (8) below:

\[
LB = \max \left\{ LB, \sum_{\omega \in \Omega} \pi_{\omega} \left[ \sum_{t \in \mathcal{T}} \lambda_t^{E(t)} p_t^{E(t)} \Delta t + \bar{C}^{R+} p_t^{R+} + \bar{C}^{R-} p_t^{R-} - \left( C^{C,F} u_t^{C,F} + C^{C,SD} u_t^{C,SD} + C^{C,SU} u_t^{C,SU} \right) \right] \right\} + z^{S*} \]  

where \( z^{S*} \) is the optimal value of the objective function (7a) of the subproblem (7).

7. If \( \frac{UB-LB}{UB} \leq \epsilon \), the algorithm stops and the solution to problem (3) is \( p_t^{R+}, p_t^{R-}, u_t^{C,F}, u_t^{C,SD}, u_t^{C,SU} \). Otherwise, go to Step 8.

8. Update the iteration counter \( \nu \leftarrow \nu + 1 \).

9. Using the results obtained in Step 5, set \( p_t^{W,A(t)} = p_t^{W,A*} \), \( k_t^{R+} = k_t^{R+*} \), and \( k_t^{R-} = k_t^{R-*} \), \( \forall t \in \mathcal{T} \).

10. Go to Step 3.

### IV. CASE STUDY

This section numerically demonstrates the performance of the proposed approach.

#### A. Data

For expository purposes, hourly price periods are considered, i.e., \( \Delta t = 1 \) h and \( |\mathcal{T}| = 24 \). As done in [9], market price scenarios are generated using data from the electricity market of Spain for the labor days from May 1st to June 17th, 2015 [34]. Therefore, uncertain market prices are represented by 34 equiprobable scenarios. Fig. 2 provides the confidence bounds and average levels for available wind power generation. Confidence bounds are obtained from [8]. For each time period, the average level for the available wind power generation is computed as the mean value of the corresponding upper and lower confidence bounds. The minimum power output and the capacity of the CPP, which is initially scheduled off, are 20 and 80 MW, respectively, whereas ramp rates are all equal to 50 MW/h. The fixed and variable cost coefficients of the CPP are $50 and $40/MWh, respectively, whereas the start-up and shut-down cost rates are both equal to $100. The lower and upper bounds for the level of stored energy of the storage unit are 0 and 100 MWh across the time span except for the last period. For \( t = |\mathcal{T}| \), the upper bound remains unchanged whereas the lower bound is set equal to the initial level, namely 50 MWh. Furthermore, the maximum charging and discharging power levels of the storage unit are both 50 MW, whereas the charging and discharging efficiencies are both equal to 90%. The minimum daily energy consumption of the flexible demand is 700 MWh and the hourly consumption limits are depicted in Fig. 3. The power levels that can be traded in the...
energy market are between -110 MW and 110 MW, whereas a 70-MW limit is considered for the participation in each reserve market. The penalty cost coefficient is set to $10^3$/MWh. Note that the results reported hereinafter remain unaltered for larger values of this coefficient.

B. Results

For illustration purposes, the results reported in this section correspond to a set of representative instances for which the wind uncertainty budget, $\Gamma^W$, is set to 12, whereas the uncertainty budget associated with the requests for reserve deployment, i.e., $\Gamma^R$, ranges between 0 and |$T$|. Thus, the robustness of the model is analyzed for different values of the uncertainty budget $\Gamma^R$ while assuming that the available wind power generation can take values different from the average at 12 time periods. The proposed approach has been implemented using CPLEX [35] under GAMS [36] on an Intel® Xeon® E7-4820 computer with 4 processors at 2 GHz and 128 GB of RAM. The convergence tolerance, $\epsilon$, as well as the optimality tolerances of CPLEX for both the master problem and the subproblem are set to 1% in all simulations. Using this stopping criterion, the average running time is 56.3 min, which is acceptable for the practical computational requirements of the day-ahead self-scheduling problem of a VPP. Moreover, the proposed model overcomes the computational issues of our previous work [11]. For all instances of this case study, the application of the KKT-based CCGA failed to solve a single subproblem after 24 hours, which is unacceptable for a day-ahead setting.

Fig. 4 shows the power scheduled by the VPP in the energy market for several values of $\Gamma^R$. As can be observed, for all instances, the participation in this market follows the profile of confidence bounds for available wind power generation. At the beginning of the scheduling horizon, the available wind power generation is higher and the demand is lower. Therefore, more power is scheduled to be sold in the energy market in order to maximize the worst-case expected profit since the cost of wind power production is null. In contrast, at the end of the scheduling horizon, the available wind power generation drops. Consequently, less power is scheduled to be sold in the energy market.

Fig. 5 depicts the scheduled participation in the up-reserve market for the same values of $\Gamma^R$ shown in Fig. 4. Note that, for $\Gamma^R = 3$, the scheduled power is at its maximum level for around half of the time periods. In contrast, for $\Gamma^R = 21$, the scheduled participation is 0 for roughly half of the time periods. However, a general pattern cannot be inferred as, for intermediate values of $\Gamma^R$, the scheduled participation increases as this parameter grows.

Finally, Fig. 6 plots the scheduled participation in the down-reserve market. As can be observed, for $\Gamma^R = 3$, the scheduled participation is at its maximum level, i.e., 70 MW, at most time periods, whereas, for $\Gamma^R = 21$, the scheduled power is different from the maximum tradable amount at six time periods. Note also that the minimum participation is featured for $\Gamma^R = 3$ at period 11. Thus, the impact of $\Gamma^R$ on the scheduled participation is case dependent.

C. Out-of-Sample Assessment

In order to validate the performance of the proposed approach, an out-of-sample assessment is carried out following the steps below:

1) Using the data reported in Section IV-A, we generate 10000 samples for market prices, available wind power generation, and up- and down-reserve request coefficients.

2) For each sample $o$ and for a given upper-level decision vector provided by the proposed model (3), namely $\{p_{E^o}^t, p_{R^+}^t, p_{R^-}^t, u_{o}^{C^o}, v_{o}^{C^o}, \bar{u}_{o}^{C^o}, \bar{v}_{o}^{C^o}, \bar{u}_{o}^{S^o}, \bar{v}_{o}^{S^o}\}_{t \in T}$, we solve the following problem:

$$
\max_{o} \sum_{t \in T} \left[ \mu_{E^o}^t p_{E^o}^t \Delta t + \left(\mu_{R^+}^t \rho_{R^+}^t + \rho_{R^-}^t \Delta t \right) p_{R^+}^t + \rho_{R^-}^t \Delta t \right] + K_{t}^{R^+} \rho_{R^+}^t \Delta t + K_{t}^{R^-} \rho_{R^-}^t \Delta t
$$
- $K_t^{R^-(o)} \mu_t^{R^-(o)} \Delta t \right) p_t^{R^+} + \left( C_{C,t} u_t^{C_s} + C_{C,V} \Delta t + \text{SUC}_C v_t^{C,SU^*} + \text{SDC}_C v_t^{C,SD^*} \right) - N (h_{t,o}^+ + h_{t,o}^-) \Delta t \right) \right]
\end{equation}

subject to:

\begin{align}
& p_{t,S}^E + K_t^{R^+(o)} p_t^{R^+} - K_t^{R^-(o)} p_t^{R^-} = p_{t,W} - p_{t,D} \\
& + p_t^{C} + p_t^{S,D} - p_t^{S,C} + h_{t,o}^+ - h_{t,o}^-, \forall t \in T \\
& \omega_t^{C} u_t^{C_s} \leq p_t^{C} \leq p_t^{C}, \forall t \in T \\
& p_{t-1}^{C} - p_t^{C} \leq \left( R_{C,t} u_t^{C_s} + R_{C,V} v_t^{C,SU^*} \right) \Delta t, \\
& \forall t \in T \\
& p_{t-1}^{C} - p_t^{C} \leq \left( R_{C,D} u_t^{C_s} + R_{C,SD} v_t^{C,SD^*} \right) \Delta t, \\
& \forall t \in T \\
& P_{t,D}^D \leq p_{t,D} \leq P_{t,D}^D, \forall t \in T \\
& \sum_{t \in T} p_{t,D} \Delta t \leq P_{t,D}^D \\
& 0 \leq p_{t,C} \leq P_{t,C}^C, \forall t \in T \\
& 0 \leq p_{t,D} \leq P_{t,D}^D, \forall t \in T \\
& s_t^S = s_{t-1}^S + \left( \frac{\sum_{t \in T} P_{t,C}^C}{P_{t,D}^D} \right) \Delta t, \forall t \in T \\
& s_t^S \leq s_{t-1}^S, \forall t \in T \\
& 0 \leq p_{t,W} \leq P_{t,W}^{W,A}, \forall t \in T \\
& h_{t,o}^+, h_{t,o}^- \geq 0, \forall t \in T
\end{align}

where set $\omega_t^{Q} = \{ h_{t,o}^+, h_{t,o}^-, p_{t,C}^C, p_{t,D}^D, p_{t,C}^S, p_{t,D}^S, p_{t,W}^{W,\Delta} \}$ where superscript “(o)” denotes the values of market prices, available wind power generation, and up- and down-reserve request coefficients for sample o. Problem (9) is a modified version of the deterministic problem (1) in which the variables related to the decisions for the energy and reserve markets, as well as to the schedule of the CPP are fixed. The infeasibility of the sample is modeled by nonnegative slack variables $h_{t,o}^+$ and $h_{t,o}^-$, which are included in the power balance constraints (9b) and penalized in the objective function (9a). The penalty cost coefficient $N$ is set to $\$10^3$/MWh. Such a choice is suitable to correctly flag infeasibility as the values of the objective function for infeasible solutions differ by several orders of magnitude from those associated with feasible solutions.

3) Finally, we compute the average sampled profit, $\Pi_{av}$, and the average sampled imbalance, $H_{av}$, as follows:

\begin{align}
\Pi_{av} &= \frac{1}{10000} \sum_{o \in O} \sum_{t \in T} \left[ \mu_t^{E(o)} p_t^{E} \Delta t + \left( \frac{\mu_t^{R^+(o)}}{\mu_t^{R^-(o)}} \right) p_t^{R^+} + \left( \frac{\mu_t^{R^-(o)}}{\mu_t^{R^+(o)}} \right) p_t^{R^-} \right] \\
&+ K_t^{R^+(o)} \mu_t^{R^+} \Delta t + R_{C,t} u_t^{C_s} \Delta t \\
&- K_t^{R^-(o)} \mu_t^{R^-} \Delta t \right) p_t^{R^-} - \left( C_{C,t} u_t^{C_s} + C_{C,V} \Delta t + \text{SUC}_C v_t^{C,SU^*} + \text{SDC}_C v_t^{C,SD^*} \right)
\end{align}

\begin{table}[h]
\centering
\caption{Results from the Out-of-Sample Assessment for the Proposed Model Using a Normal Distribution}
\label{tab:results_normal}
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{$\Gamma^R$} & \textbf{$\Pi_{av}$} [\$] & \textbf{$H_{av}$} [MW] \\
\hline
3 & 122,261 & 7.238 & 15 & 106,695 & 2.415 \\
6 & 113,786 & 3.234 & 18 & 106,251 & 2.408 \\
9 & 110,650 & 2.618 & 21 & 105,947 & 2.387 \\
12 & 106,808 & 2.422 & 24 & 105,630 & 2.051 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Results from the Out-of-Sample Assessment for the Fully Stochastic Model Using a Normal Distribution}
\label{tab:results_normal_full}
\begin{tabular}{|c|c|c|}
\hline
\textbf{Number of scenarios for $K_t^{R^+, R^-, p_{t,W,A}^W$} & \textbf{$\Pi_{av}$} [\$] & \textbf{$H_{av}$} [MW] \\
\hline
10 & 133,214 & 46.886 \\
50 & 124,822 & 11.011 \\
100 & 123,451 & 7.084 \\
150 & 123,107 & 6.083 \\
200 & 122,817 & 5.229 \\
\hline
\end{tabular}
\end{table}

\begin{align}
H_{av} &= \frac{1}{10000} \sum_{o \in O} \sum_{t \in T} \left( h_{t,o}^+ + h_{t,o}^- \right) \Delta t
\end{align}

where $h_{t,o}^+$, $h_{t,o}^-$, and $p_t^{C_s}$ respectively denote the optimal values of $h_{t,o}^+$, $h_{t,o}^-$, and $p_t^{C_s}$ resulting from (9).

Table I lists the results from the out-of-sample assessment for different values of the uncertainty budget $\Gamma^R$. Samples are generated using a normal probability distribution with an 85% confidence level. As observed in Table I, both the average sampled profit and the average sampled power imbalance decrease as the uncertainty budget $\Gamma^R$ increases. Hence, high values of $\Gamma^R$ correspond to a more conservative strategy with lower values of both metrics. It is worth emphasizing that power imbalance solely arises for those samples lying beyond the pre-specified uncertainty set.

The same out-of-sample assessment is implemented for the solution provided by a stochastic model based on [9], where all uncertainty sources are characterized by scenarios. As done in [9], 34 scenarios for market prices have been considered. In addition, the number of scenarios for available wind power generation and reserve deployment requests has been increased from 10, as originally done in [9], to 200. Further increasing the scenario set would lead to a stochastic model requiring solution times over an hour, which is the practical time limit for the day-ahead self-scheduling problem under consideration. The results shown in Table II corroborate the advantages from using ARO for such uncertainty sources. Note that the average sampled profits associated with the fully stochastic formulation are higher than those provided in Table I for the proposed model (3). Nevertheless, this economic gain comes at the expense of significantly larger and thus unacceptable infeasibility levels. As can be observed,
the lowest level of average sampled infeasibility attained for 200 scenarios is still twice as much as those featured by most instances of the proposed stochastic ARO approach.

Furthermore, we have compared the out-of-sample results for the proposed model and the fully stochastic model with those associated with the deterministic model (1). In such a model, market prices and available wind power generation levels are set equal to the mean values over the respective sets of scenarios considered in the original model. As for reserve deployment requests, three instances are analyzed: 1) the VPP is not requested to provide either upward or downward reserves, i.e., \( K_{t}^{R+} = K_{t}^{R-} = 0 \), \( \forall t \in T \), 2) the VPP is requested to provide all scheduled upward reserve capacity at all periods, i.e., \( K_{t}^{R+} = 1 \) and \( K_{t}^{R-} = 0 \), \( \forall t \in T \), and 3) the VPP is requested to provide all scheduled downward reserve capacity at all periods, i.e., \( K_{t}^{R+} = 0 \) and \( K_{t}^{R-} = 1 \), \( \forall t \in T \). The out-of-sample results are summarized in Table III. As can be observed, disregarding uncertainty yields significantly higher average sampled profits at the expense of impractical increases in the average sampled infeasibility levels by up to three orders of magnitude over those featured by the fully stochastic and stochastic ARO solutions.

Finally, in order to illustrate the impact of the probability distribution used to generate the pool of scenarios, we have implemented an additional out-of-sample analysis in which the samples are generated using a uniform distribution. The results provided in Table IV show that the proposed stochastic ARO model substantially outperforms the fully stochastic model in terms of average sampled infeasibility. These results corroborate the superiority of the proposed model regardless of the probability distribution used in the out-of-sample assessment.

### D. Impact of Demand Flexibility

The impact of demand flexibility on the VPP trading has been analyzed by considering an inflexible demand featuring identical minimum and maximum power consumption levels in every period, i.e., \( P_{t}^{D} = \overline{P}_{t}^{D} = \underline{P}_{t}^{D} \), \( \forall t \in T \). For expository purposes and a fair comparison with the results associated with the original flexible demand, the inflexible demand has been selected following the two criteria below:

1) As shown in Fig. 7, for each period of the scheduling horizon, the consumption of the inflexible demand, \( P_{t}^{D} \), is within the consumption limits of the original flexible demand.

2) The energy consumption of the inflexible demand over the scheduling horizon, i.e., \( \sum_{t \in T} P_{t}^{D} \Delta t \), is equal to 700 MWh, i.e., the minimum daily energy consumption of the original flexible demand.

The proposed approach has been run for the values of \( \Gamma^{R} \) and \( \Gamma^{W} \) and the stopping criterion adopted for the original flexible demand. As expected, for all instances, the use of the above-described inflexible demand led to lower values of the upper-level objective function maximized in (3a). This result primarily stems from the overall reduction in the participation in the up-reserve market.

For the solutions attained under the inflexible demand, Table VI summarizes the results from the out-of-sample assessment using a normal distribution. The comparison of the results provided in Table VI with those reported in Table I shows that, for all values of \( \Gamma^{R} \), the consideration of the inflexible demand yields lower average sampled profits, \( \Pi^{av} \), and higher average sampled imbalances, \( H^{av} \). Such results corroborate the beneficial impact of demand flexibility on the operation of the VPP.

### V. Conclusion

A stochastic ARO approach for the self-scheduling of a VPP in energy and reserve electricity markets is proposed in this paper. Relevant conclusions from this work are:

1) The uncertainties in market prices, available wind power generation, and requests for reserve deployment are key

<table>
<thead>
<tr>
<th>Instance</th>
<th>( \Pi^{av} ) [$]</th>
<th>( H^{av} ) [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{t}^{R+} = K_{t}^{R-} = 0 )</td>
<td>207,783</td>
<td>1,669,248</td>
</tr>
<tr>
<td>( K_{t}^{R+} = 1 ) and ( K_{t}^{R-} = 0 )</td>
<td>207,783</td>
<td>1,669,248</td>
</tr>
<tr>
<td>( K_{t}^{R+} = 0 ) and ( K_{t}^{R-} = 1 )</td>
<td>196,908</td>
<td>2,015,981</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sampled infeasibility level</th>
<th>( \Pi^{av} ) [$]</th>
<th>( H^{av} ) [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 133,501 25,601</td>
<td>50 125,674 4,313</td>
<td></td>
</tr>
<tr>
<td>100 124,410 2,301</td>
<td>150 124,068 1,663</td>
<td></td>
</tr>
<tr>
<td>200 123,834 1,516</td>
<td>250 122,530 1,016</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 7.** Inflexible demand and consumption limits for the original flexible demand.
TABLE VI  
RESULTS FROM THE OUT-OF-SAMPLE ASSESSMENT FOR THE PROPOSED MODEL USING A NORMAL DISTRIBUTION AND AN INFLEXIBLE DEMAND  

<table>
<thead>
<tr>
<th>$\Gamma^R$</th>
<th>$\Pi^w$ [S]</th>
<th>$H^w$ [MW]</th>
<th>$\Gamma^R$</th>
<th>$\Pi^w$ [S]</th>
<th>$H^w$ [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>118,236</td>
<td>7,855</td>
<td>15</td>
<td>103,773</td>
<td>2,824</td>
</tr>
<tr>
<td>6</td>
<td>110,736</td>
<td>4,174</td>
<td>18</td>
<td>103,005</td>
<td>2,790</td>
</tr>
<tr>
<td>9</td>
<td>106,646</td>
<td>2,952</td>
<td>21</td>
<td>102,757</td>
<td>2,720</td>
</tr>
<tr>
<td>12</td>
<td>104,664</td>
<td>2,841</td>
<td>24</td>
<td>102,575</td>
<td>2,615</td>
</tr>
</tbody>
</table>

factors in the decision-making problem faced by a VPP trading in energy and reserve electricity markets. The proposed combination of stochastic programming and ARO is an appropriate method because each uncertainty source is modeled with the technique that best adapts to its particular characteristics. Furthermore, the use of ARO to model the uncertainty in the requests for reserve deployment is a distinctive feature of this work.

2) The uncertainty budgets used in this model provide a flexible control of the robustness of the approach. Low values of the uncertainty budgets yield less conservative solutions while high values give rise to more robust solutions.

3) In terms of sampled infeasibility, the solutions achieved by the model presented in this paper significantly outperform those produced by both a fully stochastic model and a deterministic model.

4) Demand flexibility is useful to leverage the arbitrage opportunities arising from the participation in different markets.

Although a simple demand-side model has been presented in this paper, it should be noted that the main steps used in the proposed solution approach are readily applicable to incorporate other aspects of electricity consumption with additional operational constraints as well as some extra notation to properly index variables and parameters. We recognize that the extended model needs further numerical studies.

Future research will address the consideration of a more sophisticated demand-side model as well as additional components in the VPP, e.g., a fleet of electric vehicles and other forms of renewable-based generation. Larger VPPs will also be considered, including the characterization of the temporal and spatial correlations of renewable-based power generation. To that end, the use of ellipsoidal uncertainty sets constitutes a promising strategy which would require the development of a new solution approach to efficiently address the resulting computationally challenging problem. Furthermore, the participation of the VPP in other electricity markets with different time frames as well as the correlation of market prices will also be examined.

APPENDIX  
LINEARIZATION OF THE BILINEAR TERMS OF THE SUBPROBLEM  

This appendix presents the linearized version of the subproblem (7). The objective function (7a) comprises nonlinear terms $\alpha_t k_t^{R^+} p_t^{R(\nu)}$, $\alpha_t k_t^{R^+} p_t^{-R(\nu)}$, and $\tau_t p_t^{W,A}$. The first two terms involve products of a lower-level dual variable and a middle-level binary variable. As per (2b), the latter term $\tau_t p_t^{W,A}$ is equal to $\tau_t I_{t}^{W,A} - \tau_t u_t^{W} - \tau_t u_t^{W,A} + \tau_t u_t^{W,A}$, thereby giving rise to additional bilinear terms $\tau_t u_t^{W} - I_{t}^{W,A}$ and $\tau_t u_t^{W,A} I_{t}^{W,A}$, also involving products of a lower-level dual variable and a middle-level binary variable.

As explained in [33], a linear equivalent for the product of a binary variable $x \in \{0, 1\}$ and a continuous variable $\beta \in [\beta_{\text{min}}, \beta_{\text{max}}]$ is found as follows: 1) replace the product $\beta x$ with a new continuous variable $z$, and 2) introduce new inequalities $\beta_{\text{min}} x \leq z \leq \beta_{\text{max}} x$ and $\beta_{\text{min}} (1 - x) \leq \beta - z \leq \beta_{\text{max}} (1 - x)$. If $x$ is equal to 0, $z$ is also equal to 0 while $\beta$ is bounded by its upper and lower limits. Conversely, if $x$ is equal to 1, $z$ is set equal to $\beta$ and is bounded by the upper and lower limits for $\beta$.

Using such integer algebra results, the linearized subproblem is formulated as follows:

$$
\min_{q, \omega, \tilde{R}, S, \tilde{D}} \sum_{t \in \Omega} \left[ \sum_{t \in T} \left( \alpha_t I_{t}^{R^+} + k_t^{R^+} p_t^{R(\nu)} - \alpha_t k_t^{R^+} p_t^{-R(\nu)} \right) \Delta t - \sum_{t \in T} \left( \alpha_t I_{t}^{W,A} - \alpha_t k_t^{W,A} - \frac{\rho_t}{\alpha_t} C_t \right) \right]
$$

subject to:

Constraints (7b) – (7n)

$$
\begin{align}
\tau_t I_{t}^{R^+} &= \alpha_t k_t^{R^+}, \forall t \in T \\
\alpha_t k_t^{R^+} &\leq z_t^{R^+}, \forall t \in T \\
\alpha_t (1 - k_t^{R^+}) &\leq \alpha_t k_t^{R^+} - z_t^{R^+}, \forall t \in T \\
\alpha_t I_{t}^{R^-} &= \alpha_t k_t^{R^-}, \forall t \in T \\
\alpha_t k_t^{R^-} &\leq z_t^{R^-}, \forall t \in T \\
\alpha_t (1 - k_t^{R^-}) &\leq \alpha_t k_t^{R^-} - z_t^{R^-}, \forall t \in T \\
\tau_t I_{t}^{W^-} &= \tau_t u_t^{W^-}, \forall t \in T \\
\tau_t u_t^{W^-} &\leq z_t^{W^-} - \tau_t u_t^{W^-}, \forall t \in T \\
\tau_t (1 - u_t^{W^-}) &\leq \tau_t z_t^{W^-} - \tau_t u_t^{W^-}, \forall t \in T \\
\tau_t I_{t}^{W^+} &= \tau_t u_t^{W^+}, \forall t \in T \\
\tau_t u_t^{W^+} &\leq z_t^{W^+} - \tau_t u_t^{W^+}, \forall t \in T \\
\tau_t (1 - u_t^{W^+}) &\leq \tau_t z_t^{W^+} - \tau_t u_t^{W^+}, \forall t \in T \\
\end{align}
$$

where set $\Psi_{\text{LS}} = \left\{ \Psi^S, \left\{ z_t^{R^+}, z_t^{R^-}, z_t^{W^-}, z_t^{W^+} \right\}_{\forall t \in T} \right\}$. 

subject to:

Constraints (7b) – (7n)
The objective function minimized in (11a) is the linearized version of that minimized in (7a), expression (11b) includes constraints (7b)–(7n), and expressions (11c)–(11n) model the linearization of the nonlinear terms. More specifically, the linearizations of \( \alpha_k t^+ \), \( \alpha_k t^- \), \( \tau u \), and \( \tau u^- \) are modeled in (11c)–(11e), (11f)–(11h), (11i)–(11k), and (11l)–(11n), respectively.

REFERENCES


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