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APPROACHES FOR QUALITATIVE AND QUANTITATIVE ANALYSIS OF COMPLEX SYSTEMS: ALGORITHMS AND CASE STUDIES

by

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Under the Direction of:

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“Everybody is ignorant, only in different subjects”

(Will Rogers,1935)

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OUTCOMES

As part of this research, the following works and papers in journals and international conferences have been published:

Summary

- Journal Papers: 8 (3 JCR Published)
- Awards: 1 International
- Monographs: 2 (Pearson and Elsevier)
- Conference Papers: 9 (100% Internationals)
- Book Chapter: 3 (100% Internationals)
- Patents: 1

Journals:

Nomenclature: Authors Names, Title, Journal Details, [IF (impact factor), X/Y (Journal Position/Total number of journals into the Subject Category), Subject Category].

1 **Alberto Pliego**; F. P. García; J.M. Pinar. Optimal Maintenance Management on Offshore Wind Farms. Energies. 9(1), 46, 2016. [IF 2.072. 43/89 Energy & Fuels]. doi:10.3390/en9010046

2 F. P. García; J.M. Pinar; **Alberto Pliego**; M. Papaelias. The identification of critical components of wind turbines using FTA over the time.

Renewable Energy. Elsevier. [IF 3.361. 23/89 Energy & Fuels]. doi:10.1016/j.renene.2015.09.038

3 **Alberto Pliego**; F. P. García. A Novel Approach on Diagnostic and Prognostics in Railways: A Real Case Study. Proceedings of the Institution of Mechanical Engineers Part F-Journal of Rail and Rapid Transit. [IF 0.743. 20/32. Transportation Science & Technology]. doi:10.1177/0954409715596183

4 **Alberto Pliego**; F. P. García; J. Lorente, Decision making process via Binary decision diagram. 2014. International Journal of Management Science and Engineering Management. Online [C, Journal Rankings for ARC] http://dx.doi.org/10.1080/17509653.2014.946977.

5 **Alberto Pliego**; F. P. García; J. Lorente. Decision Making via Binary Decision Diagrams: A Real Case Study. Lecture Notes in Electrical Engineering (Springer), Vol. 180 pp. 215-222. 2014 DOI: 10.1007/978-3-642-55182-6_19. [SCImago 0.11; Source Normalized Impact per Paper 0.122]

6 F.P. García; **Alberto Pliego**; J. Lorente; J.R. Trapero. New Ranking Method Approach for Decision Making in Maintenance Management Lecture Notes in Electrical Engineering. Vol. 241, Springer, 2013, pp 23-39. [SCImago 0.11; Source Normalized Impact per Paper 0.122]

Journals under revision:

1 F. P. García; **Alberto Pliego**; Jesús María Pinar Pérez, Stuart Hillmansen, Mayorkinos Papaelias. Maintenance Management for Electrical/Electronic Components in Wind Turbines. European Journal

of Operational Research. In revision [IF 1.843. 15/79. Operations Research & Management Science]

2 **Alberto Pliego**; F. P. García. Decision making process via BDD for optimal investments under a risk environment. García; European Journal of Operational Research. In revision. [IF 1.843. 15/79. Operations Research & Management Science].

International Award

1 F.P. García; **Alberto Pliego**; J. Lorente; J.R. Trapero Arenas "Nominated Prize". 7th edition of the ICMSEM (International Conference on Management Science and Engineering Management). 7-9 Nov. 2013. Philadelphia (EEUU).

Monographs

1 F.P. García; **Alberto Pliego.** A. Manual Práctico para la Optimización de Toma de Decisiones. 2016. Pearson. ISBN:978-84-205-6472-2

2 F.P. García; **Alberto Pliego.** Decision-Making Management: A Tutorial and Applications. Elsevier. Forthcoming

International Conferences:

1 **Alberto Pliego;** F.P. García. Multivariable Analysis for Advanced Analytics of Wind Turbine Management. The 10th International Conference on Management Science and Engineering Management (ICMSEM2016). August 2016. Baku, Azerbaijan.

2 **Alberto Pliego**. F.P. García; R. Ruiz. Fault Detection and Diagnosis, and Optimal Maintenance Planning via FT and BDD. Twelfth International

Conference on Condition Monitoring and Machinery Failure, Prevention Technologies (CM 2015 and MFPT 2015). June 2015, Oxford (UK).

3 **Alberto Pliego**; F.P, Garcia. Improving the Efficiency on Decision Making Process via BDD. The 9th International Conference on Management Science and Engineering Management (ICMSEM2015). July 2015, Karlsruhe (Germany).

4 **Alberto Pliego**; F.P. Garcia. System Management for Remote Condition Monitoring in Railway Systems. Proceedings of the 6th IET (the Institution of Engineering and Technology) Conference on Railway Condition Monitoring (RCM 2014). September 2014, Birmingham (UK).

5 **Alberto Pliego**; F.P. Garcia. Decision Making via Binary Decision Diagrams: A Real Case Study. Proceedings of the eight International Conference on Management Science and Engineering Management (ICMSEM 2014). July 2014, Lisbon (Portugal).

6 **Alberto Pliego**; F.P. García. Fault-tree dynamic analysis. Proceedings of the Eleventh International Conference on Condition Monitoring and Machinery Failure Prevention Technologies (CM 2014 and MFPT 2014). June 2014, Manchester (UK).

7 **Alberto Pliego**; F.P. García. Quantitative Analysis of Probability Applied to Decision Making Process. Proceedings of the Advances in Conference on Innovation, Service and Management (ICISM2014), April 2014, Taichung (Taiwan).

8 F.P. García; **Alberto Pliego**; J. Lorente; J.R. Trapero Arenas. New Ranking Method Approach for Decision Making in Maintenance Management. The 7th International Conference on Management Science and Engineering Management (ICMSEM 2013). November 2013, Philadelphia (USA).

9 F.P. García; **Alberto Pliego**; J.M. Pinar Pérez; M. Papaelias. Fault Tree Analysis for Wind Turbine. The 5th International Conference on Management Science and Engineering Management (ICMSEM2011). November 2011, Macau (China). This conference paper has been considered as an outcome of the thesis in reference [1]. Alberto Pliego has developed the computer algorithms for obtaining the quantitative results.

Book Chapters:

1 F. P García; **Alberto Pliego**. FTA via BDD for Information Systems Design. Software Development Techniques for Constructive Information Systems Design, published by IGI Global USA, ISBN 978-1-4666-3679-8, Chapter 16, pp. 308-319. 2013. doi: 10.4018/978-1-4666-3679-8.ch016.

2 **Alberto Pliego**; F.P. García. Big Data and Web Intelligence: Improving the Efficiency on Decision Making Process via BDD. IGI Editorial. Handbook of Research on Trends and Future Directions in Big Data and Web Intelligence. IGI-Global. 2015. IGI-Global.ISBN13: 9781466685055|ISBN10: 1466685050|EISBN13: 9781466685062| DOI: 10.4018/978-1-4666-8505-5.

3 **Alberto Pliego**; F.P. García. Decision Making Approach for Optimal Business Investments., Advanced Business Analytics, Editors Fausto Pedro Garcia Marquez and Benjamin Lev, Chapter 1, pp. 1-20, Editorial Springer, ISBN 978-3-319-11414-9. 2015. DOI 10.1007/978-3-319-11415-6_1.

Patent:

1 F.P. García; **Alberto Pliego,** C.Q. Gómez. Sistema dinámico de análisis de la condición estructural mediante empleo multidireccional de ondas guiadas. 6 December 14, 2015.

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ACRONYMS

BC	Basic Cause
BDD	Binary Decision Diagrams
BFS	Breath First Search
CCO	Common Cause Occurrence
CM	Condition Monitoring
CMS	Condition Monitoring System
DFS	Depth First Search
DM	Decision Making
FT	Fault Tree
LDT	Logical Decision Tree
LT	Logical Tree
MCS	Minimal Cut-Set
MIF	Marginal Importance Factor
MOB	Multiple Occurring Branches
MOE	Multiple Occurring Event
MP	Main Problem

NN	Neural Network
OBDD	Ordered Binary Decision Diagram
O&M	Operation and Maintenance
PI	Prime Implicants
RAW	Risk Achievement Worth
ROBDD	Reduced Ordered Binary Decision Diagram
RRW	Risk Reduction Worth
SCADA	Supervisory Control and Data Acquisition
WT	Wind Turbine

1 OVERVIEW

Nowadays, Operation and Maintenance (O&M) tasks on complex systems have an essential role to ensure the correct condition of the system and to minimize losses, e.g. production, costs, etc., and increase the productivity. The new technologies and systems generate a large amount of information that needs to be analysed. As the consequence, Big Data is being generated, where the processing of these data is currently becoming researched. In this work, three methodologies are presented: The first one is based on the correlation of different variables, where redundancies are detected to guarantee that the systems do not generate false alarms and allows for the detection of abnormal behaviours; the second one evaluates some feature parameters in order to save only the essential information; finally, the third method extracts the main information in function of statistical hypothesis.

Usually the dataset corresponds to different components or events and informs about different issues, however, the dataset can be interrelated in order to gather strategic information. In this research work, the Logical Trees are employed for establishing the interrelations between the components or events, and they are used for quantitative and qualitative analysis of the system. The large Logical Trees it is a NP-Hard problem that requires to employ specific methods. A new approach to estimate the complexity of the problem is presented. Then, the Binary Decision Diagrams (BDD) are used for solving large Logical Trees.

BDD are used for operating with Boolean algebra. The ordination of the different events of a BDD is essential to guarantee the efficiency. A new method for ordering the events is proposed in this research work and it is compared with the main methods.

The information obtained is useful for System Reliability analysis and Decision Making processes. Figure 1.1 shows a schematic resume of the relation between them and the Operation and Maintenance tasks.

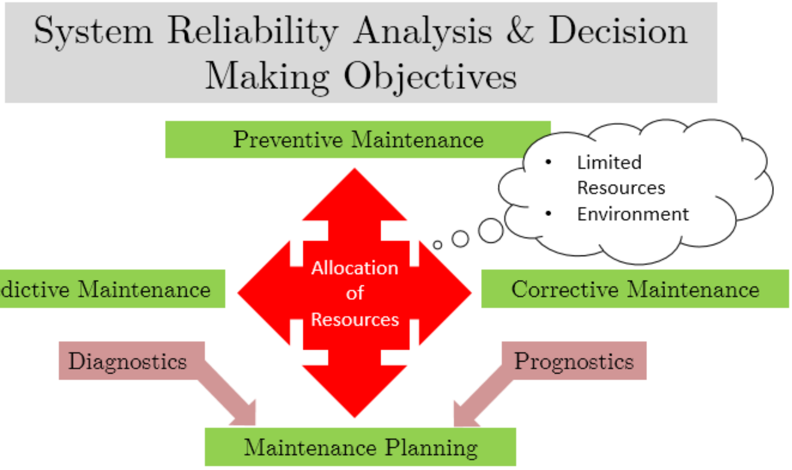


Figure 1.1 System Reliability & Decision Making

In recent years, wind energy is constantly increasing, being the most developed renewable energy. It is due to the new wind turbines, being more complexes and larges. They are considered as real case studies into the European Research Projects NIMO and OPTIMUS. A new technique is considered for carrying out a reliability analysis and a maintenance planning for offshore / onshore wind farms.

The point machines are also considered as a real case study. This mechanism is one of the most critical element in the railway super-infrastructures. A complex analysis of these system has been carried out in order to make prognostics, diagnostics and preventive maintenance planning.

Finally, a novel methodology for Decision Making has been proposed. The main objective is to perform an optimal allocation of resources. Two new methods are presented in this issue: The first one employs the results of the BDD for approaching a nonlinear programming problem; the second one uses a modified version of a known importance measure for establishing an adequate investment.

2 BIG DATA

Nowadays, the amount of data generated by all sectors of the economy and the society is growing exponentially. The information and communication technologies and the automation of the industrial processes are currently some of the most important generators of data. For instance, the internet has become the biggest producer of data in the entire history of humanity.

The comprehension of data is an activity that has always accompanied to the human beings. However, in this “Information Age”, the capacity and the necessity of acquiring, producing and generating data have reached unimaginable dimensions. As a result, the conventional data processing methodologies have become obsoletes. This is the reason why the new concept of “Big Data” is emerging. Big Data can be considered as a modern socio-technical phenomenon [2] that appears as a consequence of the current massive data generation. Some well-known companies that employ it to obtain useful information are Google, eBay, Amazon, Facebook, Twitter, IBM, LinkedIn, AOL, etc. [3]. For example, it is estimated that Google processes more than 25 petabytes ($25 \cdot 10^{15}$ bytes) every day.

The information and communication technologies have grown up with no precedents, and all aspects of human life have been transformed under this new scenario. All industrial sectors have rapidly incorporated the new technologies, and some of them have become de facto standards like supervisory control and data acquisition (SCADA) systems or Condition Monitoring Systems (CMS). Large amounts of data started to be created, processed and saved, allowing an automatic control of complex industrial systems. In spite of this progress, there are some challenges not well addressed yet. Some of them are: the analysis of tons of data, as well as continuous data streams; the integration of data in different formats coming from different sources; making sense of data to support decision making; and getting results in short periods of time. These all are characteristics of a problem that should be addressed through a big data approach.

2.1 Definition

The Big Data has been defined in the industrial field by six dimensions that can be called "The 6 Vs". This term concerns the following dimensions [4], [5]: Volume (the amount of data), Velocity (the speed at which data is created), Variety (the different natures of the data), Veracity (the certainty of data meaning), Validity (accuracy of data) and Volatility (how long the data need to be stored).

Data volume is normally measured by the quantity of raw transactions, events or amount of history that creates the data volume. Typically, data analysis algorithms have used smaller data sets called training sets to create predictive models. Most of the times, the business use predictive insight that are severely gross since the data volume has purposely been reduced according to storage and computational processing constraints. By removing the data volume constraint and using larger data sets, it is possible to discover subtle patterns that can lead to targeted actionable decisions, or they can enable further analysis that increase the accuracy of the predictive models.

Data variety came into existence over the past couple of decades, when data has increasingly become unstructured as the sources of data have proliferated beyond operational applications. In industrial applications, such variety emerged from the proliferation of multiple types of sensors, which enable the tracking of multiple variables in almost every domain in the world. Most technical factors include sampling rate of data and their relative range of values.

Data velocity is about the speed at which data is created, accumulated, ingested, and processed. An increasing number of applications are required to process information in real-time or with near real-time responses. This may imply that data is processed on the fly, as it is ingested, to make real-time decisions, or schedule the appropriate tasks.

Data veracity is about the certainty of data meaning. This feature express whether data reflect properly the reality or not. It depends on the way in which data are collected. It is strongly linked to the credibility of sources. For example, the veracity of the data collected from sensors depends on the calibration of sensors. The data collected from surveys could be truthful if survey samples are large enough to provide a sufficient basis for analysis. In resume, the massive amounts of data collected for Big Data purposes can lead to statistical errors and misinterpretation of the collected information. Purity of the information is critical for value [5].

Data validity is about the accuracy of data. The validity of Big Data sources must be accurate if results are wanted to be used for decision making or any other reasonable purpose [6]

Data volatility is about how long the data need to be storage. Some difficulties could appear due to the storage capacity. If storage is limited, what and how long data is needed to be kept. With some Big Data sources, it could be necessary to gather the data for a quick analysis [6].

These dimensions will determine the type of Big Data that is being considered. The complexity of the Big Data analysis is further defined by the volume, the velocity, the variety and the volatility. The usefulness of the analysis is usually dependent on the validity and the veracity of the data.

Besides the communication systems, social networks and companies that operate online, there is a significant source of Big Data related to the digital sensors worldwide in industrial equipment, automobiles, electrical meters and shipping crates. These sensors are capable of evaluating locations, movements, voltages, vibrations, magnetic fields and countless variables of a certain system. This concerns not only the volume of data that is generated but also the variety of such data. Figure 2.1 shows a scheme with the 6 dimensions of Big Data.

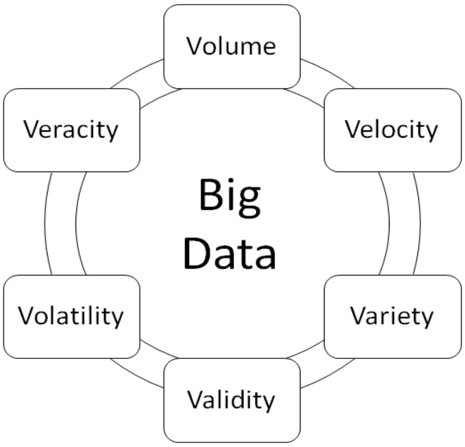


Figure 2.1 Dimensions of Big Data

This chapter presents some case studies based on the particular case of the Big Data generated by the equipment of the wind farms.

2.2 Case study: Big Data and wind turbines

Nowadays, wind energy is one of the most important renewable energy sources. The cumulative global capacity has risen from 3.5 GW in 1994 to more than 420 GW in 2016, and it is expected to be more than 1000 GW in the 2030s

[7]. The offshore wind energy presents the most important growth in the last years. In 2014, the cumulative global offshore capacity was 8.8 GW, being the European Union the world largest producer. More than 5 billion dollars were invested by the European Union in offshore windfarms in 2014 [8]. The onshore Operation and Maintenance tasks are 12% of the total system costs, being up to 23% for offshore [9].

The large number of data generated by the monitoring systems results in complex scenarios when they need to be treated. Data can come from different sources and their content can be completely random. Even so, the information can be correlated and their sorting can be useful for decision making. This situation is a common link in almost all industrial sectors where the incorporation of new technologies and the emergence of Condition Monitoring (CM) systems supported by Supervisory Control and Data Acquisition (SCADA) systems make the data processing a critical factor [10].

The field of the renewable energies is one of those sectors where the previous issue arises. The high volumes of data used in the operations and maintenance (O&M) tasks makes the introduction of Big Data a key factor. Wind farms usually divide the data analysis in three categories for decision making: descriptive analysis, post-event diagnostics and prognostics. The first category identifies the features with statistical calculations and graphics. The second category analyses the cause-effect of any change from a threshold. Finally, the prognostics predict the system changes [11].

Descriptive analysis is the basis of the following steps. Data collection must be as wide as possible to obtain a first approach. One of the first relationships that wind farms consider is the wind speed and power output connection. This is due to the fact that different wind farms can have wind turbines with similar specifications and their comparison can reveal the most efficient conditions.

The prognostic analysis is based on predictive modelling where several techniques such as regression trees or neural networks can be introduced to have an accurate model. Diverse inputs can be considered, e.g. speeds, electromagnetic data or vibration, to develop the model. The application of the techniques will entail the detection of degraded performances at earlier stages [12].

2.2.1 Condition monitoring approaches for wind turbines

Most of the wind turbines (WTs) are three-blade units [13], [14]. The energy generated by the blades is redirected from the main shaft to the generator through the gearbox. At the top of the tower, assembled on the foundation, the nacelle is found. A yaw system controls its alignment from the direction of the wind. The pitch system is mounted in each blade to position them depending on the wind. It also acts as an aerodynamic brake when needed. Finally, a meteorological unit provides information about the wind (speed and direction) to the control system.

CM is implemented from basic operations of the equipment to study [15]. The system provides the “condition”, the state of a characteristic parameter that represents the health of the component(s) being monitored. CM operates from different sensors and signal processing equipment in WTs. The main purpose is to monitor components ranging from blades, gearboxes, generators to bearings or towers.

CM reduces interferences during the features transport. Data processing, sorting and manipulation according to the objectives pursued, are usually performed by a digital signal processor. Then it can be shown, stored or transmitted to another system. One of the advantages for these systems is, therefore, that monitoring can be processed online or in certain time intervals. Thus, it is possible to maximise the productivity, to minimise downtimes, and to increase the Reliability, Availability, Maintainability and Safety (RAMS) levels [16].

Different techniques are available for CM:

- Vibration analysis [17].
- Acoustic emission [18].
- Ultrasonic testing techniques [19].
- Oil analysis [20].
- Thermography [21].
- Other methods.

The accurate data acquisition is critical to determine the occurrence of a failure and the subsequent solution. This can be achieved with the optimal type, number and placement of sensors. Data acquisition is always the first step of the CM process and includes the measurement of the required conditions (e.g. sound, vibration, voltage, temperature or speed), turning them into electronic signals. Then, signal processing introduces the handling (e.g. fast Fourier transform, wavelet transforms, hidden Markov models, statistical methods and trend analysis) and storage of data.

2.2.2 Supervisory control and data acquisition systems for wind turbines

SCADA systems are currently being introduced in WTs due to their effectiveness has been proved in other industries for detection and diagnostics of failures [22]. They are presented as an inexpensive and optimal solution to [23] control feedback for the health monitoring while reducing the O&M costs [24]. Nevertheless, they also present some minor disadvantages due to the operational or reliability conditions [23].

The SCADA system considers a large amount of measurements such as temperatures or wind and energy conversion parameters [25]. These data have raised considerable interest in different areas, e.g. wind power forecasting [26], production assessment [27] and of course, for fault detection [28].

In the case of the WTs, the introduction of SCADA systems verifies the efficiency when their components deteriorate. This degradation can indicate problems of different nature such as misalignments in the drive-train, friction caused by bearing or gear faults. The basic elements of the performance monitoring consist of a first collection of raw values by the sensors. After the application of the appropriate filters, anomalies are detected. Finally, a diagnosis will be provided. The anomaly detection includes a series of techniques that range from simple threshold checks to statistical analyses [29].

2.3 Data reduction techniques

As aforementioned, the wind farms are becoming a source of massive data. The purpose of these data is to describe the condition of the systems. However, the data are useless by themselves, they are only valuable when information can be gathered from them. It is necessary to process the data in order to extract useful information, but this is an arduous task when there is a very large amount of data. For this reason, it is essential to employ some techniques that allow for reducing the amount of data without losing the main information that they can provide. With this purpose, three procedures are proposed in the following sections. The first one uses the Pearson correlation coefficient to detect possible redundancies and false alarms. The second one is to analyse a continuous signal coming from a CMS by extracting feature parameters and, the last one provides a reduction for SCADA systems by filtering the unnecessary data.

2.3.1 Pearson correlation method

A correlation is a statistical procedure used to determine a relationship between two variables. The methodology proposed hereby employs the Pearson correlation coefficient in order to correlate the different variables that have been collected by the SCADA. The Pearson correlation coefficient (r) between a signal *x* and a signal *y* is the covariance of the two signals divided by the product of their standard deviations. This coefficient can be calculated by equation 2.1:

$$r = \frac{N\left(\sum_{n=1}^{N} xy\right)-\left(\sum_{n=1}^{N} x\right)\left(\sum_{n=1}^{N} y\right)}{\sqrt{\left(N\sum_{n=1}^{N} x^2-\left(\sum_{n=1}^{N} x\right)^2\right)\left(N\sum_{n=1}^{N} y^2-\left(\sum_{n=1}^{N} y\right)^2\right)}} \tag{2.1}$$

This coefficient can vary from -1 to 1 and represent the degree of correlation between the signals, being 1 a perfect positive correlation (if *x* increases, then y increases) and -1 a perfect negative correlation (if *x* increases, then y decreases). Figure 2.2 shows that there is an interrelationship in a) and c), however, the variables considered for b) are totally independent of each other. This measure will be used as an indicator of the goodness of data from a SCADA system and as a detector of abnormal behaviours of the WT.

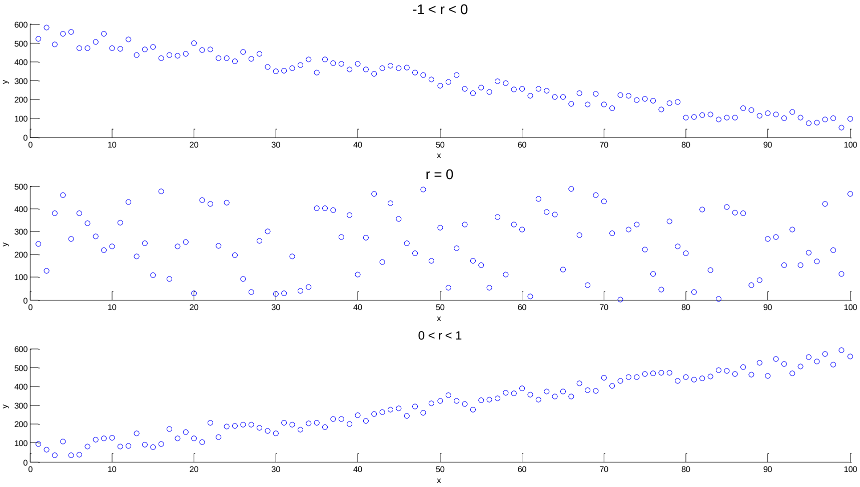


Figure 2.2 Possible correlation coefficients: a) negative correlation.
b) no correlation. c) positive correlation.

The methodology proposed in this paper has two main goals. The first one is to verify that the data processing does not generate false alarms and, therefore, the data can be used for planning a certain maintenance schedule on a wind turbine, e.g. repairs, replacements, resets, etc. The second purpose of this methodology is to detect abnormal behaviours based on the correlations of the variables and the redundancies. In summary, a statistical treatment of the data is done in order to ensure that the SACADA system is working properly and that the alarms generated are real.

The data of a SCADA system can be disposed as shown in Table 2.1

Table 2.1 Data structure from SCADA

	Date 1	Date 2	…	Date *j*	…	Date m-1	Date m
Variable1	$P_{1,1}$	$P_{1,2}$		$P_{1,j}$		$P_{1,m-1}$	$P_{1,m}$
Variable 2	$P_{2,1}$	$P_{2,2}$		$P_{2,j}$		$P_{2,m-1}$	$P_{2,m}$
…							
Variable *i*	$P_{i,1}$	$P_{i,2}$		$P_{i,j}$		$P_{i,m-1}$	$P_{i,m}$
…							
Variable *n-1*	$P_{n-1,1}$	$P_{n-1,2}$		$P_{n-1,j}$		$P_{n-1,m-1}$	$P_{n-1,m}$
Variablen	$P_{n,1}$	$P_{n,2}$		$P_{n,j}$		$P_{n,m-1}$	$P_{n,m}$

The first step of the method is to create a correlation matrix (**R**) in order to determine the correlations between the collected variables. **R** is a *nxn* matrix as shown in Table 2.2 shows.

Table 2.2 Correlation Matrix (R) for SCADA data

	Variable 1	Variable 2	…	Variable *i*	…	Variable n-1	Variable n
Variable 1	1	$r_{1,2}$		$r_{1,i}$		$r_{1,n-1}$	$r_{1,n}$
Variable 2	$r_{2,1}$	1	…	$r_{2,i}$	…	$r_{2,n-1}$	$r_{2,n}$
…	…	…	1	…	…	…	…
Variable *i*	$r_{i,1}$	$r_{i,2}$	…	1	…	$r_{i,n-1}$	$r_{i,n}$
…	…	…	…	…	1	…	…
Variable n-1	$r_{n-1,1}$	$r_{n-1,2}$	…	$r_{n-1,i}$	…	1	$r_{n-1,n}$
Variable n	$r_{n,1}$	$r_{n,2}$	…	$r_{n,i}$	…	$r_{n,n-1}$	1

The second step is to identify the most important correlations. The miscorrelation between some variables can determine either a certain operational failure of the WT or a failure in the data acquisition system [23].

2.3.2 Supervisory control and data acquisition systems for wind turbines

The CMSs installed in WTs are employed to evaluate variables such as vibration, lubrication oil or generator current signal. These systems usually provide a continuous monitoring of the variables. For this reason, it is important to develop algorithms capable of detecting possible abnormal behaviours of the variables over the time [30].

The main goal of this section is to perform a statistical study of the historical data of a CMS in order to achieve some feature parameters. These parameters facilitate to focus the analysis on the information that is really significant. Consequently, an important reduction of the amount of data is obtained. The feature parameters that will be used in this chapter are explained below ([31], [32], [33], [34], [35] and [36]):

- *Average:* the average can be useful for those signals without abrupt changes, i.e. signals that are almost constant. For example, it could be useful for humidity or temperature signals.
- *Peaks:* The more representative peaks are usually those that correspond to a maximum value of the signal within a certain time interval. These peaks can be referred to the time domain or to the different harmonics in the frequency domain. Other feature parameter related to the peaks is the peak to peak value that is defined as the distance between the maximum and the minimum amplitude of the signal.
- *Correlation coefficient* (r): See section 2.3.1.
- *Root Mean Square (RMS)*: This is a time analysis feature that corresponds to the measure of the signal power. It can be useful for detecting some out-of-balance in rotating systems. It can be calculated by equation 2.2:

$$RMS = \sqrt{\frac{\sum_{n=1}^{N}\left(y(n)\right)^2}{N}} \tag{2.2}$$

being N the total number of discrete values of the signal y. Other common parameter is the Delta RMS that is the difference between the current RMS and the previous value.

- *Standard Deviation*: This parameter is used to obtain the dispersion of a data set. It can be calculated by equation 2.3:

$$SD = \sqrt{\frac{\sum_{n=1}^{N}(y(n)-Mean)^2}{N-1}} \tag{2.3}$$

- *Skewness*: This parameter is an indicator of the signal symmetry. It is defined by equation 2.4:

$$Skewness = \sqrt{\frac{\sum_{n=1}^{N}(y(n)-Mean)^3}{(N-1)S^3}} \tag{2.4}$$

- *Kurtosis*: This parameter corresponds to the scaled fourth moment of the signal. It is a measure of how concentrated the data are around a central zone of the distribution. It is calculated by equation 2.5:

$$Kurtosis = \frac{\sum_{n=1}^{N}(y(n)-Mean)^4}{(N-1)S^4} \tag{2.5}$$

- *Crest Factor*: This parameter is capable of detecting abnormal behaviours in an early stage. It is defined by equation 2.6:

$$Crest\ Factor = \frac{Peak}{RMS} \tag{2.6}$$

- *Shape Indicator*: This factor is affected by the shape of the signal but it is independent of its dimensions. It is obtained by equation 2.7:

$$Shape\ Indicator = \frac{RMS}{\frac{1}{N}\sum_{n=1}^{N}|y(n)|} \tag{2.7}$$

- *Other parameters*: Other parameters are widely used such as enveloping, demodulation, FM0, NA4, FM4, M6A, M8A, NB4, sideband level factor, sideband index, zero-order figure of merit, impulse indicator, clearance factor etc.

These parameters can be only evaluated on finite signals. For this reason, it is necessary to choose some pieces of the continuous signal. The goal is to obtain the main features of the entire signal analysing only some pieces. Therefore, there are two factors why the data are reduced: firstly, a continuous signal is converted into several finite signals and, secondly some parameters of these finite signals are saved. Table 2.3 shows a general structure of the data using the method proposed.

Table 2.3 Association of data of the CMS and the condition of the WT

	Signal 1			Signal k			Signal M			WT Condition
	P1	Pj	PJ	P1	Pj	PJ	P1	Pj	PJ	
Date1	e_{11}^{1}	e_{1j}^{1}	e_{1J}^{1}	e_{11}^{k}	e_{1j}^{k}	e_{1J}^{k}	e_{11}^{M}	e_{1j}^{M}	e_{1J}^{M}	C_1
Date2	e_{21}^{1}	e_{2j}^{1}	e_{2J}^{1}	e_{21}^{k}	e_{2j}^{k}	e_{2J}^{k}	e_{21}^{M}	e_{2j}^{M}	e_{2J}^{M}	C_2
Date$i$	e_{i1}^{1}	e_{ij}^{1}	e_{iJ}^{1}	e_{i1}^{k}	e_{ij}^{k}	e_{iJ}^{k}	e_{i1}^{M}	e_{ij}^{M}	e_{iJ}^{M}	C_i

The element e_{ij}^{k} corresponds to the j parameter of the k piece collected at the time (date) i.

The main objective of this method is to determine the condition of the WT by making a comparison between the historic data and the data that is being receiving. With this purpose, the historical data will be subjected to a pattern recognition analysis to determine what features are significant. There are a lot of models for pattern recognition analysis, i.e. statistical model, structural model, template matching model, neural network based model, fuzzy based model, hybrid models, etc. [37], [38].

A neural network (NN) based model will be implemented to analyse the data in this chapter. The NN are complex structures based on the biological neurons.

These structures provide a good solution for those problems that cannot be analytically defined. Basically, the NN receives a dataset that is used into a training process to recognise the parameters. In this process, some weights are adapted to provide an adequate output. The different parameters of the signals will be considered as inputs, whereas the condition of the WT will correspond to the desired output of the NN. Further information about NN can be found in references [39] and [40]. A case study is developed in section 2.4.2 in order to clarify the procedure hereby explained.

2.3.3 Data analysis for the SCADA system

Besides the evaluation of the variables cited in the previous section, other signals can be collected to complete the data acquisition of a CMS, such as power, pressures, speeds and temperatures among others. With all these data, it is possible to track and analyse the set from the emergence of incipient failures. A SCADA system consisting of different processing tools that transform the data received into real-time analysable information is involved. The displays that comprise the system are configurable to obtain the information when and where it is needed (*see Figure 2.3*).

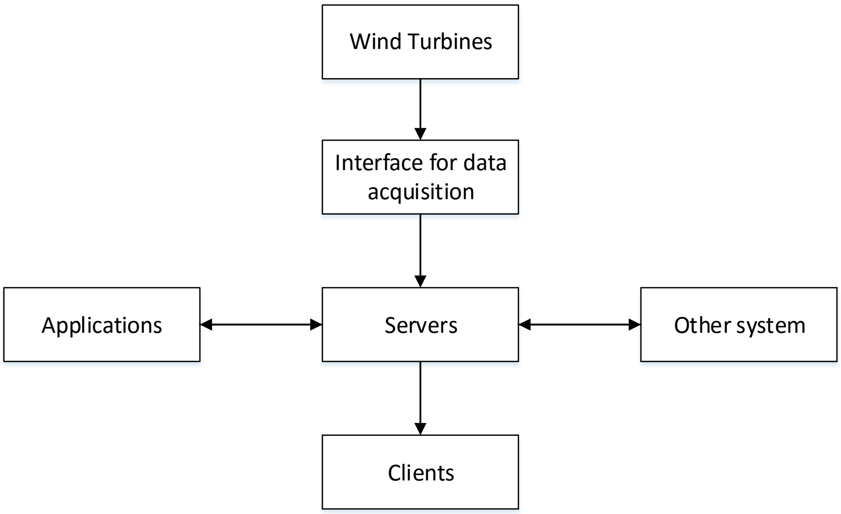


Figure 2.3 SCADA System

One of the main advantages of the SCADA system that will be presented for the cases studies is that allows almost infinite storage data in the original resolution. The software included can create and analyse process flow diagrams and graphics. The settings can be adapted to any operating system through menus and toolbars. In addition, the information can be exported to other formats, such as spreadsheets.

The second purpose in this research is to identify alarms from their location in a power curve. Likewise, it is interesting to know how many of those alarms go unnoticed by the system for being within the prediction bounds. The main problem associated to this task will be the definition of the curve. Due to the high number of data, a previous pre-processing will be done to remove non-significant data. This case could also be extended to other stored signals besides the wind speed and the power.

2.4 Case studies for Big Data

In the former section, three methodologies for processing the Big Data coming from WTs have been proposed and explained. All the methodologies are aimed to reduce the amount of data without losing the main information. This section presents two case studies in order to clarify these procedures.

2.4.1 Case study: Pearson correlation method

The SCADA system used for this case study has a tipical sampling rate of 10 min., i.e. each 10 minutes the system provides a set of data. A total of 37 variables have been taken into account for the analysis.

In order to classify the obtained correlations, four correlation levels have been set based on some literature [41] [42] and being very restrictive in order to ensure that the strong correlations are correct:

- Weak correlation $0.3 \le |r| < 0.5$

- Moderate correlation: $0.5 \leq |r| < 0.7$
- Strong correlation: $|r| \geq 0.7$
- Perfect correlation $|r| = 1$

ANNEX VII shows the correlations between the variables considered in this case study. There is a letter and a colour to determine if the correlation is weak (w, yellow), moderate (m, blue), strong (s, red) or perfect (p, black). For this case study, there is a total of 92 weak correlations; 95 moderate correlations; 46 strong correlations; and 3 perfect correlations. These numbers are calculated taking into account that the correlation matrix is symmetric, i.e, the correlation between the variables i and j are repeated twice, but it is only counted once.

In this case, the perfect correlations are referred to the pitch angle of each blade, a miscorrelation between these angles would indicate a failure. Some strong correlations can be observed between different temperature values or wind speed variables.

It is not possible to indicate what each variable is measuring for reasons of confidentiality. However, it is demonstrated that many of the variables are interrelated and these interrelationships can be used to identify abnormal behaviours of the WT or possible failures in the SCADA that can cause false alarms.

Depending on the variable, different conclusions can be extracted. For example, in a certain moment, the variable 27 reach an abnormal value and it is miscorrelated with variables 26, 29 and 30. However, the last three variables are maintaining a good correlation between them. Then, it would be a clear symptom of a false alarm. This kind of analysis can result very useful and it is facilitated by the methodology proposed.

2.4.2 Case study: CMS signals

A drive-train CMS is considered for this case study. This system provides a continuous vibration signal of 8 different points of the drive-train, attending to the point of the drive train that is being monitoring. The sampling rate of the CMS is 1000 samples/s. Therefore, a total of 8000 samples are received per second. The data have been collected during two years, therefore, more than $5 \cdot 10^{11}$ have been generated by this CMS along that period of time.

In order to apply the methodology explained in section 3.1, pieces of one second each three hours have been considered. Considering the sampling rate of the CMS, a total of 4.6$\cdot$ 10^7. As can be observed, this is the first reduction of the amount of data and it corresponds to a reduction of 99.99%. Therefore, the computational costs will be drastically reduced.

Once the set of pieces has been chosen, the following parameters are calculated attending to the definitions in section 3.1: RMS, average, standard deviation, maximum peak, kurtosis, crest factor, shape factor and impulse indicator. The evaluation of these parameters allows for a further reduction of the amount of data to analyse. Concretely, a total of 46720 data will be used to determine the patterns in the CMS data.

The different conditions of the WT are defined in an alarm report where the state of the WT is collected along the last two years. In this case study, the NN designed is able to differentiate between 4 possible states: “Alarm 1”, “Alarm 2”, “Alarm 3” or “No Alarms”. Each set of inputs is associated with a specific state of the WT and the relationships are established by the NN. Therefore, the purpose of the NN is to determine the state of the WT when a new set of data is available, i.e. to predict the condition of the WT attending to a new set of inputs. The following Figure 2.4 shows the NN designed for this case study.

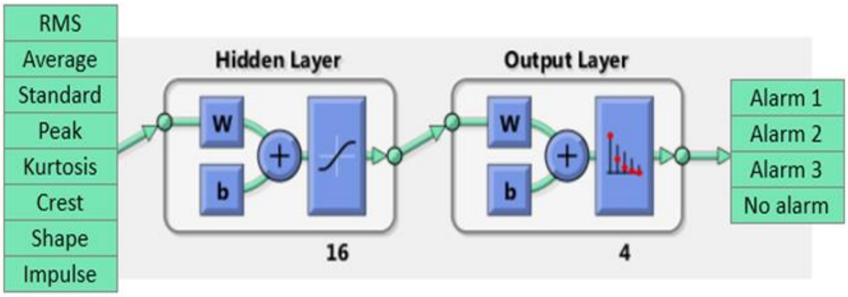
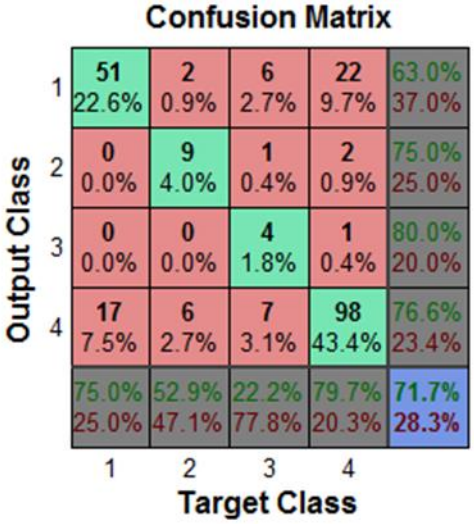


Figure 2.4 Neural network designed for the case study

The NN is formed by three layers. The input layer has 64 neurons that corresponds to the amount of inputs (8 signals by 8 parameters). The output layer is composed by 4 neurons according to the possible outputs considered for this case study. Finally, the hidden layer is composed by 16 neurons because the pyramid rule has been applied [43]. The pyramid rule suggests that the number of neuron of the hidden layer must be equal to the square root of the product between the number of input neurons and the number of output neurons.

Figure 2.5 shows the outcomes of the NN through a confusion matrix. The confusion matrix indicates the output provided by the NN (output class) compared with the real condition of the system (target class). The diagonal of the points those cases in which the outcomes of the NN are right (green cells). The values placed in the grey cells provide the percentages of successes and error for each type of output. The percentages in the fifth row provide information about how many conditions of each type the NN is detecting. However, the percentages in the fifth column express the degree of success when a certain state has been detected. Finally, the blue cell shows a summary of the results that determine the goodness of the NN.



Confusion Matrix

Output Class \ Target Class	1	2	3	4	
1	51 22.6%	2 0.9%	6 2.7%	22 9.7%	63.0% 37.0%
2	0 0.0%	9 4.0%	1 0.4%	2 0.9%	75.0% 25.0%
3	0 0.0%	0 0.0%	4 1.8%	1 0.4%	80.0% 20.0%
4	17 7.5%	6 2.7%	7 3.1%	98 43.4%	76.6% 23.4%
	75.0% 25.0%	52.9% 47.1%	22.2% 77.8%	79.7% 20.3%	71.7% 28.3%

Figure 2.5 Confusion Matrix. Results of the Neural Network

Figure 2.5 shows that the real condition of the WT can be successfully determined by using this method in 71.7% of cases. This is a very good result considering that only the 0.00001% of the total available data have been employed.

Once the patterns have been recognised by the NN, the new data from CMS can be pre-processed in order to achieve the mentioned parameters. These new data should be introduced in the NN and the output can provide information of the state of the WT. In this process the amount of data will be reduced from 8000 samples/s to only 64 samples/s. This technique can reduce the 99.2% of the data. Therefore, this method can result very useful to treat Big Data.

2.4.3 Case study: the SCADA systems using wind speed-power curves

This second case study will be focused on the information related to the wind speed and the power. Both features will be connected from the power curve. The power curve of a wind turbine indicates the electrical power that is available for these devices depending on the wind speed. It is usually close to zero for low speeds. Then, it quickly increases until reaching 10-15 m/s. From those speeds, the curve keeps constant as the result of the limitation devices attached to the turbine. This maximum power is often referred as the nominal power. Once speeds of 20-25 m/s are reached, the wind turbine operation is cancelled due to the activation of protection mechanisms. Therefore, power curves are often not represented at speeds exceeding these limits. In short, it can be said that the power curve is a useful indicator to evaluate the efficiency of a wind turbine.

Power curves are obtained from actual measurements on a wind turbine where an anemometer is strategically positioned. It must be located at certain distance from the rotor to avoid turbulences and therefore, to lose reliability for the stored speed. One of the main constraints of any wind-power curve is that, in practice, the speed fluctuates; so it is important to work with mean values to represent the curve effectively. A non-proper designed curve may show errors of up to 10% between the wind-power ratios.

Regarding the study, the SCADA system stores signals of wind speed and power every ten minutes, i.e. 52560 samples per year; and subdivides them into sampled, maximum, minimum and average collections, as well as the standard deviation. Once the data are extracted and converted into a readable format by software, it is reordered, from lowest to highest, to get the curve (see Figure 2.6). The first representation should fit to the theoretical model expected with minor exceptions (high wind speeds and power outputs).

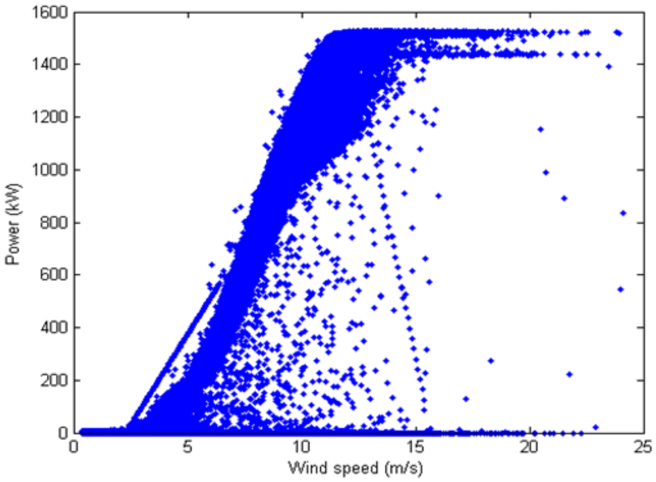


Figure 2.6 Initial scenario

Figure 2.7 (left) is the result of introducing Big Data in the case study. This task has been carried out with a curve fitting tool, doing a previous data selection where the appropriate samples are identified from statistical calculations. An exploratory data analysis is used to remove outliers (alarms in some cases) as well as redundant information. This way, it can be seen a reduction of the initial 52560 to an 841 samples, representing a decrease of the processed data up to 80% of the total amount (Table 2.4). Figure 2.7 also represents the data resulting from the descriptive analysis (left) versus the 904 samples indicating the occurrence of an alarm (right). It can be noted that the sum of both graphics still gives an accurate insight to the data registered by the sensors.

Table 2.4 Descriptive analysis

	Initial data	Data after the exploratory data analysis	Alarms
Samples	52560	841	904
Percentage (%)	100	1,6	1,72

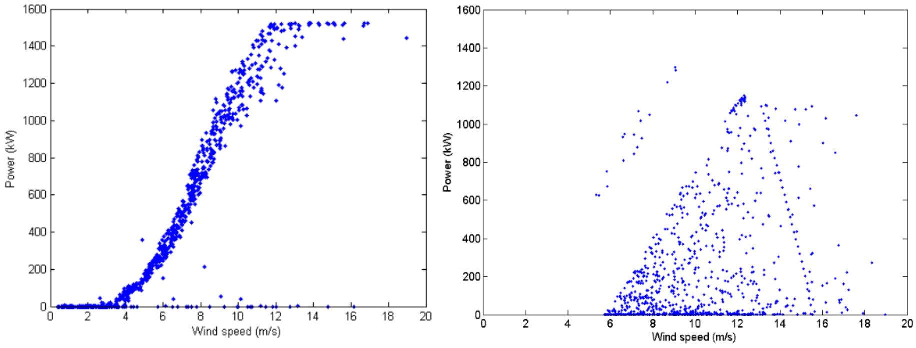


Figure 2.7 Post-processed curve (left) vs. alarms (right)

A second regression analysis is conducted to finally obtain Figure 2.8. Once the curve that best describes data series is selected, a post processing analysis can be performed. This enables the creation of a graphic with prediction bounds and the calculation of the 95% confidence intervals for the coefficient estimates.

The prior step is critical for the development of further analysis where alarms and operating states are linked to the power curve. The importance of this research is that some of the considered alarms have been found when the drive-train was monitored. The idea, still in development, is to create a pattern recognition where alarms can be identified from their location. Something similar could happen with the information that is not detectable for being within the prediction bounds.

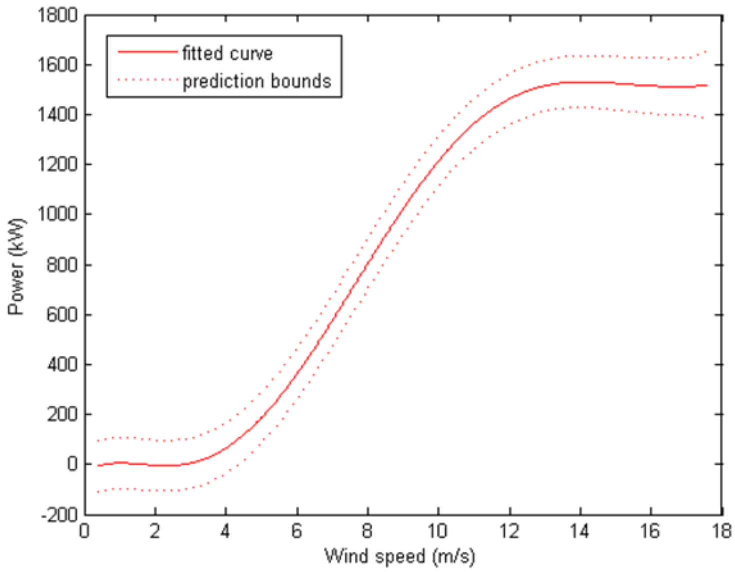


Figure 2.8 Wind-power curve

Through a first approach, some unusual performances have been found such as data being positioned above the power curve. This behaviour corresponds to alarms where currents and temperatures are involved and it results in an uncommon speed-power ratio. However, this situation occurs in the 2% of the cases studied. The general trend is to locate the failures up to 500 kW and from 8 m/s to 15 m/s, but usually below the curve. In quantitative terms, this can be translated into up to the 58% of the failures detected in terms of wind speed, and up to the 35% in terms of power. Moreover, it should be mentioned that approximately the 53% of the failures are within the prediction bounds and may go unnoticed if they are based on this technique.

3 LOGICAL TREE ANALYSIS

The Logical Tree Analysis (LTA) is a technique based on symbolic logic that is applied in the study of complex systems. It is a deductive method that considers a set of events that forms the structure function of a system.

The structure function is a logical function that defines the condition of the system. This condition is provided by the state of the events that compose the system $x = (x_1, x_2 \ldots x_n)$, where x_i are logical variables that represent the state of each event. In this thesis, the structure function is considered as a binary function, i.e. the events only can get one of the two possible states: occurrence and non-occurrence. The term *"event"* can represent a specific component (Chapter 5) of certain system or a cause of a problem (Chapter 6).

The Logical Trees (LT) will be the tool used for build these structure functions.

3.1 Logical Trees

LT is a graphical representation of a structure function. A LT structure consists of a root node (top event) that is broken down into various nodes located below it, where the nodes can be events, logical gates and branches. A specific set of symbols is used to represent each type of node (*see ANNEX III*), called LT system of symbols.

The following types of events can be identified in a LT:

- *Top event:* This is the event placed at the highest level of the LT. It represents the main cause, or the success that is pretended to be studied.
- *Basic events*: They cannot be broken down into more elementary events.
- *Intermediate events*: They can be broken down into more elementary events and are located under a logical gate.

Figure 3.1 shows a LT composed of seven non-basic events and nine basic events. Basic Events are those that are not possible to be broken down into simpler ones. All these events are linked by logical gates, in particular by one ‘OR’ gates and three ‘AND’ gates. This LT provides useful information about how a failure could be generated and what the most conflictive events are. For instance, Figure 3.1 shows a LT composed of seven non-basic events and nine basic events. It shows that the event ‘e_7’ is one of the most important events, due to it is directly related with the system failure. In other words, if e_7 occurs, the top event will occur.

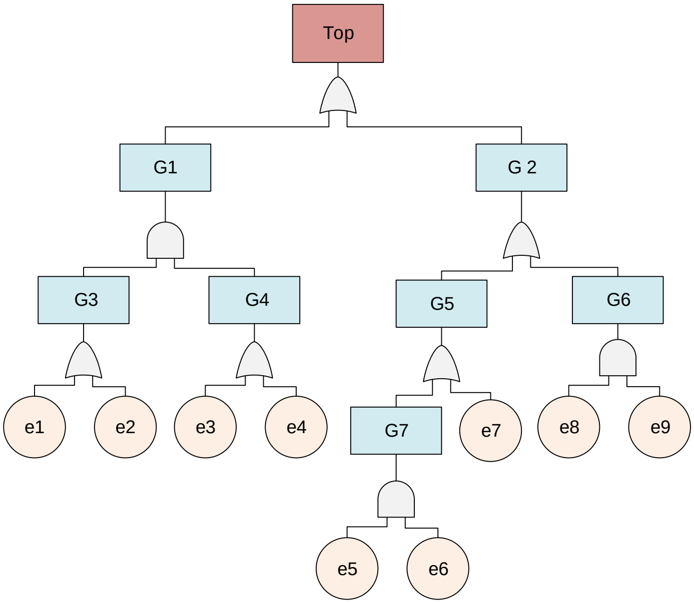


Figure 3.1 Example of LT for definition

LT s can be classified according to their size and complexity, the amount of logical gates, etc. The following classification is done in function of the number of events [44] :

- Small LTs (number of events < 100)
- Medium size LT s (100 < number of events < 1000)
- Large LTs (number of events > 1000)

The classification regarding to the logical gates is:

- *Static LTs*: They are only formed by conventional logical gates: AND, OR and VOTING OR (k-out-of-n).
- *Dynamic LTs*: They are composed by conventional gates and other logical gates with dynamic nature such as: Exclusive OR (XOR), Priority AND (PAND), Sequence Enforcing (SEQ) and SPARE gates.

A LT is coherent when a certain event and its negation don't appear simultaneously. According to the coherence, the LTs are classified as follow:

- *Coherent LTs*: A LT is coherent when a certain event and its negation do not appear simultaneously, i.e. a coherent LT cannot include any NOT gate. The study presented is focused on coherent LTs because otherwise the operations become very complex.
- *Non-Coherent LTs*: They include NOT gates and, therefore, a certain event and its negation could appear simultaneously. In a non-coherent LT. the top event can occur due to a certain event has not occurred.

A LT could be formed by multiple occurring events (MOEs). It is said that these LTs present redundancies. It is possible that a certain LT appears more than once in the LT, i.e. there are multiple occurring branches (MOB). All the events that belong to same MOB are MOE.

Very large and/or complex trees can be simplified into smaller and/or less complex ones called modules. A module only can have one output to the rest of the LT and cannot have any input from it. Two modules are considered s-independents when there is not any event that belongs to both of them.

A module is considered to be dynamic if:

- The top gate of the module is dynamic
- The top gate of the module is not dynamic but there are LT s that are not s-independents in this module.

It is important to remark that LTs will be used for three different purposes in this research work. The first purpose is to presents examples of the LT resolution, the second purpose is the failure analysis and the last one is the decision making analysis. A specific nomenclature will be used for each purpose.

- *Logical Tree examples.* In this case, the LT are employed in a generic manner. Each event will be called *'e_i'* and its probability of occurrence will be noted as *'q_i'*. The probability of the top event will be noted as $Q_{top.}$
- *Failure analysis*. When the objective of the analysis is to determine the occurrence of possible failures in a system, the LT used will be called Fault Tree. In this case the events that compose the Fault Tree will represent the failure of a determined component of the system. The failure probability assigned to each component will be noted as *'q_i'*. The probability of the Top event will be named as system failure probability and noted as *'Q_{sys}'*.
- *Decision Making Analysis*. In this case, the LT will be called "Logical Decision Tree". The events that compose the tree will represent different Basic Causes and the probability of the Top event will be called Main Problem Probability (Q_{MP}).

3.2 Stages of the Logical Tree Analysis

LTA process can be defined by different approaches according to the method set. Some references consider the analysis by emphasizing the interrelation between the different stages and different feedbacks, for example Stamatelatos [45] proposes the scheme given in Figure 3.2.

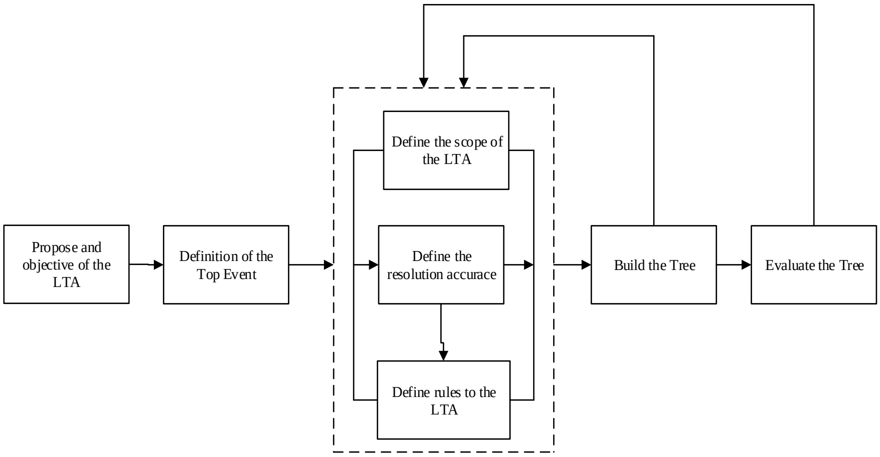


Figure 3.2 Stages and feedback of Logical Tree Analysis

However, the analysis proposed in this research work consists in the following steps [46]:

- *Definition of the system*: To understand and design the problem, employing flow charts, drawings, instrumentation, operating procedures, etc. Definition of the top event.
- *Construction of the tree*: To develop the LT taking into account the interrelationship between the different events.
- *Evaluation of the tree*: To collect the dataset, defining models to describe the behaviour of each component, employing revisions, interviews, etc. Qualitative and quantitative assessment of the LT, where the numerical

values of the variables are obtained in certain conditions. (The data used in this step can be obtained through the methods presented in chapter 2)

This procedure can be observed in Figure 3.3.

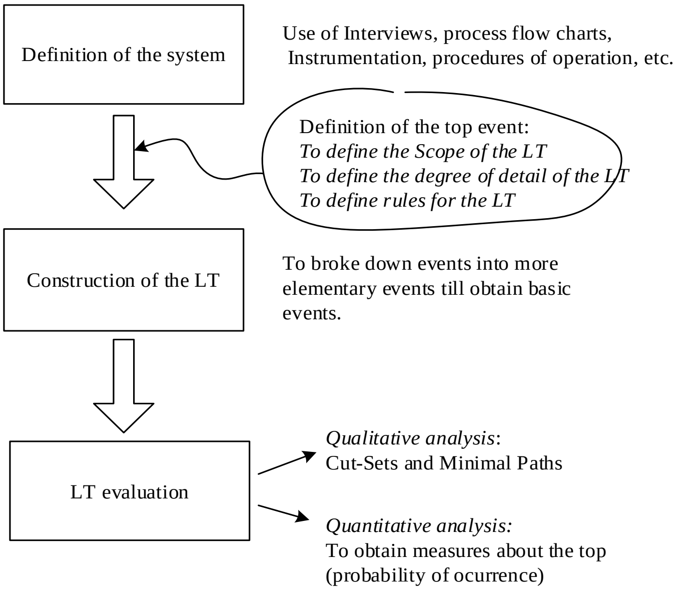


Figure 3.3 Stages of the LTA

3.2.1 Definition of the system

This stage is essential because it defines the objective of the analysis, which has a direct impact on the design and construction of the LT. It is collected the information of the system, e.g. a description of all system events. Each of these system components will be disaggregated until the required degree of detail. This system description can be done using:

- Flowcharts or block charts.
- Functional diagrams.
- Definitions of objectives and modes of operation.
- etc.

The flowcharts and functional diagrams require the information about the events and the relationships between them. It is important to set the boundary conditions of the system, e.g. the level of detail, internal and external conditions, etc. External conditions determine the scope of the analysis, and the level of detail set the accuracy of the analysis.

3.2.2 Construction of the LT

The construction method depends on the objective, the scope and the level of resolution. The following main steps can be taken into account for the construction:

- Identification of the objective of the LTA.
- Definition of the top event.
- Scope the LTA.
- Definition of the level of detail of the LTA.
- Defining rules for the LTA.

The top event can be set once the purpose of the analysis is identified. Once it is defined, the construction process can be then start. The top event can be delay in orders, a strategic decision, an investment, loss of landing gear of an airplane, etc.

The scope definition is used for choosing the events that will be included in the LT. The scope includes designs of the system, the time period and the boundary conditions for the analysis (initial states of the events and the system inputs), i.e. the LT A is done in a certain time and conditions.

The LT structure will be detailed according to the detail or accurate of the results, therefore it will define the number of the events, levels and logical gates. The construction of the LT is done from the top event to the events connected via logical gates. The events are broken down into more elementary events, etc., and finally into the basic events. The main construction methods can be classified into the following criteria:

- How the system is modeled?
- The algorithm to obtain the LT
- How the knowledge is used?

The construction process is an iterative process that can be divided into the following three stages:

- The “father” events (causes) must be identified for each event. They are necessary and sufficient events that cause the “son” event (effect).
- To categorize each causal event as primary or secondary. If there are different categories of causal events, it is habitual to use the rule of including an OR gate in the events from left to right in the order listed.
- To define the status of each event, e.g. the probability of the event.

The basic events at the same level under a logical gate must be independent between them. Other rules and conventions used in the LT construction are:

- When there are more than two branches for a gate, they must be set converging into a single input branch to the gate.
- An event must be located between two gates in order to avoid the direct connection of two logical gates.

The construction of a LT is a tedious task that can be very complex when a large and/or complex system is treated. However, there have been important advances in order to facilitate it, e.g. software that facilitates this work. Although the software for the LT construction is useful, it is necessary and unavoidable to make a final detail review of the LT by an expert.

3.2.3 Evaluation of the LT

The qualitative analysis provides information about the interrelationships between the occurrences of basic events and the top event. The LT shows a graphical representation of a structure function that can be expressed in different equivalent forms. For example, it is usually defined as any logic function that can be expressed in the first canonical form, also called normal dilemma form (NDF), or sum of disjoint products (SDP), that is a sum of products *("minterms")* of variables logic. Each of these terms represents a sufficient combination of basic events that causes the occurrence of the top event. The information gathered from a qualitative analysis is:

- Cut-sets (CS)
- Minimal paths.

A CS is a combination of basic events whose simultaneous occurrence causes the occurrence of the top event. A Minimal CS (MCS) is a "minimum" combination of basic events whose simultaneous occurrence causes the occurrence of the top event. A "minimum" combination is defined as a necessary and sufficient combination to cause the occurrence of the top event, i.e. a MCS would not be a CS if any of its basic events did not happen. For non-coherent structures, the MCSs are called prime implicants (PIs). In case of coherent systems, they are called MCSs.

A minimal path set is a "minimum" combination of events whose simultaneous operation (non-occurrence of events) ensures the system operation (no occurrence of the top event), i.e. if any of the events of a minimal path set occurs, then the top event occur.

The PIs (or MCSs) are unique and independent of the equivalent forms that a structure function can have. The structure function can be expressed as a sum of PIs (or MCSs). This is shown by equation 3.1:

$$\varphi(e_1, e_2, \dots, e_n) = \sum_{j=1}^{m} P_j \,, where\ P_j = \prod e_i \qquad (3.1)$$

In order to determine the PIs (or MCSs) from the LT, there are direct methods applied to the LT, and indirect methods in which equivalent graphs are used to study the structure function. The methods employed ever are based on previous identification of the CS. It is not possible to produce a complete list of all groups in large LT s. In order to address this problem, it is usual to choose only the CSs that contribute most significantly to the occurrence of the top event. The direct methods include:

- *Simulation methods*: random conditions are generated for the basic events. The structure function is evaluated for each condition vector.
- *Deterministic methods:* the structure function is transformed applying mathematical expressions to obtain a NDF form that consists on PIs or MCSs. The main algorithms to achieve this objective are: Mucus, factoring-division, addition & test logic, OR-EOR, truncation and modularization.

The main indirect method is the use of BDDs. They are data structures that represent the structure function and work directly with logical expressions rather than with CSs. A detailed description is presented in the Chapter 4.

The quantitative evaluation is normally used to obtain information about the top event. The main methodologies to carry out a quantitative analysis can be divided into three sections:

- *Methods based on the use of PIs*. A qualitative analysis is done to determine the CSs, and then it is used to perform probabilistic approaches.
- *Direct evaluation methods.* A previous qualitative evaluation is not required because they work directly with the structure function.

- *Methods based on BDDs.* In this research work, BDDs will be used as a tool for quantitative evaluation.

3.3 Importance analysis

The Importance Measures (IMs) are used to quantify the influence of a certain event in the behavior of a system. They provide an index of how that event contributes to the occurrence of the top event. IMs provide a quantitative index of the influence of each basic event on the occurrence of the top event. Generally, an event is more relevant in the occurrence of the top event when its IM is higher.

The main applications of the IMs are:

- In the design, research and development of a particular system, a product or a process: There are often limited resources to improve a system, i.e. it is essential to identify the more important events to allocate the resources in an optimal way. The IMs provide useful information to assign the resources because they show which events generate the greatest when they are improved.
- In the conversion from LTs to BDDs: A ranking of events generated using the structural importance will establish a proper ranking to obtain smaller BDDs.
- In predictive and/or preventive maintenance and inspections: A list of importance is useful for prioritizing inspections and planning checkups.

From a probabilistic point of view, the IMs can be divided into two categories:

- Deterministic. They determine the importance of an event without taking into account its probability of occurrence. These IMs are useful in the design phase when the information about the occurrence of the events is limited. Such measures depend on the position of the event within the LT structure.

- Probabilistic. They provide much more information about the system than the deterministic IMs. In this case, the importance of each event depends on its probability of occurrence and its position within the system.

It is very useful to get a list where events are ranked from highest to lowest importance measures. There are different criteria to generate this ranking, called “events ranking". Generally, these criteria or methods are used in the IM calculus. Each criterion usually provides a certain ranking.

The IM are applied to the LTs once it has been developed, where all the probabilities of occurrence are known, i.e. these IMs cannot be applied in the design and development stages. There are simple methods with a low computational cost that are based on the structure of the LT.

If the same ranking is obtained applying different methods, the obtained ranking is more reliable. However, it does not usually happen in practice. When different IMs provide different rankings, it is not clear which is the most appropriate IM. In these cases, an average of the most important measures, e.g. Fussell, Birnbaum and Criticality, can be considered, and the obtained ranking will be a consensus between the three measurements.

A set of examples of importance analysis is considered in ANNEX IV.

3.3.1 Fussell-Vesely

The Fussell-Vesely IM was introduced by the teachers J. Fussell and W.E. Vesely in the early seventies [47]. Sometimes it is called diagnostic importance factor (DIF). This IM is constructed using MCSs. The probability of the union of all the MCSs is the probability of occurrence of the top event.

An important measure associated with each basic event is the probability of union of all the MCSs that contain the event. This is probability that the top event occurs due to the occurrence of any CS that contains the event. This IM does not measure the probability of occurrence of the top event due to a basic event; however, it is a good indicator of the future importance of this such event.

The Fussell-Vesely IM is defined for coherent LTs as the quotient between the probability of the union of the CSs that contain an event and the probability of the top event. The Fussell-Vesely IM, I^{FV}, is given by equation 3.2.

$$I_i^{FV} = \frac{P(e_1^i \cup e_2^i \cup e_2^i \dots e_n^i)}{Q_{Top}} \tag{3.2}$$

where:

- $P(e_1^i \cup e_2^i \cup e_2^i \dots e_n^i)$is the probability of the union of all the CSs that contains the event *i*.
- Q_{Top} is the probability of the top event.

The numerator is usually approximated by the sum of the probabilities of all the MCSs that contain the basic event *i*.

3.3.2 Birnbaum

The Birnbaum IM was introduced by Professor Z.W. Birnbaum in 1969, during his research on the reliability of multicomponent systems [48].The Birnbaum IM is also called marginal importance factor.

The Birnbaum IM is defined as the probability that the system is in a critical condition respect to a certain event, i.e. the system varies from a state of non-occurrence to a state of occurrence due to the occurrence of the event [49]. Birnbaum developed this measurement, I^{Birn}, for coherent LT s, defined by equation 3.3. (Birnbaum IM for non-coherent trees can be found in reference [50])

$$I^{Birn} = \frac{\partial Q_{Top}}{\partial q_i} \tag{3.3}$$

Where

- Q_{Top}=Q(q1, q2,…, qn) is the occurrence function of the top event.
- q_i is the probability of occurrence of the event 'i'.

The first expression to be set is the probability of the top event(Q_{Top}) as a function of the probabilities of occurrence of the events, q_i. Then this expression is derived with respect to the corresponding variable. The procedure to find the expression of the probability of occurrence of the top event is as follows: To simplify the logical expression that defines the system using the Boolean algebra rules. This allows obtaining a logical expression given by the sum of the MCSs. The probability of occurrence of the top event is the probability of the union of these MCSs, calculated using the principle of inclusion-exclusion (see ANNEX II). Then, the principle of idempotent will be applied in those terms that include probability of intersections and MOEs, i.e. the terms that contain factors xi n, being a linear function Q_{Top} for each q_i.

The Birnbaum IM can be expressed as a difference of conditional probabilities of occurrence. It is defined by equation 3.4.

$$I^{Birn} = P\left(\frac{Q_{Top}}{q_i}\right) - P\left(\frac{Q_{Top}}{\overline{q_i}}\right) \tag{3.4}$$

where

- Q_{Top}: Probability of the top event
- q_i: Probability of occurrence of the basic event i
- $\overline{q_i}$: Probability of non-occurrence of the basic event i

The derivative of the expression (3.4) can be written as shown by equation 3.5:

$$I^{Birn} = \frac{\partial Q_{Top}}{\partial q_i} = \frac{Q_{Top}(1_i,q) - Q_{Top}(0_i,q)}{1-0} \tag{3.5}$$

In other words, it can be expressed as shown in equation 3.6

$$I^{Birn} = Q_{Top}(1_i, q) - Q_{Top}(0_i, q) \tag{3.6}$$

Where

$$Q_{Top}(1_i, q) = Q_{Top}(q_1, q_2, \dots, q_{i-1}, 1, q_{i+1}, \dots, q_n) \tag{3.7}$$

$$Q_{Top}(0_i, q) = Q_{Top}(q_1, q_2, \dots, q_{i-1}, 0, q_{i+1}, \dots, q_n) \tag{3.8}$$

The following example shows the equivalence between the different definitions of the Birnbaum IM and explains how this measure can be calculated.

Example

A system is composed by two events A and B, they are linked by an OR gate, so that the logical function is f = A + B. The probabilities of occurrence for each event are $\mathrm{q_A} = 0.1$ and $\mathrm{q_B} = 0.2$

The probability of occurrence of the top event is provided by the union of both events:

$$Q_{Top} = P(A \cup B) = P(A) + P(B) - P(A \cap B) = q_A + q_B - q_A q_B$$

Using the equation (3.5):

$$I^{Birn} = \frac{\partial Q_{Top}}{\partial q_i} = 1 - q_B = 1 - 0.2 = 0.8$$

Considering the equation (3.6) and assuming that the event A has occurred, the probability of occurrence of the top event is 1 because *f=1+B=1*. Assuming that the event A has not occurred, the probability of occurrence of the top event is the probability of occurrence of B, because *f=0+B=B*.

$$P\left(\frac{Q_{Top}}{A}\right) = 1$$

$$P\left(\frac{Q_{Top}}{\bar{A}}\right) = q_B$$

$$I_A^{Birn} = P\left(\frac{Q_{Top}}{A}\right) - P\left(\frac{Q_{Top}}{\bar{A}}\right) = q_B$$

Employing the equations 3.7 and 3.8:

$$Q_{Top}(1_A, \boldsymbol{q}) = 1 + q_B - q_B = 1$$

$$Q_{Top}(0_A, \boldsymbol{q}) = 0 + q_B - 0 = q_B$$

$$I_A^{Birn} = Q_{Top}(1_A, \boldsymbol{q}) - Q_{Top}(0_A, \boldsymbol{q}) = 1 - q_B = 1 - 0.2 = 0.8$$

3.3.3 Criticality

A disadvantage of the Birnbaum IM is that it does not consider the probability of occurrence of the analysed event. This can lead to assign a high importance to rare events (Rare events are those events whose probability of occurrence is very low). The Criticality IM [51] can be used to correct it. This measure modulates the Birnbaum IM using a factor to measure the weight of the event in the system that considers the occurrence probability of the event. The definition of the Criticality IM for a certain event *i* is expressed by equation 3.9 [52].

$$I_i^{Crit} = \left(\frac{q_i}{Q_{Top}}\right) \cdot \left(\frac{\partial Q_{Top}}{\partial q_i}\right) = \left(\frac{q_i}{Q_{Top}}\right) \cdot I_i^{Birn} \tag{3.9}$$

3.3.4 Structural importance

Starting from the definition of the Birnbaum IM given by equation 3.6 and following the Lambert’s method [51], the Structural IM is defined by equation 3.10:

$$I_i^{Struct} = Q_{Top}(1_i, 1/2) - Q_{Top}(0_i, 1/2) \tag{3.10}$$

where

$$Q_{Top}(1_i, 1/2) = Q_{Top}(q_1 = \frac{1}{2}, \dots, q_{i-1} = \frac{1}{2}, q_i = 1, q_{i+1} = \frac{1}{2}, \dots, q_n = 1/2)$$

$$Q_{Top}(0_i, 1/2) = Q_{Top}(q_1 = \frac{1}{2}, \dots, q_{i-1} = \frac{1}{2}, q_i = 0, q_{i+1} = \frac{1}{2}, \dots, q_n = 1/2)$$

3.3.5 AND method

This method establishes the importance of each basic event by counting the number of AND gates on the path from the event to the top event [53]. It is applicable to LTs with events whose probabilities are unknown. The advantage of using this approach is that the probabilities of occurrence of the events are not necessarily known.

This approach considers that the events under AND gates are less "important" than the events under OR gates, due to a state 1 at the output of an AND gate requires all the input events to occur. Assuming double-input gates, each event needs at least other event at the same level to occur. According to this method the events with a lower number of AND gates have more priority, i.e. more importance.

Events will be ordered according to the number of AND gates in the path to the top event that can be set by the following steps:

- To count the number of AND gates in the path to the top event for each basic event.
- To classify the events into categories according to the number of AND gates to the top event.
- Events in the same category will be decreasingly ordered according to their probability of occurrence.
- The events that are closer to the top are more important if there are events with the same probabilities of occurrence being in the same category.

- The same importance will be assigned to the events that are in the same category and are linked by an OR gate.
- If there is any repeated event located in different places of the LT, the more restrictive one will be considered to determine its importance.

3.3.6 Sensitivity analysis and uncertainty analysis

The sensitivity analysis aims to measure changes in the system caused by variations in the probabilities of the events. The effects on the top event are evaluated when the probabilities of the basic events change.

A sensitivity analysis can answer the following questions:

- What are the weaknesses of the system?
- How do variations in the input parameters affect to the results?
- What event is better to invest in to improve the decision making?

The system is considered to be sensitive to a particular event when a variation in the probability of that event leads the system to vary considerably.

An uncertainty analysis is done to analyse the variation in the probability of occurrence of the top event due to the dispersion on the data of certain basic events. Some statistics and probabilistic techniques can be used to perform this type of analysis [54].

3.3.7 Other importance measures

The Barlow-Proschan IM [55] provides the number of cases that the top event occurs by the occurrence of the event *i* in the period [0, t]. It is given by the equation 3.11.

$$I_i^{BP} = \int_0^t [Q_{Top}(1_i, \boldsymbol{q}) - Q_{Top}(o_i, \boldsymbol{q})] \cdot w_i(t) dt \tag{3.11}$$

The Risk Achievement Worth (RAW) is an IM that provides a measure of the degradation of the system assuming the occurrence of a certain event. It is defined by equation 3.12:

$$RAW_i = \frac{P\left(\frac{Q_{Top}}{x_i}\right)}{Q_{Top}} \tag{3.12}$$

It is calculated (equation 3.13) normally dividing the sum of the probabilities of the MCSs containing the basic event *i*, assuming a probability of 1 (occurrence) for this even, and the total probability of the top event:

$$RAW_i = \frac{\sum_{j=1}^{m} P(e_{j,q_i=1}^{i})}{P(\bigcup_{k=1}^{n} e_k)} \tag{3.13}$$

The Risk Reduction Worth (RRW) measures how much the probability of the top event decreases when the event *i* does not occur [56]. It identifies the basic events that cause the greatest reductions in the probability of the top event when their probabilities of occurrence decrease. It is calculated by equation 3.14.

$$RRW_i = \frac{Q_{Top}}{P\left(\frac{Q_{Top}}{x_i}\right)} \tag{3.14}$$

It is also calculated (equation 3.15) dividing the probability of the top event and the sum of the probabilities of all the MCSs, assuming that the probability of occurrence of the event i is 0.

$$RAW_i = \frac{P(\bigcup_{k=1}^{n} e_k)}{\sum_{j=1}^{m} P(e_{j,q_i=0}^{i})} \tag{3.15}$$

3.4 Common cause occurrences

The logical model is built considering that the basic events that are located under the same gate are considered independent. This is a limitation because possible implicit dependencies between them may not be considered.

In case of two independent basic events, the probability of the intersection is given by equation 3.16:

$$P(A \cap B) = P(A) \cdot P(B) \tag{3.16}$$

However, if both events are dependent, it is given by equation 3.17:

$$P(A \cap B) > P(A) \cdot P(B) \tag{3.17}$$

Therefore, the contribution to the occurrence of the top event will be greater when there are some dependencies, and they must be studied to obtain a representative logic model of the system.

The common cause occurrences (CCOs) happen when two or more events occur simultaneously (or in a relatively short period of time) due to a "common cause" [57].

The CCOs are not included in the initial model of LT because they are usually caused by dependencies that are difficult to identify a priori. In this case previous qualitative analysis of the initial LT is required to identify them.

According to Mosleh (1998) the dependencies can be classified into [58]:

- Endogenous to the system. The occurrence of a certain event is affected by the occurrence of other event. In this type of dependencies there are several subclasses:
    - *Functional requirement dependence*. The condition of a specific event determines whether the occurrence of an event B is required. There are four cases: the occurrence (or non-occurrence) of B is required when the event A occurs, and the occurrence (or non-occurrence) of B is required when the event A does not occur.

    - *Functional occurrence between events.* The occurrence of a certain event A may cause the occurrence of an event B.
    - *Cascade of occurrences.* The occurrence of one event may cause the occurrence of several events.
- Other. There are other endogenous dependencies as a result of combinations of the above dependencies.
- Exogenous to the system. They are considered as non-functional features of the system, being:
    - *Physical & environmental dependencies.* They can be: external environmental conditions (fire, storm, flood, earthquake, etc.); extreme physical operating conditions (pressure, temperature, humidity, vibration, etc.). It includes conditions caused by the occurrence of an event that creates an abnormal environment.
    - *Dependencies due to human factors.*

The CCOs are often used to model systems redundancies because their probabilities increase when there are multiple events with the same specifications. The redundancies are defined as a set of identical elements that can perform the same function. They can cause that the probability of occurrence of the top event decreases, e.g. the use of four engines on a plane. A simple example of a subsystem composed by three identical events (3 redundant events) is showed to demonstrate the importance of the CCOs, where the three events should occur to cause the occurrence of the subsystem top event. The logical model is presented in Figure 3.4.

Considering that the probabilities of occurrence of the events are as shown in equation 3.18:

$$q_1 = q_2 = q_3 = q = 10^{-3} \tag{3.18}$$

The probability that the 3 events occur independently is given by equation 3.19:

$$Q_{indep} = q_1 = q_2 = q_3 = q^3 = 10^{-9} \tag{3.19}$$

The probability that the 3 events occur at the same time is very low (one in a billion).

Taking into account the case where the possibility of a CCO for the three components, and considering that the 1% of occurrences are due to a common cause, the probability that the three components occur because of a CCO is defined by equation 3.20:

$$Q_{CCO} = 1\%(q) = 10^{-2} \cdot 10^{-3} = 10^{-5} \tag{3.20}$$

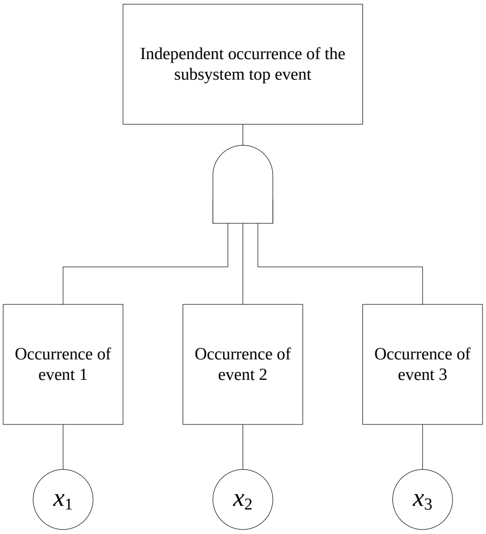


Figure 3.4 . LT of a redundant subsystem

Then the probability of occurrence due to CCO is 104 times bigger than the probability of occurrence of the three independent events. It is demonstrated that the CCOs have a great importance in the LT modelling. Figure 3.5 shows how to include CCOs.

The probability of occurrence of the top event including the CCO is given by equation 3.21:

$$Q_{Top} = Q_{indep} + Q_{CCO} - Q_{indep}Q_{CCO} = 10^{-5} \cdot 10^{-9} - 10^{-14} \cong 10^{-5} \quad (3.21)$$

Once the CCO has been identified, the logical models must be chosen. The factor β method is the easiest to model the CCOs. This method considers that the total rate of occurrence of an event can be decomposed into the following two rates as shown in equation 3.22:

$$\lambda = \lambda_{indep} + \lambda_{CCO} \quad (3.22)$$

The factor β is defined as the quotient between the rate of occurrence of an event that corresponds to a CCO and the global rate of occurrence.It is given by equation 3.23 .

$$\beta = \lambda_{CCO}/\lambda \quad (3.23)$$

Other methods used to model CCOs are: the alpha factor method, Multiple Greek Letters method, etc.

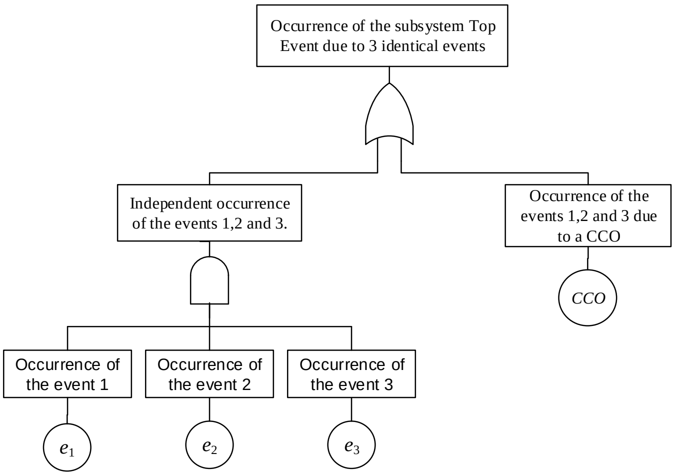


Figure 3.5 LT of a redundant subsystem including CCO

3.5 Limitations of conventional methods for LT analysis

The conventional methods present some drawbacks in the evaluation. These limitations are due to:

- *LT size*: The number of MCSs (PIs, for coherent structures) increases exponentially with the number of basic events, being an NP-complete problem. Therefore, the calculation of the probability of the top event is an exponential problem too. Poincaré's formula is used to achieve the probability of the union of the PIs and to address the calculation of the probability of the top event. However, this formula does not allow operate with large LTs. This computational problem requires approximations to simplify the problem. These approximations compute only those most relevant PIs.
- *Complexity of the LT*: The redundancies increase the computational cost because it is necessary to make simplifications of the logical expression. Idempotent and absorption rules are used to eliminate redundant terms. Large combinations of AND / OR gates difficult calculations and increase the computational cost
- *Computational limitations*
    - Processor speed
    - Memory size
    - Programming language
    - Etc.

3.6 Estimations for reducing the computational cost of LTA

The LTA is a NP-hard type problem and, therefore, for a large number of events, or a complex topology, it can be not recommended to find a solution. This chapter presents a novel approach for estimating the necessity of minimizing the computational cost. This approach is based on the logical gates, especially the

AND gates, the number and the position on the tree (level), and their effects to the solution and the computational cost of the system. The reference solutions, or experimental solutions, are obtained in simple systems, where it can be extrapolated to complex systems via polynomial regression functions. These functions are setting according to the reference solutions, where it will be more precise with more reference solutions.

Table 3.1 shows the probabilities and CSs for different LT cases studies in ANNEX XI. The probability of occurrence of the Top and the CSs are obtained for different amounts of AND gates in each level. The LT has been calculated for the cases marked in black. Red values correspond to estimated results. The estimations have been obtained through polynomial expressions, where the polynomial degree depends on the number of experimental points obtained. The experimental solutions have been obtained using the algorithms developed in reference [59].

Table 3.1 Experimental results and estimations

Probability of Occurrence of MP						Number of AND Gates	Number of Cut-Sets					
Level 1	Level 2	Level 3	Level 4	Level 5	Level 6		Level 6	Level 5	Level 4	Level 3	Level 2	Level 1
0,0756	0,291	0,3864	0,4313	0,4531	0,4638	1	63	64	72	112	288	1024
	0,0436	0,2836	0,3846	0,4308	0,453	2	63	72	144	672	4608	
		0,1637	0,3341	0,4077	0,4419	3	65	104	536	5840		
		0,027	0,2795	0,3837	0,4306	4	71	208	2528	15616		
			0,2205	0,3587	0,4191	5	85	528	7400			
			0,1578	0,3326	0,4074	6	115	1496	16432			
			0,0911	0,3036	0,3954	7	177	3673	30904			
			0,0204	0,2752	0,3832	8	303	7836	52095			
				0,2458	0,3727	9	527	14952				
				0,2154	0,3604	10	896	26177				
				0,184	0,3479	11	1463	42860				
				0,1516	0,3352	12	2294	66544				
				0,1182	0,3223	13	3462	98961				
				0,0838	0,3092	14	5048	142036				
				0,0484	0,2959	15	7144	197888				
				0,012	0,2824	16	9850	268825				
					0,2687	17	13276					
					0,2548	18	17540					
					0,2407	19	22770					
					0,2264	20	29103					
					0,2119	21	36684					
					0,1972	22	45669					
					0,1823	23	56221					
					0,1672	24	68513					
					0,1519	25	82729					
					0,1364	26	99058					
					0,1207	27	117701					
					0,1048	28	138869					
					0,0887	29	162778					
					0,0724	30	189658					
					0,0559	31	219744					
					0,0392	32	253283					

Figure 3.6 shows the results of probabilities found exactly (E) by BDD and the predicted (P) results found by new approach. It is observed that the probability is indirectly proportional to the number of AND gates, and proportional to the level, which is expected. Moreover, the consequences of adding a new AND gate is indirectly proportional to the level. In Figure 3.6 is also plotted (black curve) the absolute deviation expressed as abs((E-P)/P). The deviation is proportional to the number of gates, and with values always inferior to 0,45 %. It demonstrates that the accuracy of the solutions founds by the new approach is in every case very good.

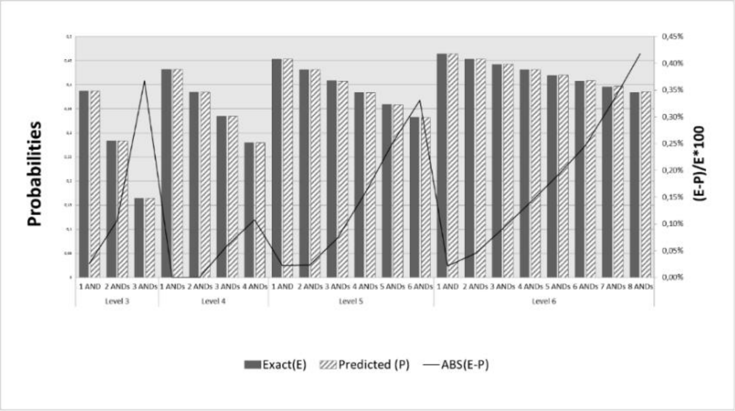


Figure 3.6 Probability Analysis

The deviation has been estimated for different levels and number of AND gates and presented in Figure 3.7.

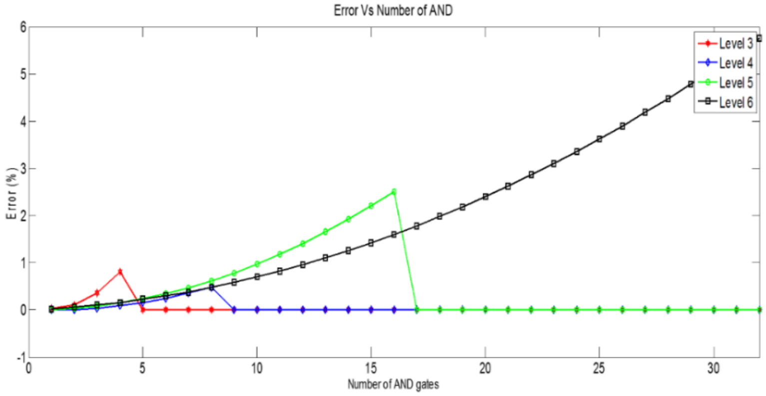


Figure 3.7 Deviation vs. Number of AND

It has been estimated through quadratic polynomial expression. It is useful in order to know approximately the accuracy of the probability estimated in Table 3.1.

A similar study presented in Figure 3.6 has been done taking into account the number of CSs, and showed in Figure 3.8.

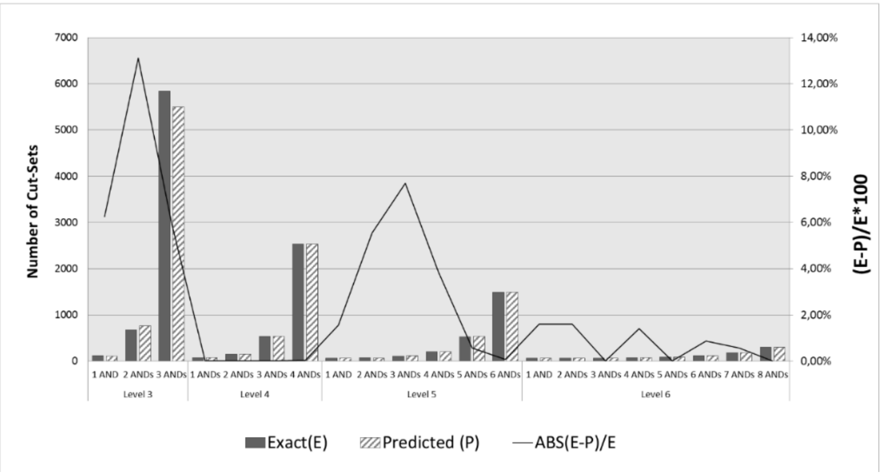


Figure 3.8 CS Analysis

The number of CSs is larger in each level when the number of AND gates increase, and the number of CSs is smaller when the level is larger taking into account the same number of AND gates. The error is not as relevant for CSs than for the probabilities, because it is the same independently of the number of CSs. It is relevant in order to estimate the computational cost for solving the problem. Exponential expressions have been used to evaluate the size of the CSs.

4 BINARY DECISION DIAGRAMS

The BDDs are presented as an indirect method for those LTs that cannot be directly solved because of their complexity. They are data structures used to represent Boolean functions. They were proposed in the 70s by Akers as models for decision making [60]. Later, they were popularized by Randal E. Bryant [61] who represented these structures in its canonical form. This form establishes a set of constrains and provides great advantages to operate with the BDDs.

An Ordered Binary Decision Diagram (OBDD) is a BDD where all the variables are ordered. All the paths from the root node to a terminal node find the variables in such order.

A Reduced Ordered Binary Decision Diagram (ROBDD) is an OBDD where certain principles have been applied to get a smaller BDD. It contains neither repetitive sub-LTs nor redundant vertex. All the variables must be ordered in advance and this order cannot be changed during the generation of these diagrams. A ROBDD is associated with a set of operative algorithms [62].

Nowadays, the BDDs are applied in many fields, e.g. they are used to represent LTs because they are very efficient to operate with Boolean expressions [63,64].

The main advantage of the BDDs is the possibility of evaluating the top event using implicit formulas. It avoids the exponential growth of the number of PIs.

The BDDs use less computational memory than the explicit representations for representing Boolean functions.

4.1 Definition

A BDD is a directed acyclic graph that simulates a logical function. It is a structure **G(V, N)** composed by vertices (**V**) that are connected by branches (**N**). Each vertex can be a terminal or a non-terminal vertex:

- *Non-terminal vertex*: It is a vertex associated with a basic event. A non-terminal vertex is followed by two branches: with value 1 if the event occurs, and 0 if the event does not occur. The nodes under a non-terminal vertex are called "sons" of that vertex. Therefore, each non-terminal vertex has two “sons”: the upper one and the lower one. The digital function associated with a non-terminal vertex is given by the Shannon’s theorem (equation 4.1):

  $$f_v(e_1, e_2, \dots, e_i, \dots e_n) = e_i f_{low(v)}(e_1, e_2, \dots, 1, \dots e_n) + \overline{e_i} f_{up(v)}(e_1, e_2, \dots, 0, \dots e_n) \tag{4.1}$$

  where

  e_i: Logical variables: Occurrence ($e_i = 1$) or non-occurrence (ei = 0).

  f_{low} *(v):* Lower “son” of the vertex **V** that corresponds to the branch if (case e_i =1).

  $f_{up}(v)$*:* Upper “son” of the vertex **V**, that corresponds to the branch else (case e_i =0).

  A special non-terminal vertex is the vertex located at the top of the graph. This is not a “son” of any other vertex. This vertex is called the “root vertex” of the BDD.
- *Terminal vertex*: It is a node that is located at the end of a path and is not followed by any branch, i.e. it does not have “sons”. The terminal vertices

only can be associated with values 1 and 0 that correspond to the possible condition of the system, where:

$f_v=1$, when $(V)=1$

$f_v=0$, when $(V)=0$

The non-terminal vertices usually have sons (at least one) that represent logical expressions. However, the two “sons” of every terminal vertex are directly logical values [1, 0].

All the paths of a BDD begin in the “root vertex” and end in “a terminal vertex”. The paths that end in a vertex with value 1 determine the CSs of the original LT. Figure 4.1 shows an example of BDD where two CSs can be identified.

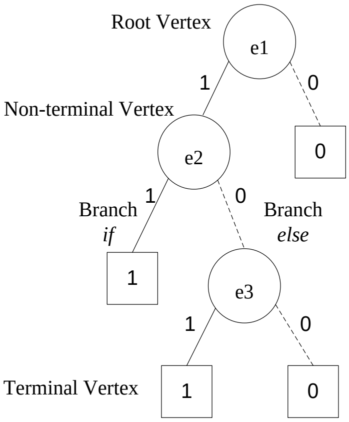


Figure 4.1 example of BDD

$CS_1: e_1e_2$

$CS_2: e_1\overline{e_2}e_3$

The structure function is determined as the sum of all the CSs:

$f = e_1e_2 + e_1\overline{e_2}e_3$

4.2 Conversion from LT to BDD. Construction of the BDD

The conversion from LT to BDD provides some advantages in terms of efficiency and accuracy for quantitative analysis. When the LT has a large amount of basic events, the direct analysis of the decision tree is often impossible. In these cases, it is necessary to use some truncation techniques and consequently, a loss of accuracy is produced [65]. BDDs provide an exact analytical expression of the occurrence probability of the main problem. Therefore, the main reason that leads to hereby study to convert LT into BDD is due to the ability to deal a large number of events using both computational time and resource management in a reasonably computational cost.

The conversion is achieved by applying some mathematical algorithms such as the Rauzy method (used in this paper) [66] or the simple component-connection method [67]. In this study, the ITE (If-Then-Else) conditional expression is one of the BDD's cornerstones [68],[66] *(see Figure 4.2)*:

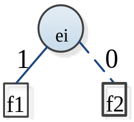


Figure 4.2 ITE applied to BDD

Figure 4.2 could be described as: "If e_i variable occurs, then f_1, else f_2". The solid line always belongs to the 1-branches and the dashed lines to the 0-branches. Taking into account Shannon's theorem [69], it can be obtained by equation 4.2:

$$f = e_i \cdot f_1 + \bar{e}_i \cdot f_2 = ite(e_i, f_1, f_2) \tag{4.2}$$

The left branch in Figure 4.2 is associated with the state 1 (occurrence of e_i), and the right branch is associated with a state 0 (non-occurrence of e_i). This notation will be used in the following figures.

A logical variable can be expressed by equation 4.3

$$e_i = ite(e_i, 1, 0) \tag{4.3}$$

It is necessary to establish a correct sequence of the events for the conversion from LT to BDD (further detailed information about the conversion and variable ordering methods can be found in [70], [71], [72]). Figure 4.3 shows this conversion considering that the following sequence for the events: e_1, e_2, e_3, e_4

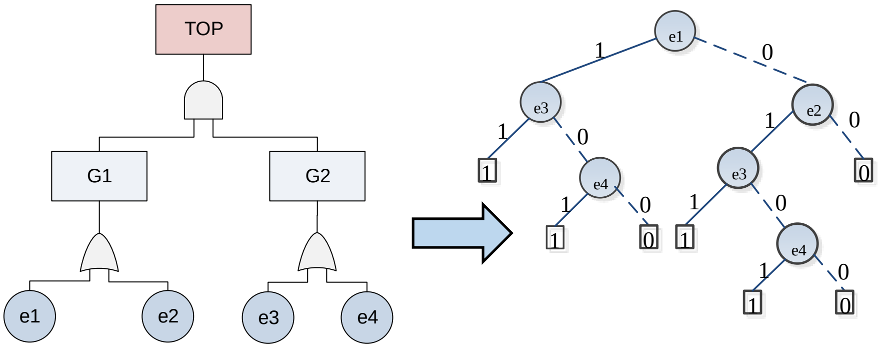


Figure 4.3 ITE applied to BDD

A set of operations and rules about the conversion from LT to BDD can be found in ANNEX V.

4.3 Ranking of events

Different BDDs can be generated from the same LT depending on the way to carry out the Shannon's expansions. The size of the resulting BDD will strongly depend on the order assigned to the basic events. An improper order of the variables can produce an exponential growth of the BDD size. The selection of the variable ordering is one of the most important problems in the use of BDDs, and many researchers have focused their efforts on search effective ranking methods.

4.3.1 Importance of ordering the events

An inefficient variable ranking usually produces a large BDD. Different rankings generate different BDD sizes. Figure 4.4 shows a LT that corresponds to the logical function: $Top = e_1 + e_2 + e_3e_4$. It will be analysed to demonstrate the importance of a right variable ordering.

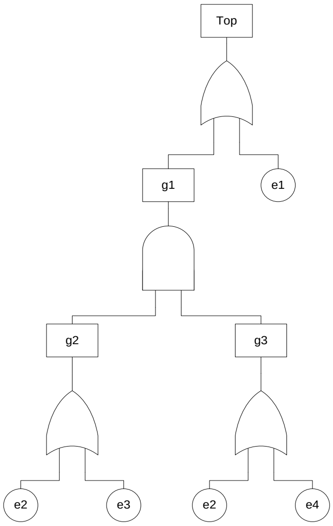


Figure 4.4 Example of a LT. Importance of order

Once a variable ranking has been set, e.g. $e_1 < e_2 < e_3 < e_4$, the conversion is carried out according to that ranking.

$$G_2 = ite(e_2, 1\ ,0) + ite(e_3, 1\ ,0) = ite\big(e_2, 1 + ite(e_3, 1\ ,0), 0 + ite(e_3, 1\ ,0)\big)$$
$$= ite(e_2, 1\ , ite(e_3, 1\ ,0))$$

$$G_3 = ite(e_2, 1\ ,0) + ite(e_4, 1\ ,0) = ite\big(e_2, 1 + ite(e_4, 1\ ,0), 0 + ite(e_4, 1\ ,0)\big)$$
$$= ite(e_2, 1\ , ite(e_4, 1\ ,0))$$

$$G_1 = ite\big(e_2, 1\ , ite(e_3, 1\ ,0)\big) \cdot ite\big(e_2, 1\ , ite(e_4, 1\ ,0)\big)$$
$$= ite\big(e_2, 1, ite(e_3, 1\ ,0) \cdot ite(e_4, 1\ ,0)\big)$$
$$= ite\left(e_2, 1\ , ite\big(e_3, ite(e_4, 1\ ,0), 0 \cdot ite(e_4, 1\ ,0)\big)\right)$$
$$= ite\big(e_2, 1\ , ite(e_3, e_4\ ,0)\big)$$

$$Top\ = ite(e_1, 1\ ,0) + ite\big(e_2, 1\ , ite(e_3, e_4\ ,0)\big)$$
$$= ite\big(e_1, 1 + ite(e_2, 1\ , ite(e_3, e_4, 0)), 0$$
$$+ ite(e_2, 1\ , ite(e_3, e_4, 0))\big) = ite\big(e_1, 1\ , ite(e_3, e_4, 0)\big)$$

If the logical function is simplified, it is proved that:

$$f(e_1, e_2, e_3, e_4) = (e_2 + e_3)(e_2 + e_4) + e_1 = e_2 + e_2 e_3 + e_2 e_4 + e_1$$
$$= e_2 + e_3 e_4 + e_1$$

Therefore, the MCSs are:

$CS_1\ = e_1$
$CS_2\ = e_2$
$CS_3\ = e_3\ e_4$

And employing expansions in the logical function according to the selected ranking, the BDD can be represented as shown in Figure 4.5:

$$f = ite\big(e_1, 1, (e_2 + e_3)(e_2 + e_4)\big) = ite\big(e_1, 1, ite(e_2, 1, e_3 e_4)\big)$$
$$= ite(e_1, 1, ite(e_2, 1, ite(e_3, e_4, 0\ )))$$

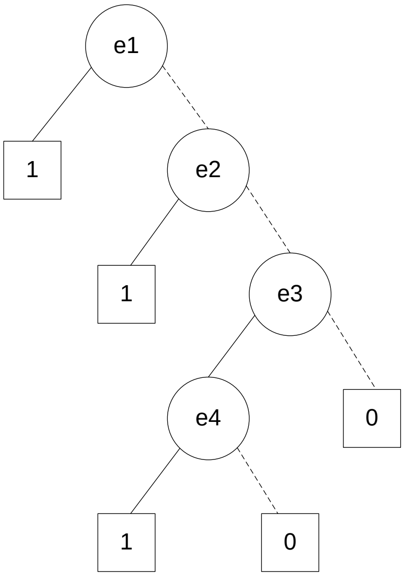


Figure 4.5 Resulting BBD for e1 < e2 < e3 < e4

A different variable ranking has been taken into account in the following example:

Ranking $e_4 < e_3 < e_2 < e_1$

The expressions obtained using the ITE formats are:

$$G_2 = ite(e_2, 1\,,0) + ite(e_3, 1\,,0) = ite\big(e_3, 1 + ite(e_2, 1\,,0), 0 + ite(e_2, 1\,,0)\big)$$
$$= ite(e_3, 1\,, ite(e_2, 1\,,0))$$

$$G_3 = ite(e_2, 1\,,0) + ite(e_4, 1\,,0) = ite\big(e_4, 1 + ite(e_2, 1\,,0), 0 + ite(e_2, 1\,,0)\big)$$
$$= ite(e_4, 1\,, ite(e_2, 1\,,0))$$

$$G_1 = ite(e_3, 1\,, ite(e_2, 1\,,0)) \cdot ite(e_4, 1\,, ite(e_2, 1\,,0))$$

$$= ite\,(e_4, ite(e_3, 1\; ite(e_2, 1, 0)), ite(e_2, 1,0) \cdot ite(e_3, 1\,, ite(e_2, 1,0)))$$

$$= ite\,(e_4, ite(e_3, 1\; ite(e_2, 1, 0)), ite(e_3, ite(e_2, 1,0), ite(e_2, 1,0)$$

$$\cdot\, ite(e_2, 1,0)))$$

$$= ite(e_4, ite(3_3, 1, ite(e_2, 1,0)), ite(e_3, ite(e_2, 1,0), ite(e_2, 1,0)))$$

$$Top$$

$$= ite(e_1, 1,0)$$

$$+\, ite\,(e_4, ite(e_3, 1, ite(e_2, 1, 0)), ite(e_3, ite(e_2, 1, 0), ite(e_2, 1, 0)))$$

$$= ite\,(e_4, ite(e_3, 1, ite(e_2, 1, 0))$$

$$+\, ite(e_1, 1, 0), ite\,(e_3, ite(e_2, 1, 0, ite(e_2, 1, 0)) + ite(e_1, 1, 0)))$$

$$= ite\,(e_4, ite(e_3, 1, ite(e_2, 1,0) + ite(e_1, 1,0)), ite(e_3, ite(e_2, 1,0)$$

$$+\, ite(e_1, 1,0), ite(e_2, 1,0)) + ite(e_1, 1,0))$$

$$= ite(e_4, ite\,(e_3, 1, ite(e_2, 1, ite(e_1, 1, 0))),$$

$$ite(e_3, ite(e_2, 1, ite(e_1, 1, 0)), ite(e_2, 1, ite(e_1, 1, 0))))$$

The BDD can be represented as follows:

$$f = ite(e_4, (e_2 + e_3) + e_1, (e_2 + e_3)e_2 + e_1)$$

$$= ite(e_4, ite(e_3, 1, e_2 + e_1), ite(e_3, e_2 + e_1, e_2 + e_1))$$

$$= ite(e_4, ite(e_3, 1, ite(e_2, 1, e_1)), ite(e_3, ite(e_2, 1, e_1), ite(e_2, 1, e_1)))$$

Figure 3.16 shows that the ranking $e_4 < e_3 < e_2 < e_1$is not efficient due to the BDD obtained is larger than the BDD presented in Figure 4.6. This ranking produces 7 non-terminal vertices, and the previous one only produces 4 non-terminal vertices. The BDD presented in Figure 4.6 has the minimum size that can be obtained for the logical function of the example.

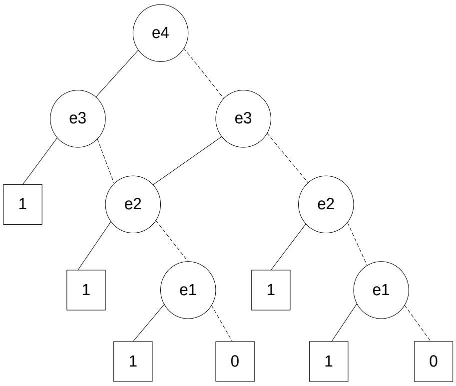


Figure 4.6 Resulting BBD for e4 < e3 < e2 < e1

4.3.2 Ranking methods

The problem of finding the optimal variable ranking is NP-complete and it cannot be solved on a reasonable time. Heuristic methods are widely used in order to find an efficient ranking. These methods do not provide an optimal solution but a good enough one. The main methods are described below:

- *Topological Heuristic Methods*: They are the simplest methods and the easiest to implement. No calculations are performed in these methods, but a procedure to read the LT is chosen and the variables are ranked in the order in which they are found.
    - *Top-Down Left-Right (TDLR):* The LT is read as the method itself says, i.e. from the top to bottom and from left to right. The ranking is generated according to the order in which the events are found [73]. An example is shown in Figure 4.7.

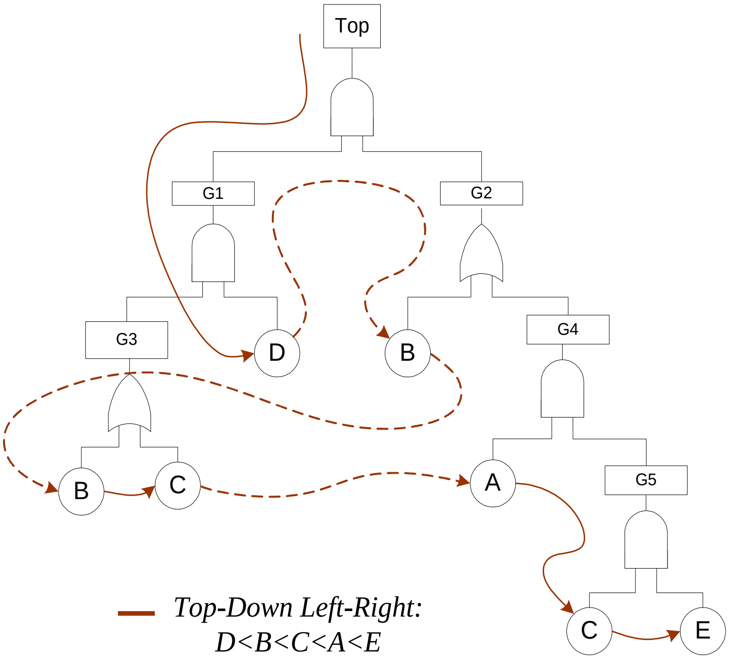


Figure 4.7 Top-Down Left-Right ranking method

- ○ *Depth First Search (DFS):* The LT is read from top to bottom and in each level the left LTs are read firstly. An example is shown in Figure 4.8.

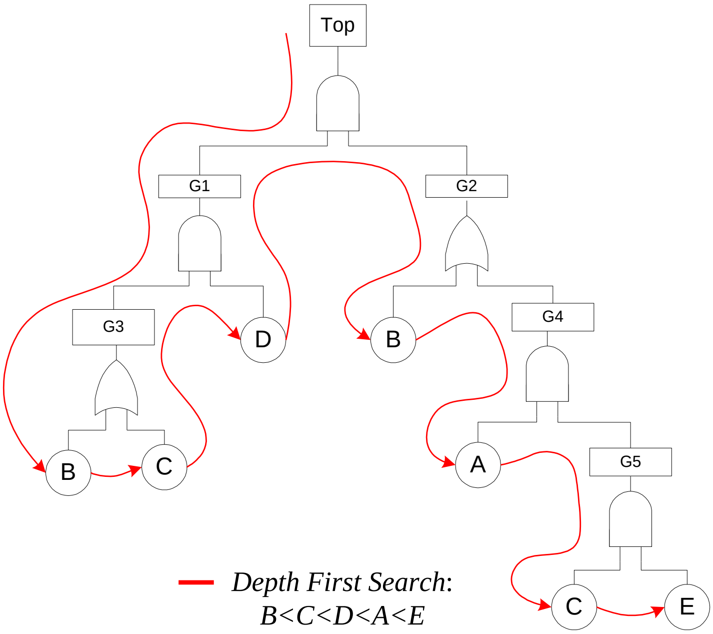


Figure 4.8 Depth First Search ranking method

  - *Breadth-First Search (BFS):* The LT is read from left to right and the events are ranked according to the order in which they are found. It must be stated that if a repeated event is found it must be ignored.

- *Weights method, or Minato's heuristic* [74]: Unitary weights are assigned to the non-terminal vertices and the weight of each "father" will be the sum of the weights of its "sons". Finally, the events are reordered in decreasing weights. There are some variants of this heuristic method such as Weight-Op that assigns different weight values in function of the logical gates [75]. An example is shown in ANNEX VI.
- *Method of flows* [66]: Flow values are assigned to each branch of the LT. The root node that is assigned a unitary flow and the flow values are propagated downward. The flow of a "son" depends on the sum of the

flows of the “fathers”. Finally, the events are reordered in decreasing weights.

- *Method of fathers* [76]: The total number of “fathers” is counted for each variable. The “sons” are ranked in descending order of number of “fathers”.
- *Level Method* [77]: This method is not so simple and direct. It makes a difference between the basic events depending on where they are located, i.e. it is directly related with the number of gates there are above them. The MOEs are listed first when there are events in the same level.
- *Heuristic method based on the structural importance* [73]: This ranking method is based on the Structural Birnbaum Measure. This measure can be achieved using the equation (3.6), where the probability of occurrence of the events is required. It is a difficult task when the LT is large. Bartlett (2001) proposes an alternative way to achieve an approximation of this measure [73].

1. Generate a list of events using TDLR method
2. Choose an event i and follow the next steps:

2.1. Consider the probability of the event i as 1 and the probabilities of the rest of events as 1/2.

2.2. Take the probability of the event i as 0 and the probabilities of the rest of events as 1/2.

The gates that only have basic events are chosen. The output probabilities of these gates are achieved:

a)If it is an AND gate: $\prod q_i$

b) If it is an OR gate: $\coprod q_i = 1 - \prod(1 - q_i)$

3. The probabilities obtained in steps 2.1 and 2.2 are obtained.

- *The AND method*, explained in section 3.3.5, can be also used as a ranking method [53]. Bartlett's method provides an appropriate importance analysis when there is much uncertainty in the input data. In general, topologic heuristics methods are not robust. Although they produce good results, sometimes they are completely unpredictable and produce inefficient rankings. These methods can become more robust by applying some restrictions (weights, flows, etc.).

4.3.3 Novel Ranking method proposed

A new ranking method has been defined by the authors of this book. It aims to reduce the size of the BDDs taking into account the following considerations:

- Each logical gate of the LT needs an appropriate weighting
- An importance value is assigned to each event evaluating the multiplication of the weighting of the gates from the event considered to the Top Event
- The basic events are sorted in decreasing values of importance
- The weighting of each logical gate will depend on its nature (OR or AND gates), and the number of events under the logical gate
- If there is "*n*" events through an AND logical gate, the failure could only be extended through the gate if all the "n" events occur, i.e. only 1 state of the 2^n possible states will cause

If there is "*n*" events through an AND logical gate, the occurrence of the "*father*" event of that gate could only occur if all the events are given, i.e. there is only one state of the 2^{n} possible states that will cause the occurrence of the "*father*" event. Therefore, the weight assigned to an AND logical gate will be given by equation 4.4:

$$P_{and}(n) = \frac{1}{2^n} \tag{4.4}$$

The "*father*" event of an OR gate will occur in all cases when any of the events is not cero, i.e. only one of the 2^{n} states will not cause the occurrence of the "*father*" event, and consequently, the occurrence of the top will not be caused by these events. The mentioned state is the one in which the state of all the events is 0. Therefore, the OR logic gate weighting is given by equation 4.5:

$$P_{or}(n) = \frac{2^n - 1}{2^n} \tag{4.5}$$

The new approach for ranking the events is summarized in the scheme given in Figure 4.9.

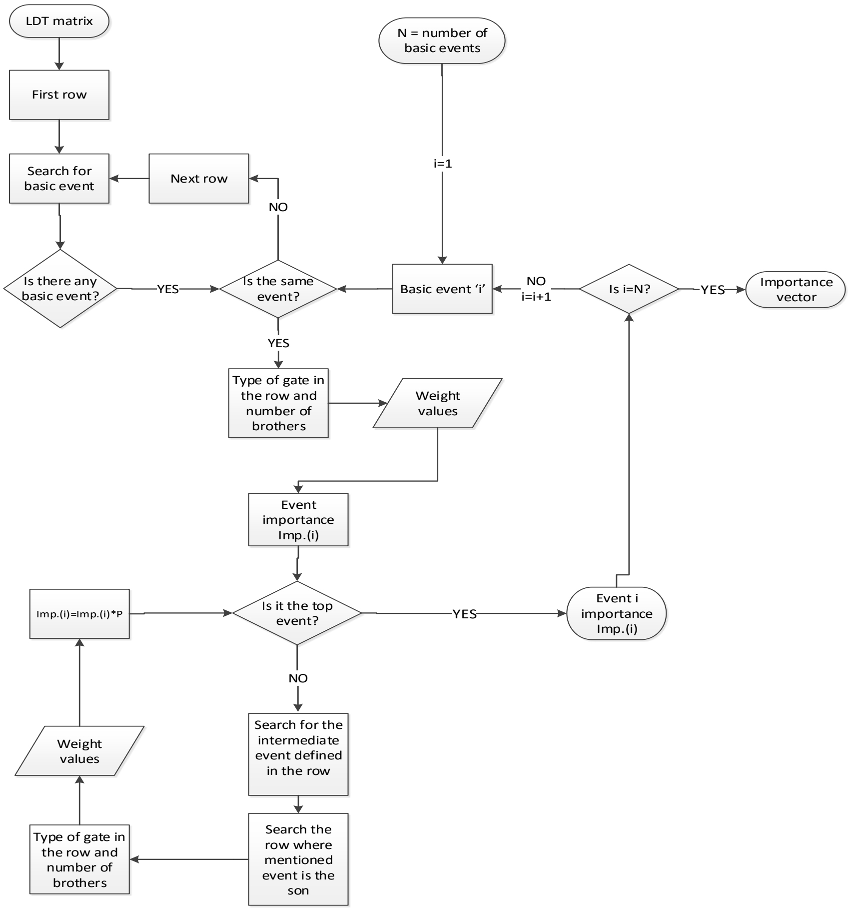


Figure 4.9 Scheme of the new approach for ranking events

In Figure 4.10 is presented a LT as an example for ranking the event employing the new approach (Figure 4.9).

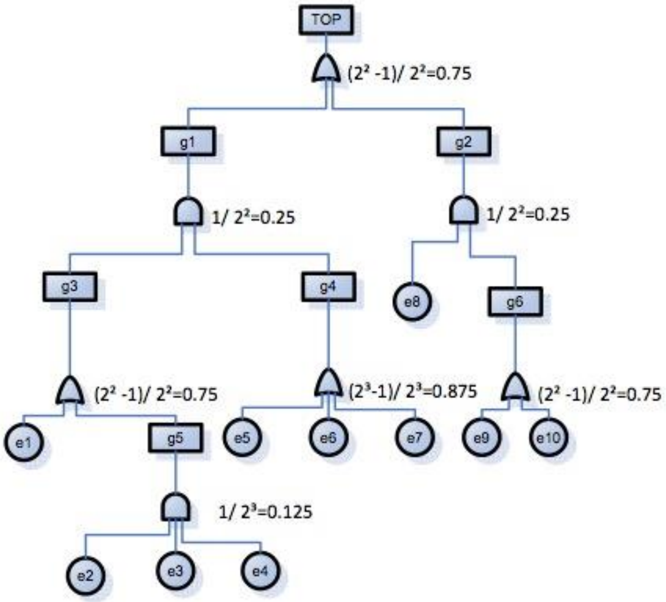


Figure 4.10 Weighting of the logic gates by the new ranking method

For each basic event, there is a single path to the top event. The importance of the event e_1 will be given by all the weights of the gates that are in the path from that event to the top event. For example, the importance for the events e_1 and e_2 (Figure 4.10) will be:

$$I_{e1} = 0{,}75 * 0{,}25 * 0{,}75 = 0{,}140625$$

$$I_{e2} = 0{,}125 * 0{,}75 * 0{,}25 * 0{,}75 = 0{,}01757813$$

The importance measurements of the basic events employing the new approach are given in Table 4.1, being the ranking: $\mathrm{e}_8 < \mathrm{e}_5 < \mathrm{e}_6 < \mathrm{e}_7 < \mathrm{e}_1 < \mathrm{e}_9 < \mathrm{e}_{10} < \mathrm{e}_2 < \mathrm{e}_3 < \mathrm{e}_4$, obtaining 20 cut-sets, where 22 cut-sets are obtained by employing the AND criterion with the ranking $\mathrm{e}_8 < \mathrm{e}_1 < \mathrm{e}_5 < \mathrm{e}_6 < \mathrm{e}_7 < \mathrm{e}_9 < \mathrm{e}_{10} < \mathrm{e}_2 < \mathrm{e}_3 < \mathrm{e}_4$. The main reason that the new approach provides better results than the AND criterion is because the importance of e_1 is the same to e_5, e_6 and e_7 according to the AND criterion. That means e_1 is more important due to its location in the FT.

Table 4.1 Importance of basic events

Event	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
Importance	0,14	0,01	0,01	0,01	0,16	0,16	0,16	0,18	0,14	0,14

The new method approach considers that e_5, e_6 and e_7 are connected by an OR logic gate, which means that the failure is more probable to happen through it, i.e. e_5, e_6 and e_7 are given more importance than e_1.

A set of LTs have been considered for evaluating the ranking events. The number of basic events, intermediate events, OR and AND gates and levels is defined for each LT in the following Table 4.2

Table 4.2 LTs characteristics

	Basic Events	**Intermediate Events**	**OR gates**	**AND gates**	**Levels**
FT 1	5	5	3	3	3
FT 2	15	13	10	4	8
FT 3	11	9	5	5	6
FT 4	25	21	16	6	12
FT 5	20	15	10	6	5
FT 6	12	7	5	3	4
FT 7	10	7	7	1	5
FT 8	20	17	12	6	11
FT 9	31	25	16	10	11

The methods aforementioned have been employed for ranking the events of the LTs showed in Table 4.2. The number of cut-sets obtained by using each method is given in Table 4.3.

Table 4.3 Cut-sets obtained by the ranking events

	TDLR	**DFS**	**BFS**	**Level**	**AND**	**Approach**
FT 1	2	2	2	2	2	2
FT 2	30	30	155	30	30	30
FT 3	12	24	36	12	12	12
FT 4	64	142	176	64	22	28
FT 5	99	207	257	99	55	55
FT 6	9	7	7	9	9	12
FT 7	9	12	21	9	9	9
FT 8	44	76	192	44	44	44
FT 9	1012	1292	3456	1012	1012	**924**

BFS provides poor results in most of the cases, especially when the LT has a large number of events, levels and “or” and “and” gates. The Level and AND methods generate the ranking of the events with a minimal cut-sets. The conclusions regarding to Level, DFS and TDLR methods should be studied for each LT.

The new approach proposed in this paper provides the minimal cut-sets in most of the cases, i.e. for LT 1-3, 5, 7-9, being the number of cut-sets close to the minimal cut-sets found for LT 4 and 6. The new approach could improve the minimal cut-sets for LT 9, the most complex LT taken into account.

There is not a specific heuristic method appropriate for all the LTs. Some methods are more appropriate than others depending on the logical function. The most appropriate method should be chosen for each case. The heuristic methods described hereby are static. There are also dynamic heuristic methods, however, they are not suitable for large or complex LTs. They present some drawbacks such as they need to store in memory the BDD or a part of it [75].

4.4 Benefits and drawbacks of BDDs

Previous sections have stated the numerous advantages provided by mentioned conversion in hereby project. Besides already mentioned advantages, the equivalent BDD obtained from the LT will provide the necessary basis to obtain a ranking of the most important events over the whole LT.

When any business seeks to improve a department or inner issue in a certain department at any given time, numerous causes related with those departments involved are found. There will be several tens of events under the most favourable conditions which will lead to the MP. Nonetheless, and specifically when speaking about large companies, the regular scenario is to face hundreds or even tens of hundreds of events. In other words, this introduces a challenging scenario in computational terms due to the fact that to handle mentioned data is not straightforward.

The main reason that leads to hereby chapter to convert LT to BDD is the ability to deal with hundreds or thousands of events using both computational time and resource management in a remarkable way. Moreover, in order to be able to simulate the IMs in a reasonable computational time, the CSs obtained from the BDD seems to be the feasible manner. Once the conversion from LT to BDD is done and the CSs are achieved, it is possible to obtain the Importance Measures. In fact, the CSs as well as the probability associated to each event provide the needed data for the Importance Measures afterward described. Exact calculations to obtain the Top probability are carried out and approximations are not needed when using BDDs. That turns out into one of the biggest advantages of the BDDs. Table 4.4 shows some other reasons that have influenced in the final decision to use BDDs as a formal solution to respond to the needs asked in hereby project.

The way the BDDs are able to handle all kind of trees, regardless whether they are small, medium or large size is one of the biggest advantages of using BDDs that has made them so particularly appropriate. It has been proved that when big size trees must be faced there could be some issues and there exist some different techniques. For instance, a technique successfully used is to convert the tree into small ones in a way that the software is able to simulate each single small part and then combine all the obtained results [59]. Pretty good results have been achieved.

BDDs makes possible to calculate the occurrence probability of a Top occurred in a business given a certain LT. On this occasion, BDDs based algorithms do not use approximation techniques such as truncation to calculate the occurrence probability of a MP. Nonetheless, BDDs could have an incredibly high time and memory consumption, chiefly when many events are involved. At this point and as previously stated, a particular emphasis must be done when dealing with the variable ordering. BDD arises when a low computational time and reliable results are sought for solving LTs problems. Thanks to BDDs is possible to achieve a good solution in an efficient and effective manner, whenever the variable ordering is tackled with special care.

In resume, BDDs present advantages that are listed below:

- The computational cost is independent of the number of PIs and the way in which the LT is built.
- All the PIs are taken into account.
- They provide exact qualitative and quantitative information.
- The computational speed is between 100-1000 times higher than using classic methods.
- Typical operators of Boolean algebra can be evaluated with quadratic complexity.

- The cost of the analysis using BDD depends on the LT size. Large Boolean functions can be represented with relatively small diagrams.
- Operations with "Products" over time are linear respect to the BDD size.
- Great efficiency in the treatment of non-coherent LTs.

A comparison between LT and BDD is presented in Table 4.4

Table 4.4 Benefits and drawbacks

Logical Trees	**Binary Decision Diagrams**
Right depiction of a DM problem	Poor depiction of a DM Problem
Mathematical issues when trying to find the solution	Great efficiency finding the solution*
Poor software implementation	Well-grounded background to achieve mathematical solutions
Lack of reliable software to treat this kind of problems	Accurate software and Low Computational time*
No qualification needed by employees to depict a DM problem	Great complex associated and software is essential to obtain it

A set of didactic examples of the conversion from LT to BDD can be found in ANNEX VI.

5 RELIABILITY ANALYSIS

Modern technologic systems are usually formed by a large number of components. Each component has an essential role and importance into the system. The reliability of the system depends of the complexity of the system, being indirectly proportional. Generic systems are taken into account but the techniques presented in this paper can be used to analyse the reliability of a generic system. LTs are used in this paper as a useful tool to facilitate the analysis of the system from a qualitative and quantitatively point of view.

Quantitative techniques are obtained by mathematical models from the probabilities of the events. Firms may dispose of a large amount of data for reliability coming from multitude and different components (see chapter 2). The analysis purposed in this paper establishes the probability of the top event taking into consideration this dataset.

As mentioned in section 3.1, a LT used for reliability analysis are called Fault Tree (FT). In this case, the events that compose the FT are representing the failure of a certain component of the system. The Top Event is related to a global system failure.

5.1 Reliability analysis over the time

One of the main reasons for using the conversion from LT to BDD is to obtain the analytical expression provided by the BDD. This expression allows saving much computational time when an analysis over time is required due to it is not necessary to solve the LT at any time. This chapter explains how to employ this expression to collect the occurrence probability of a top event over time. It is important to remark that this book focuses on the study of static LT and not dynamic ones, i.e. the topology of the tree remains immutable at any period of time.

The same procedure could be employed if a reliability analysis is required not over the time but in a certain point of time. The only difference between the two type of analysis is that the probability is given by time dependant functions for dynamic analysis and by scalar values for static analysis.

5.1.1 Probability Set

As aforementioned topology of the static LTs does not change over time, however, the occurrence probability of the basic events can be variable over time. The occurrence probability of the events can be gathered from statistical studies and some mathematical models can be created to adapt the behaviour of the event over time to a certain analytical model. Four models have been taken into account in this section in order to consider events with different behaviour over time, they are defined as follows:

- Constant probability (equation 5.1):

$$q_i(t) = K \tag{5.1}$$

where ‘K’ is a constant valued, K $\in$ [0, 1].

- Exponential increasing probability (equation 5.2):

$$q_i(t) = 1 - e^{-\lambda t}, \lambda > 0 \tag{5.2}$$

- Linear increasing probability (equation 5.3):

$$q_i(t) = mt, m > 0, mt < 1 \tag{5.3a}$$

$$q_i(t) = 1, m > 0, mt \geq 1 \quad (5.3b)$$

- Periodic probability (equation 5.4)

$$q_i(t) = 1 - e^{-\lambda(t-n\alpha)} \quad (5.4)$$

where n=1, 2, 3…, $\lambda > 0$, and $\alpha > 0$ determines the period.

Figure 5.1 shows an example of the aforementioned mathematical models. They can be modified or combined in order to emulate better a certain event. Figure 5.1 has been created from the following data:

- Constant probability →$q_i(t) = 0.3$
- Exponential increasing probability →$q_i(t) = 1 - e^{-2t}, \lambda > 0$
- Linear increasing probability→$q_i(t) = 0.06t$
- Periodic probability→ $q_i(t) = 1 - e^{-0.4(t-n2)}$

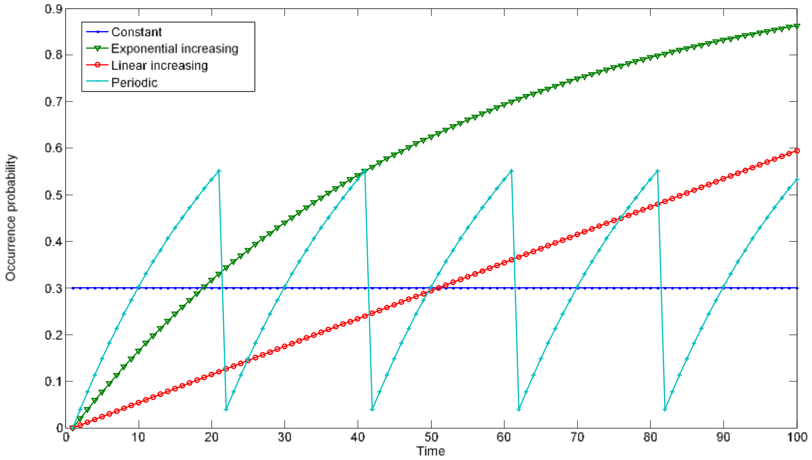


Figure 5.1 Dynamic occurrence probabilities

This set of probability data can be used as a variable input for the probability expression obtained by the BDD.

Once the behavior of the events is analytically modeled the dynamic analysis is to develop an iterative process. The occurrence probability of the i event at the iteration t is defined as q_i^t. These probabilities are collected in a n x m matrix ***PV*** *(i,t))* where *n* is the number of events and *m* number of iterations. Therefore, this matrix is defined as follows:

$$PV(i,t) = \begin{bmatrix} q_1^1 & q_1^2 & \dots & q_1^{m-1} & q_1^m \\ q_2^1 & q_2^2 & & q_2^{m-1} & q_2^m \\ \vdots & & \ddots & \vdots & \\ q_{n-1}^1 & q_{n-1}^2 & \dots & q_{n-1}^{m-1} & q_{n-1}^m \\ q_n^1 & q_n^2 & & q_n^{m-1} & q_n^m \end{bmatrix}$$

Where each component q_i^t corresponds to the occurrence probability of the event 'i' at the iteration 't'. Figure 5.2 shows the flowchart to obtain **PV** in order to save the probabilities automatically.

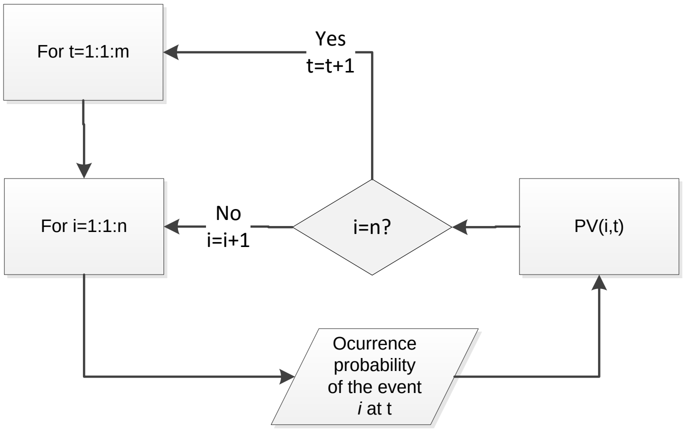


Figure 5.2 Dynamic occurrence probabilities

Once ***PV****(i,t)* is obtained, it is possible to use this matrix as an input in the iterative process. The main advantage of using BDDs for this dynamic analysis is that the LT has to be solved only one time to extract its probability function. The iterative process is based on the evaluation of this probability function using the corresponding probabilities provided by PV at each iteration.

For example, the data shown in Figure 5.1 is collected for the first five iterations by the following **PV** matrix:

$$PV(i,t) = \begin{bmatrix} 0.00 & 0.04 & 0.08 & 0.11 & 0.15 & 0.18 \\ 0.30 & 0.30 & 0.30 & 0.30 & 0.30 & 0.30 \\ 0.00 & 0.18 & 0.33 & 0.45 & 0.55 & 0.63 \\ 0.00 & 0.06 & 0.02 & 0.02 & 0.02 & 0.03 \end{bmatrix}$$

Therefore, at each iteration the probability vector corresponds to a different column of the PV matrix.

5.1.2 *Dynamic analysis case study*

An example of the proposed dynamic analysis is presented in order to clarify the procedure.

Figure 5.3 shows a LT that represents a logical structure of system where the objective is to minimize the probability of occurrence of the Top Event.

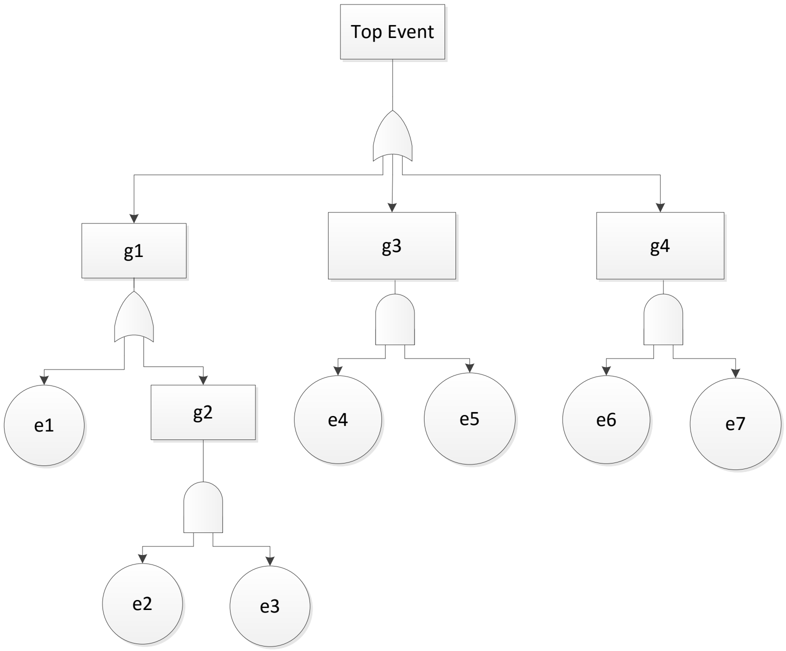


Figure 5.3 LT dynamic analysis example

The resultant BDD is achieved from a LT to BDD conversion software. Thus, the resulting CSs are

$CS_1 = e_1$

$CS_2 = e_6 \cdot \overline{e_1}$

$CS_3 = e_7 \cdot \overline{e_6} \cdot \overline{e_1}$

$CS_4 = e_5 \cdot e_4 \cdot \overline{e_7} \cdot \overline{e_6} \cdot \overline{e_1}$

$CS_5 = e_3 \cdot e_2 \cdot \overline{e_5} \cdot e_4 \cdot \overline{e_7} \cdot \overline{e_6} \cdot \overline{e_1}$

$CS_6 = e_3 \cdot e_2 \cdot \overline{e_4} \cdot \overline{e_7} \cdot \overline{e_6} \cdot \overline{e_1}$

And therefore the analytical expression that rules the LT for dynamic analysis is:

$$\begin{aligned} Q_{sys}(t) = {} & q_1(t) + q_6(t) \cdot \left(1 - q_1(t)\right) + q_7(t) \cdot \left(1 - q_6(t)\right) \cdot \left(1 - q_1(t)\right) \\ & + q_5 \cdot q_4 \cdot \left(1 - q_7\right) \cdot \left(1 - q_6\right) \cdot \left(1 - q_1\right) + q_3 \cdot q_2 \\ & \cdot \left(1 - q_5\right) \cdot \left(1 - q_7\right) \cdot \left(1 - q_6\right) \cdot \left(1 - q_1\right) + q_3 \cdot q_2 \\ & \cdot \left(1 - q_4\right)\left(1 - q_7\right) \cdot \left(1 - q_6\right) \cdot \left(1 - q_1\right) \end{aligned}$$

The probability assignments are based on the parameters collected in the following Table 5.1

Table 5.1 Probability assignment example

	Mathematical model	**Parameters**
Event 1	Constant	q=0.3
Event 2	Linear increasing	m=0.06
Event 3	Exponential increasing	λ=2
Event 4	Periodic	λ=0.4, α=2
Event 5	Constant	q=0.2
Event 6	Periodic	λ=0.6, α=3
Event 7	Exponential increasing	λ=3

Figure 5.4 shows the probabilities of the events throughout 100 samples. Each sample may represent a certain increment of time.

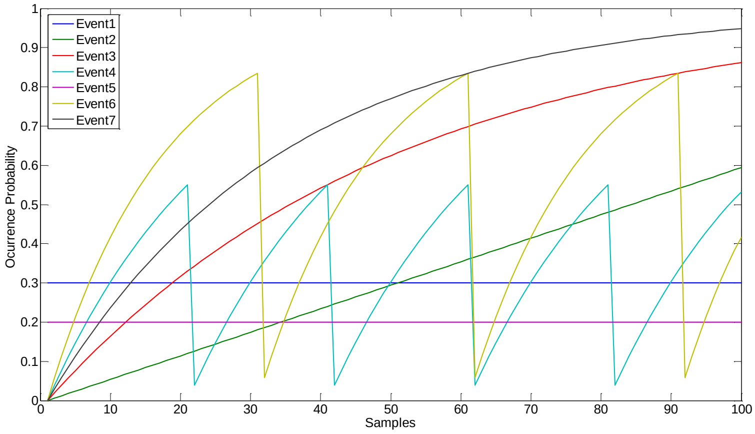


Figure 5.4 Probability assignment. Example

Once the probability assignment has been done, it is possible to obtain the occurrence probability of the top event (Q_{Sys}) over time. Figure 5.5 plots this probability function, with a general rising trend. The reason that it is not always rising is because there are events with periodic probability functions. This probability has been obtained for 50 samples. The curve in Figure 5.5 can be employed in order to do prognostics, to fit the operations thresholds, etc.

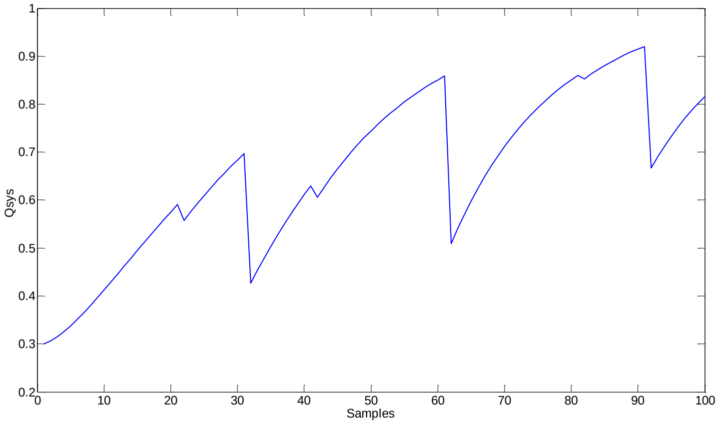


Figure 5.5 *Qsys* over time. Example

The dynamic analysis proposed in this book can facilitate to establish a maintenance planning because the probability of the top event is available over time. It leads to keep the reliability of this event under control.

5.1.3 Dynamic Importance Measure

The same procedure is used to carry out a dynamic importance analysis. The Importance measures are calculated for all the events and iterations. This information is very useful to establish a variable ranking over time and, consequently, to determine the most significant events in a certain period of time.

Continuing the same example of section 5.1.2, Figure 5.6 shows the Birnbaum importance calculated for 100 samples.

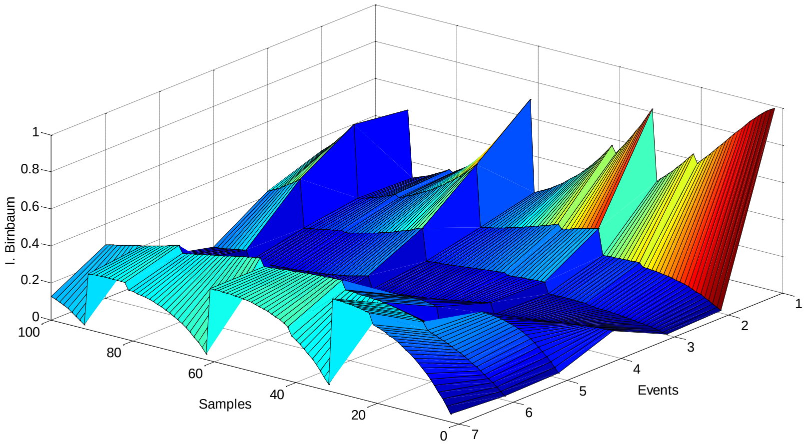


Figure 5.6 Dynamic Birnbaum IM. Example

The Birnbaum importance is variable over time and the ranking of the most critical events also varies, e.g. the event e_1 is the most important event at the first sample, however event e_7 is the most important one at the sample 60th. This means that in function of the period of time different events should be taken into account to optimize the resource allocation.

Figure 5.7 shows the Criticality IM calculated for 100 samples. Similar conclusions can be gathered taking into account that this IM considers the occurrence probabilities of the events and this causes differences respect to the Birnbaum IM.

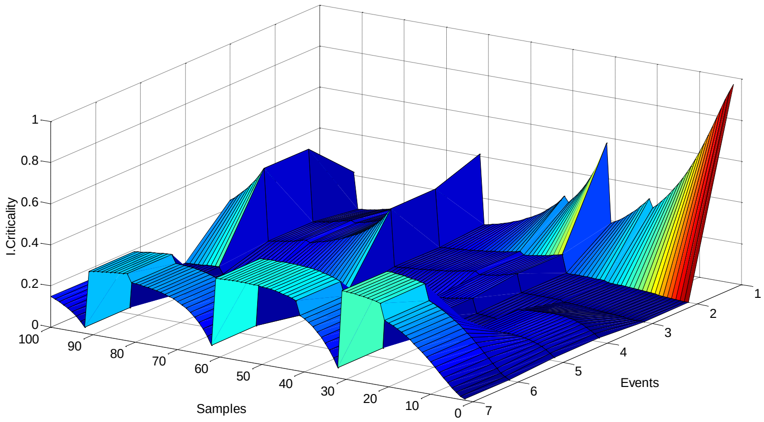


Figure 5.7 Dynamic Criticality IM. Example

Figure 5.8 shows the Fussel – Vesely IM calculated for 100 samples.

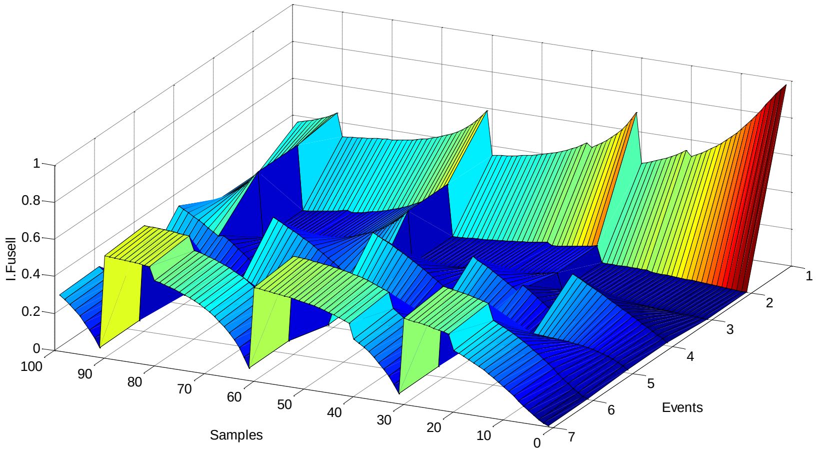


Figure 5.8 Dynamic Fussel – Vesely IM

In general, the shape of the graph is similar in the three cases. Each measure has a different meaning and therefore one of them should be selected in function of the objectives of the decision making process.

To apply temporary variables allows determining operational strategies that will raise the reliability of the system. It will lead also the following issues:

- To determine the availability of the system and their components at a certain moment.
- To identify critical operating states of the system and their components.
- To determine the optimal time to carry out a preventive task and to choose the components to be repaired or replaced.
- To determine the repairs or replacements necessaries to ensure a certain availability of the system for a period of time.

5.1.4 Procedure for maintenance

Firms frequently need to guarantee certain availability for their systems or products because a low reliability can lead to unprofitable situations. For instance, the image of a firm could be seriously affected by unreliable products. For this reason, firms usually establish a degree of availability for their systems or products. Figure 5.9 shows a simple threshold (dash line) for unavailability of the system depicted in Figure 5.5. The first intersection between the two lines provides the limit that the system has an acceptable unavailability rate. This point also fixes the moment in which certain repairs should be done in order to maintain an desired unavailability. Once this point has been localized (time=0.7 in Figure 5.9), the next step is to select the components to repair from the IMs obtained previously. Once components have been repaired or replaced, the unavailability will be acceptable again. This iterative process constitutes a good strategy for ensure a certain availability of the system for a certain period of time.

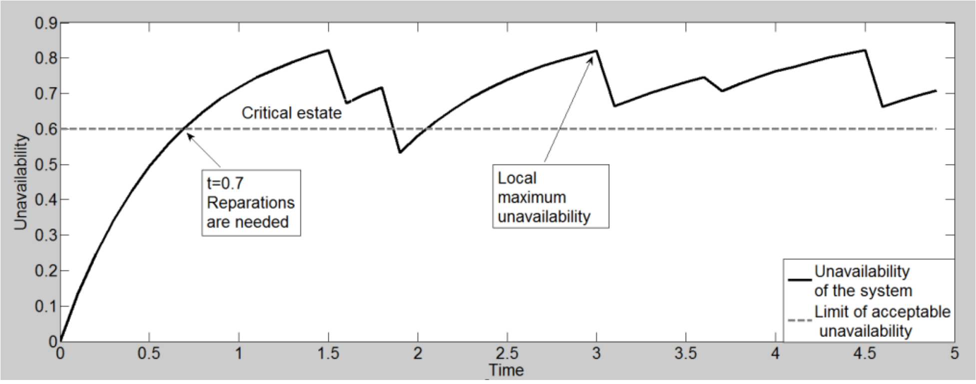


Figure 5.9 Unavailability threshold.

Moreover, the identification of maximum unavailability points allows determining the degree of criticality by evaluating the distance between these points and the corresponding threshold.

5.1.5 ***New method for reducing the system failure probability***

The following section presents a new method to optimise the actuation (replacement, repair, maintenance) upon a group of events in order to ensure a certain reliability to the system and doing an appropriate use of the available resources. The purpose of this study is to demonstrate that the individual importance of each event is not determinant when a group of components are going to be repaired or replaced. A method is presented in order to establish a new ranking for the case study in ANNEX VIII that leads to a faster reduction of the system failure probability. The probability assignment for the 159 events considered is as shown in Figure 5.10.

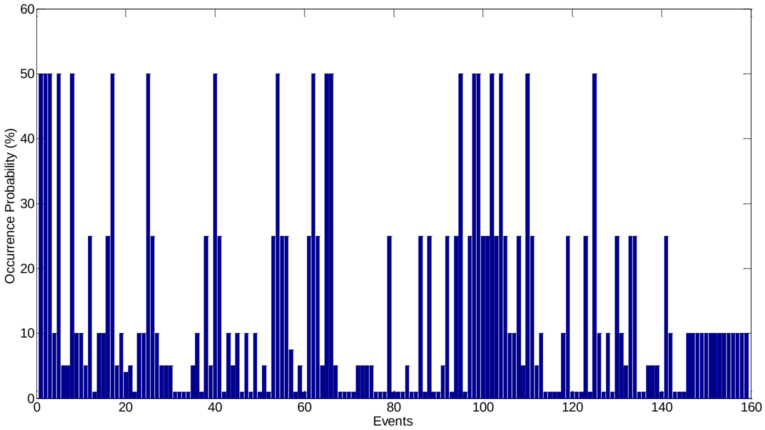


Figure 5.10 Probability assignment for probability reduction method

Figure 5.11 shows the Birnbaum importance value obtained for each event using the FT and the probabilistic values given in ANNEX VIII. As can be observed, there are a big gap between the importance of different events. This ranking provided by Figure 5.11 will be the starting point of the method proposed in the following section.

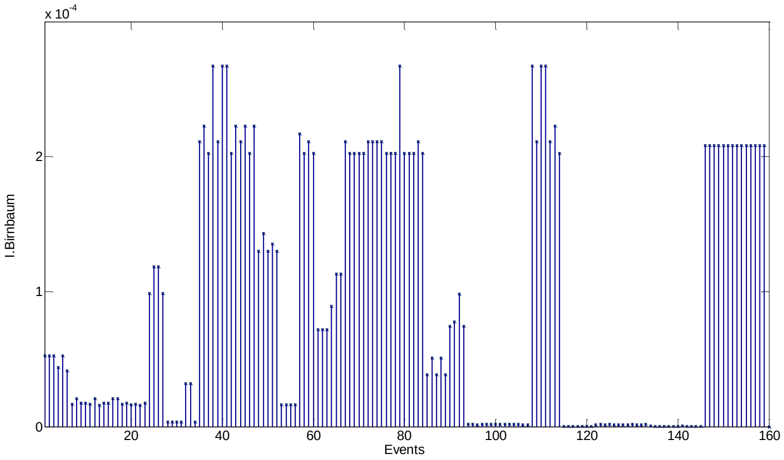


Figure 5.11 Birnbaum importance for probability reduction method

Figure 5.12 shows the Q_{sys} of the system when the probabilities of a certain number of events are set to zero. This figure uses the Birnbaum ranking gathered from Figure 5.11 to determinate which events are taken into account in each case. The abscissa represents the total of events whose probabilities are set to zero, i.e. the value 1 means that the probability of the first event, according to the Birnbaum importance, has been set to zero. The maximum group of events would correspond to the total of events that forms the FT in the ANNEX VIII, in this case a total of 159 events.

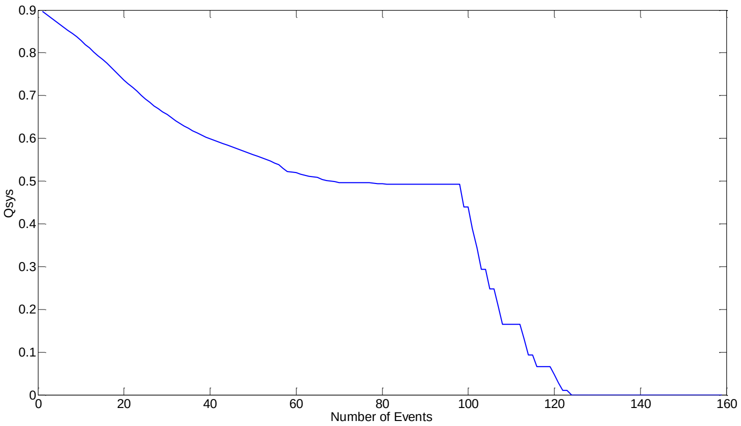


Figure 5.12 System Probability reduction vs group of events

The objective of this analysis is to find an appropriate ranking of events that allows reducing the Q_{sys} by acting upon the smallest possible number of events. The method is based on the evaluation of the slopes in Figure 5.11. Each event is ordered in function of the reduction of the Q_{sys} generated by them. An importance parameter named reduction importance (I_{red}) is assigned to each event as follows:

Let $\boldsymbol{I_{ord}^{Birn}}$ be a vector where the events are ranking in a descendant order according to their Birnbaum importance and $\boldsymbol{I_{red}}\left(I_{ord}^{Birn}(x)\right)$ be the reduction importance assigned to the event that occupies the position x in the vector $\boldsymbol{I_{ord}^{Birn}}$. It is defined by equation 5.5:

$$\boldsymbol{I_{red}}\left(\boldsymbol{I_{ord}^{Birn}}(x)\right) = Q_{sys}(x) - Q_{sys}(x-1) \tag{5.5}$$

where $Q_{sys}(x)$ is the system failure probability when the probabilities of a total of x events have been reduced to zero.

The events are reordered in descendant values of $\boldsymbol{I_{red}}$ and a new ranking is obtained. In order to clarify the method, the flowchart in Figure 5.13 shows the proposed procedure to find this new ranking.

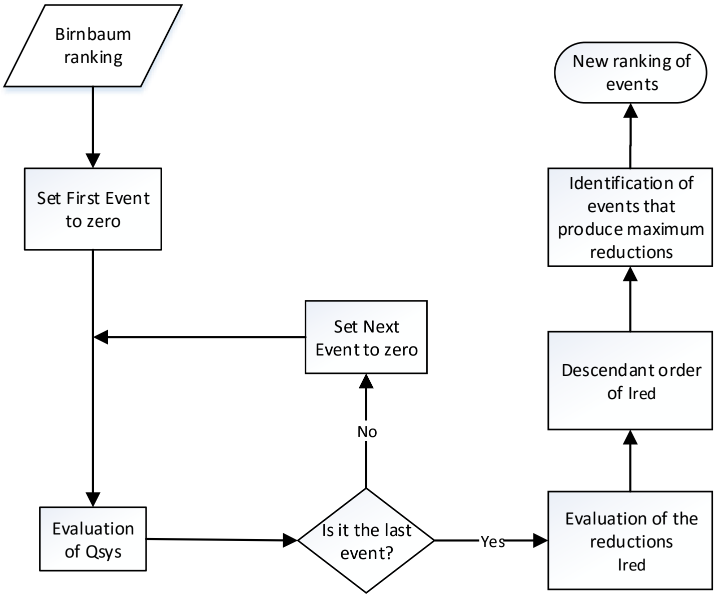


Figure 5.13 Proposed method flowchart

Figure 5.14 shows the application of the proposed method to the case study presented in ANNEX VIII. It is demonstrated that the reduction of the failure probability is better when the importance of a group of events is taking into account and not only considering the individual importance assigned by the IMs.

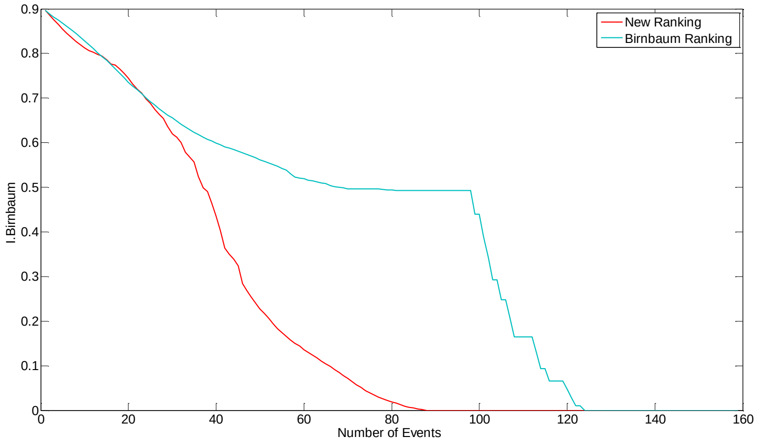


Figure 5.14 Group of events reduction using the proposed method

Therefore, the new ranking provides a faster reduction of the system failure probability. This can result in a saving of resources when the objective is to keep the availability of a system under control.

5.2 Case study. Fault Tree Analisys for Wind Turbines

This section is based on results of the European projects in references [78], [79] and [80]. The FTs and the failure modes have been already considered in the thesis of reference [1]. The contribution of this research work is the achievement of the quantitative results showed in section 5.2.2. This results have been also employed for establishing a novel maintenance planning explained in section 5.2.3.

The main components of the WTs are illustrated in Figure 5.15. The blades, connected to the rotor via the hub, are moved by the wind blowing on them. The rotor transmits the mechanical energy via the low speed shaft through the gearbox to the high speed shaft, ending in the generator. The low speed shaft is supported by the main bearing. The alignment to the direction of the wind is controlled by a yaw system that turns the housing (or “nacelle”) for that purpose. The nacelle is mounted at the top of a tower, and the tower is assembled on a base or foundation. The pitch system in each blade is a mechanism that turns the blade to control the wind power captured. This can be employed as an aerodynamic brake as well as for increasing the efficiency of power production. The WT has also a hydraulic brake to stop the WT. The meteorological unit, or weather station, provides the weather data (e.g. wind speed and direction) to the control system. The data from the meteorological unit provide the required information for controlling effectively the pitch system, brake, yaw, etc.

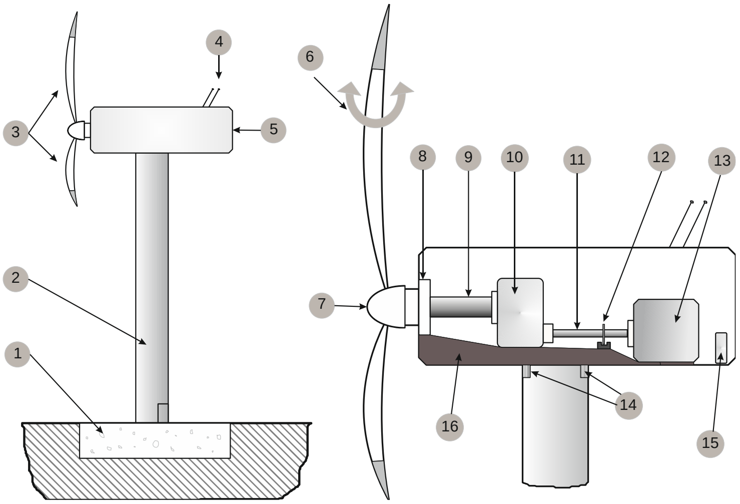


Figure 5.15 Components of the WT: 1-Base/Foundations; 2-Tower; 3-Blades; 4-Meteorological unit (vane and anemometry); 5-Nacelle; 6-Pitch system; 7-Hub; 8-Main bearing; 9- Low speed (main) shaft; 10-Gearbox; 11- High speed shaft; 12-Brake system; 13-Generator; 14-Yaw system, 15-Converter, 16-Bedplate. N.B. Drive train = 9+11.

A study of failure modes and effects analysis (FMEA) for WTs in 2010 (RELIAWIND project) collected the causes of failure and failure modes of a specific WT of 2MW with a diameter of 80 m [81] and [78]. Some causes of failures (or root causes) are summarised in Table 5.2. These main causes of the failures can be due to environmental conditions (e.g. lightning, ice, fire, strong winds, etc.) or to defects, malfunctions or failures in the components of the WT (e.g. braking system failure, or be struck by blade, etc.) [82]. Table 5.3 shows some of the principal component failure modes of the WTs [81] and [83].

Table 5.2 Root causes of the failures of the components of a WT

Structural	**Wear**	**Electrical**
Design fault	Corrosion	Calibration error
External damage	Excessive brush wear	Connection failure
Installation defect	Fatigue	Electrical overload
Maintenance fault	Pipe puncture	Electrical short
Manufacturing defect	Vibration fatigue	Insulation failure
Mechanical overload	Overheating	Lightning strike
Mechanical overload–collision	Insufficient lubrication	Loss of power input
Mechanical overload–wind		Conducting debris
Presence of debris		Software design fault

Table 5.3. Failure modes of the failures of the components of a WT [25] and [28].

Mechanical	**Electrical**	**Material**
Rupture	Electrical insulation	Fatigue
Uprooting	Electrical failure	Structural
Fracture	Output inaccuracy	Ultimate
Detachment	Software fault	Buckling
Thermal	Intermittent output	Deflection
Blockage		
Misalignment		
Scuffing		

The construction of the illustrative FT studied herewith is focused on a three-blade, pitch controlled geared WT. The WT has been divided into four major groups of elements for a better FTA:

- The foundation and tower;
- The blades system;

- The electrical components (including generator, electrical and electronic components), and;
- The power train (including speed shafts, bearings and a gearbox).

The elements are connected by AND and OR gates. The faults considered in this paper are set by an exhaustive review of the literature and the support of member experts in the NIMO and OPTIMUS FP7 European projects [79] and [80].

Table 5.4 shows a summary of the failures from the literature taken into account for this paper. It can be seen that gearboxes, generators, blades and electric and control systems have been extensively studied in the literature. Nonetheless, there are not many references which analyse other components of a WT such as brakes, hydraulic and yaw systems.

Table 5.4 Failures of the main elements of a WT

Foundation and tower	Structural fault [84],[82],[85],[86]	
	Yaw system failure [87]	
Critical rotor	Blade failure	Structural failure [88],[89],[90], [91], [92], [93],[94],[95] Pitch system failure [96] Hydraulic system fault [97],[98]
		Meteorological unit failure [97],[99]
	Rotor failure	Rotor hub [84],[87] Bearings [98],[87],[86]
Power train	Low speed train failure [100],[87]	
	Critical gearbox failure [95],[100],[87],[101],[102],[103]	
	High speed train failure	Shaft [100],[84],[87]
		Critical brake failure [104], [84]
Electrical components	Critical generator failure [100],[87],[105],[106],[107],[101]	
	Power electronics and electric controls failure [100],[98],[101]	

5.2.1 Fault Tree for Wind Turbines

The following sections show the FT for the aforementioned main components of the WT. It is very important to mark that they could be simplified or extended, but the authors, following the opinion of the experts, have set them in order to show the most relevant events.

5.2.1.1 Foundation and Tower

The tower supports the nacelle which is located at a suitable height in order to minimize the influence of turbulence and to maximize the wind energy. The tower is assembled by relatively thin-wall steel cylindrical elements welded together along their perimeters in three sections and joined by bolts. This is done in order to enable the transportation of the large structural elements to the wind farm where they need to be assembled in-situ [108]. The base section of the tower is installed on a reinforced concrete foundation comprising a round base.

Structural defects associated with the tower, foundation, blades and hub, in the form of fatigue cracks, delamination etc., can initiate and evolve with time. The main causes for structural failures are fatigue induced crack initiation and propagation, extreme wind speeds and distribution, extreme turbulences, maximum flow inclination and terrain complexity [83], and also ice accumulation, hail, bird strikes, dust particle impacts, or lightning bolt strikes. Material fatigue [82] (tower-based fatigue damage has been shown to decrease significantly when using active pitch for the blades [85]), impact of blades on the tower, faulty welding and failure of the brakes [86] are the main representative failure modes.

The literature shows that the major defects found on WT towers are [79]: cracks in the concrete base, corrosion [84], gaps in the foundation section, loosen studs joining the foundation and the first section, loosen bolts joining first/second and second/third sections and welding damages [82].

On the top of the tower, the yaw system turns the nacelle in an optimum angle with respect to the wind direction. Powered by electromechanical or hydraulic mechanisms (in this paper the electromechanical mechanism is considered), the yaw systems can seize to operate due to the failure of the yaw motor or the meteorological unit failure [87] resulting in a wrong yaw angle. Structural failures could appear when the yaw motor is damaged or it does not have power supply [109], in addition to extreme wind speed or turbulences and some structural faults. These structural failures can cause the collapse of the tower [82]. Design load cases (DLC) must be taken into account for different design situations and wind or other conditions. The IEC 61400-1 relative to design requirements for wind turbines shows some DLCs that shall be considered as minimum [83]. For example, the event e012 (High wind speed/ turbulence) will occur when DLCs are exceeded. Table 5.5 presents the basic and intermediate events for the FT of the foundation and tower illustrated in Figure 5.16.

Table 5.5. Principal events in the foundation and tower.

Yaw system failure	g005	Yaw motor fault	e001
Structural failure	g006	Abnormal vibration I	e002
Yaw motor failure	g007	Abnormal vibration H	e003
Wrong yaw angle	g008	Cracks in concrete base	e004
Severe structural fault (foundation and tower)	g009	Welding damage	e005
No electric power for yaw motor	g010	Corrosion	e006
Meteorological unit failure	g011	Loosen studs in joining foundation and first section	e007
Structural fault (foundation and tower)	g012	Loosen bolts in joining different sections	e008
		Gaps in the foundation section	e009
		Vane damage	e010
		Anemometer damage	e011
		High wind speed/ turbulence	e012
		No power supply from generator	e013
		No power supply from grid	e014

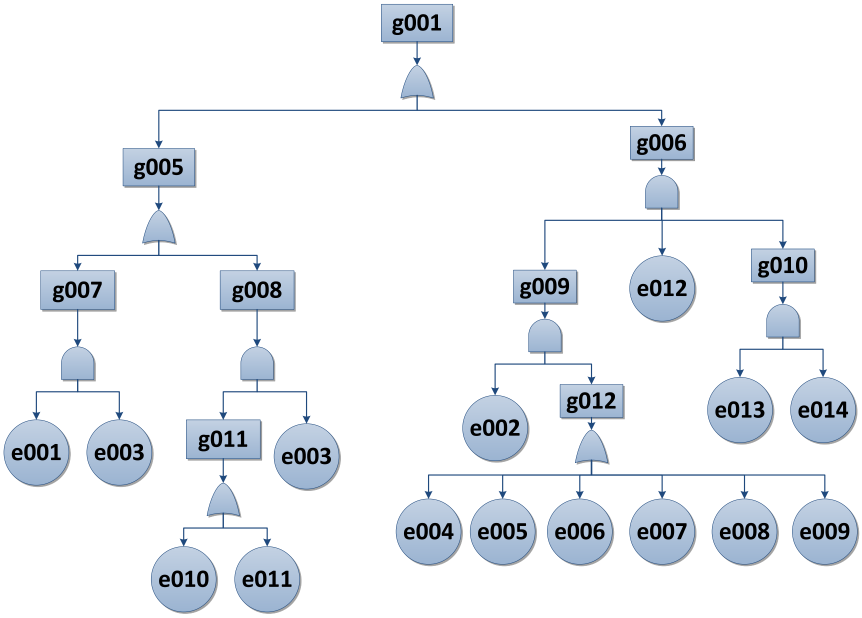


Figure 5.16. Fault tree of the foundation and tower

5.2.1.2 Blade System

The rotor is located inside the nacelle. The blades are attached to the rotor shaft by the hub and they are mounted on bearings in the rotor hub. The blades are the components of the WT with the highest percentage of failures and downtimes [110]. Ciang *et al.* in 2008 done a review of damage detection methods, particularly considering the blades [84]. The rotor hub supports heavy loads that can lead faults such as clearance loosening at the blade root, imbalance, cracks and surface roughness. Bearings between blades and hub can be damaged by wear produced by pitting, deformation of outer face and rolling elements of the bearings [87], spalling and overheating [98]. Cracks can appear due to the fatigue [98]. Fatigue, wear, faults in lubrication and corrosion are typically the main failure cause of bearings.

The blades faults are predominantly related to structural failures, e.g. strength [88] and fatigue of the fibrous composite materials [48]. Other faults, e.g. cracks, erosion, delamination and debonding, could appear in the leading and trailing edges of the blades [90] and [91]. Delamination, debonding or cracks are found in the shell [91] and [92], and also in the root section of the blades [93]. The tip deflections (a structural failure of the blade [94]) increase drag near the end of the blades [95].

A common fault of the blades is associated with the failure of the pitch control system [96]. In pitch-controlled turbines, the pitch system is a mechanism that turns the blade, or part of the blade, in order to adjust the angle of attack of the wind. Turbulence of wind is an important cause for pitch system faults [111]. Pitching motion can be done by hydraulic actuators or electric motors. The hydraulic system leads stiffness of bearings, a little backlash and a higher reliability than the electric motors [100]. The hydraulic system can suffer from possible defects such as leakage, overpressure and corrosion [98].

The weather station or meteorological unit provides information about some characteristics of the wind (direction and speed) to the control system of the WT. The main failures found in the WT weather station are related to the vane and anemometer [99]. These can result in adjusting the pitch of the blade to a sub-optimal angle [97]. Table 5.6 collects the main faults given in blades, and Figure 5.6 shows the FT for the blade system.

Table 5.6. Principal events in the blade system.

Severe blade failure	g013	High wind speed/turbulence	e015
Blade failure	g014	Blade angle asymmetry	e016
Pitch system failure	g015	Abnormal vibration A	e017
Structural failure of blades	g016	Hydraulic motor failure	e018
Hydraulic system failure	g017	Leakages in hydraulic system	e019
Wrong blade angle	g018	Over pressure in hydraulic system	e020
Hydraulic system fault	g019	Corrosion in hydraulic system	e021
Meteorological unit	g020	Vane damage	e022
Structural fault of blades	g021	Anemometer damage	e023
Leading and trailing damage	g022	Abnormal vibration B	e024
Shell damage	g023	Root cracks in the structure of blades	e025
Tip damage	g024	Cracks in edges of blades	e026
Rotor system failure	g025	Erosion in edges of blades	e027
Rotor system fault	g026	Delamination in leading edges of blades	e028
Rotor bearings fault	g027	Delamination in trailing edges of blades	e029
Rotor hub fault	g028	Debonding in edges of blades	e030
Wear in bearings of the rotor	g029	Delamination in shell	e031
Imbalance of blade system	g030	Crack with structural damage (shell)	e032
		Crack on the beam-shell joint	e033
		Open tip	e034
		Lightning strike on tip	e035
		Abnormal vibration C	e036
		Cracks in bearings of rotor	e037
		Corrosion of pins in bearings of rotor	e038
		Abrasive wear in bearings of rotor	e039
		Pitting in bearings of rotor	e040
		Deformation in bearings of rotor	e041
		Lubrication fault in bearings of rotor	e042
		Clearance loosening at root (hub)	e043
		Cracks in the hub	e044
		Surface roughness in the hub	e045
		Mass imbalance in the hub	e046
		Fault in pitch adjustment	e047

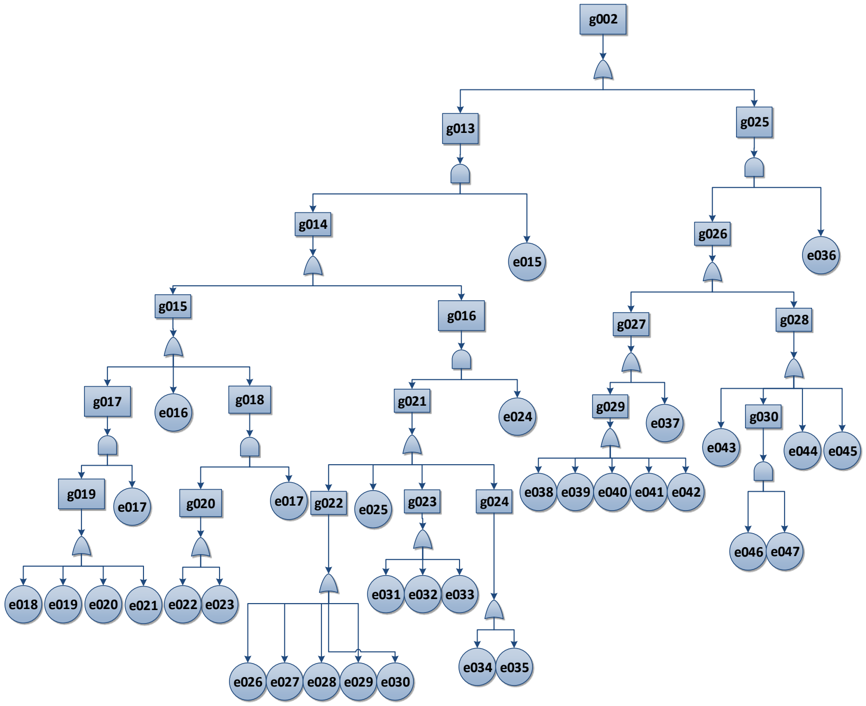


Figure 5.17 FT for Blade System

5.2.1.3 *Generator, electrical and electronic components*

The generator, electrical and electronic components are installed inside the nacelle. The high speed shaft drives the rotational torque to the generator, where the mechanical energy is converted to electrical energy. This conversion needs a specific input speed, or a power electronic equipment to adapt the output energy from the generator to the characteristics of the grid.

Faults in generators can be the result of electrical or mechanical causes [107]. The main electrical faults are due to open-circuits or short-circuit of the winding in the rotor or stator [100] that could cause overheating [87]. Many research works have demonstrated that bearings, rotors and stators involve a high failure rate in WTs [105]. The bearing failures of the generator are usually caused by wear, fatigue cracks, asymmetry and imbalance [112]. The rotor and stator failures can be produced by broken bars [106], air-gap eccentricities and dynamic eccentricities, among other failures [100]. Rotor imbalance and aerodynamic asymmetry can have their origin in the non-uniform accumulation of ice and dirt over the blades system [100]. Short-circuit faults, open-circuit faults and gate drive circuit faults are the three major electrical faults of the power electronics and electric controls in WTs [100]. Corrosion, dirt and terminal damage are the main mechanical defects [98]. The group formed by generator, electrical system and control system, has a relevant rate of failures and downtime in WTs. Table 5.7 shows the main elements and failures in the generator, electrical and electronic components.

Table 5.7. Principal faults in the generator, electrical and electronic components.

Critical generator failure	g031	Abnormal vibration G	e048
Power electronics and electric controls failure	g032	Cracks	e049
Mechanical failure (generator)	g033	Imbalance	e050
Electrical failure (generator)	g034	Asymmetry	e051
Bearing generator failure	g035	Air-Gap eccentricities	e052
Rotor and stator failure	g036	Broken bars	e053
Bearing generator fault	g037	Dynamic eccentricity	e054
Rotor and stator fault	g038	Sensor T^a error	e055
Abnormal signals A	g039	Temperature above limit	e056
Overheating generator	g040	Short circ (generator)	e057
Electrical fault (power electronics)	g041	Open circ (generator)	e058
Mechanical fault (power electronics)	g042	Short circ (electronics)	e059
		Open circ (electronics)	e060
		Gate drive circ	e061
		Corrosion	e062
		Dirt	e063
		Terminals damage	e064

Figure 5.18 presents the FT for the main elements of the generator, electrical and electronic components given in Table 5.7.

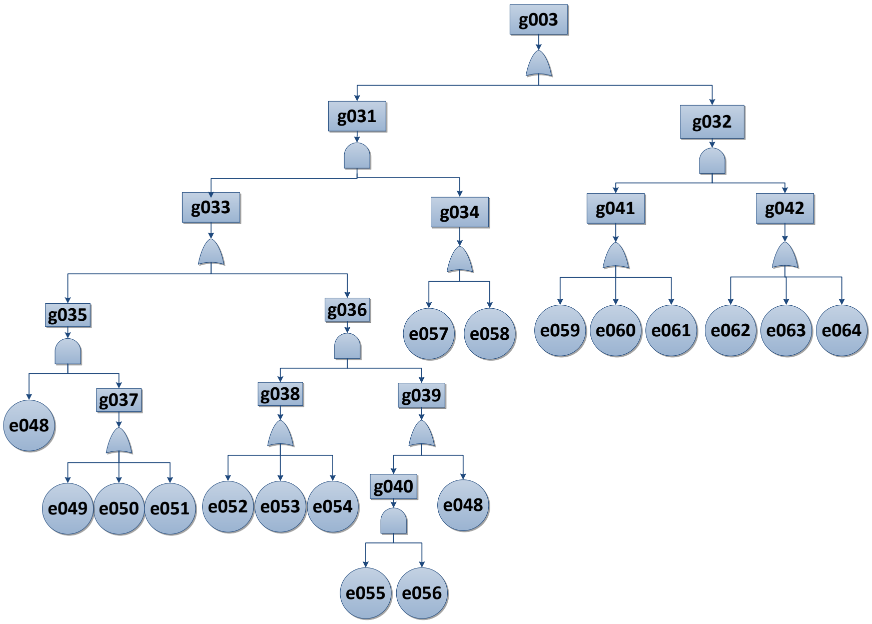


Figure 5.18 Fault tree of the generator, electrical and electronic components

5.2.1.4 Power train

The power train, or drive train, is installed in the nacelle and consists of the main bearing, main (low speed) shaft, the gearbox and the generator. Through the main bearing, the rotor is attached to the low speed shaft that drives the rotational energy to the gearbox. The rotational speed of the rotor is generally between 5 and 30 RPM, and the generator speed is from 750 to 1500 RPM, depending on the type and size of generator. A gearbox is mounted between the rotor and the generator in order to increase the rotational speeds. The gearbox output is driven to the generator through the high speed train. A mechanical brake powered by a hydraulic system is usually mounted in the high speed train as a secondary safe breaking system.

The low speed train failure includes main bearing [98] and low speed shaft defects. Severe vibrations can appear due to impending cracks in any component, or to the mass imbalance in the low speed shaft [100]. The gearbox failure is one of the most typical failures [95]. There are many studies about gearboxes in the literature because their failure causes significant downtimes in the system [113]. The most common faults were found in gear teeth and bearings due to lubrication faults [100], e.g. contamination due to defective sealing [96] or loss of oil [102], wear or fatigue damage which can generate pitting, cracking, gear eccentricity, gear tooth deterioration, offset or other potential faults [95] and [87].

Overheating can appear in shafts due to the rotational movement of the high speed train. The wear and fatigue, that can initiate cracks [87] and mass imbalance [100], are the principal source of failures in the high speed shaft. The main failure causes of brakes are overpressure or oil leakages [84], cracking of the brake disc and callipers [104]. Figure 5.19 shows the FT for the main elements of the power train described in Table 5.8

Table 5.8 Principal faults in the power train

Low speed train failure	g043	Abnormal vibration D	e065
Critical gearbox	g044	Cracks in main bearing	e066
High speed train failure	g045	Spalling in main bearing	e067
Main bearing failure	g046	Corrosion of pins in main bearing	e068
Low speed shaft failure	g047	Abrasive wear in main bearing	e069
Main bearing fault	g048	Deformation of face & rolling element (main bearing)	e070
Wear in main bearing	g049	Pitting (main bearing)	e071
Low speed shaft fault	g050	Imbalance of low speed shaft	e072
Wear in low speed shaft	g051	Cracks in low speed shaft	e073
Gearbox failure	g052	Spalling (low speed shaft)	e074
Bearings (gearbox)	g053	Abrasive wear in low speed shaft	e075
Lubrication of the gearbox	g054	Pitting (low speed shaft)	e076
Gear failure	g055	Abnormal vibration F	e077
Wear bearing gearbox	g056	Corrosion of pins (bearing gearbox)	e078
Gear fault	g057	Abrasive wear (bearing gearbox)	e079
Tooth wear (gears)	g058	Pitting (bearing gearbox)	e080
Offset of teeth gears	g059	Deformation of face & rolling element (gearbox bearing)	e081
High speed shaft fault	g060	Oil filtration (gearbox)	e082
Critical brake failure	g061	Particle contamination (gearbox)	e083
High speed structural damage	g062	Overheating gearbox	e084
Wear of high speed shaft	g063	Abnormal vibration E	e085
Brake failure	g064	Eccentricity (gear)	e086
Abnormal signals B	g065	Pitting (gear)	e087
Hydraulic brake system fault	g066	Cracks in gears	e088
Abnormal signals C	g067	Gear tooth deterioration	e089
Overheating brake	g068	Poor design of teeth gears	e090
		Tooth surface defects	e091
		Abnormal vibration J	e092
		Cracks in high speed shaft	e093
		Imbalance (high speed shaft)	e094
		Overheating (high speed shaft)	e095
		Spalling (high speed shaft)	e096
		Abrasive wear (high speed shaft)	e097
		Pitting (high speed shaft)	e098
		Cracks in brake disk	e099
		Motor brake fault	e100
		Oil leakage (hydraulic brake)	e101
		Over pressure (hydraulic brake)	e102
		Abnormal speed	e103
		T^a sensor error (brake)	e104
		T^a above limit	e105

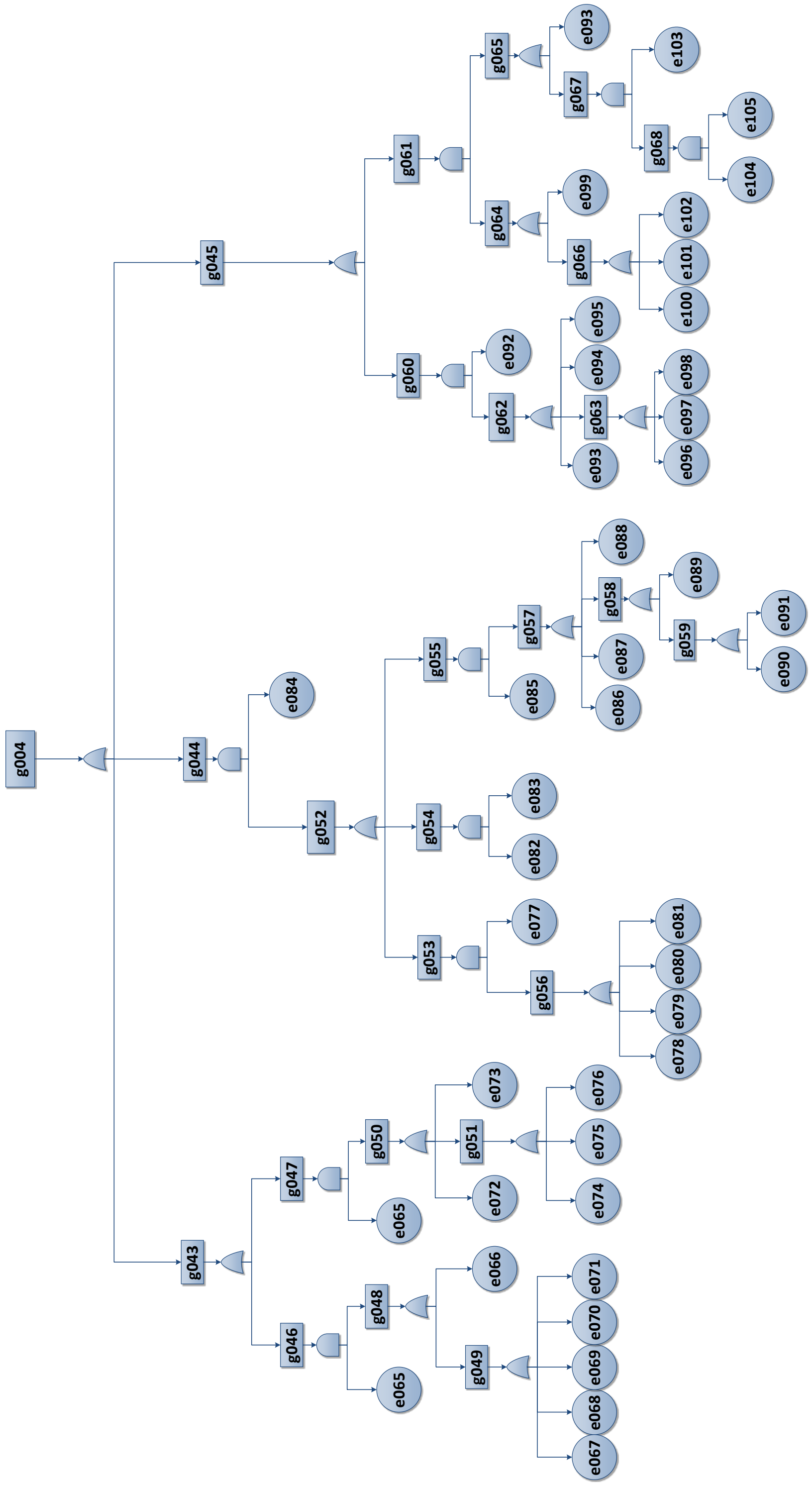


Figure 5.19 FT for Power train

5.2.2 Results

The most important events according to IM values obtained with the methods Birnbaum, Criticality, Structural and Fussell-Vesely can be identified in Figure 5.20. In this case, the most important events are e_{001}, e_{003}, e_{017}, e_{018}, e_{019}, e_{036}, e_{057}, e_{058}, e_{059}, e_{062}, e_{065}, e_{084}, e_{092} and e_{093}, i.e. the events "yaw motor failure" and "abnormal vibration H" must be studied with detail because they probably cause a tower or foundation failure; the events "abnormal vibration A", "hydraulic motor failure", "leakages in hydraulic system" and "abnormal vibration C" are usually involved in a critical rotor failure; the events "short circuit (generator)", "open circuit (generator)", "short circuit (electronics)" and "corrosion" are prone to be the cause of an electrical failure; the occurrence of "abnormal vibration D", "overheating gearbox", "abnormal vibration J" and "cracks in high speed shaft" are the most probably causes of a power train failure.

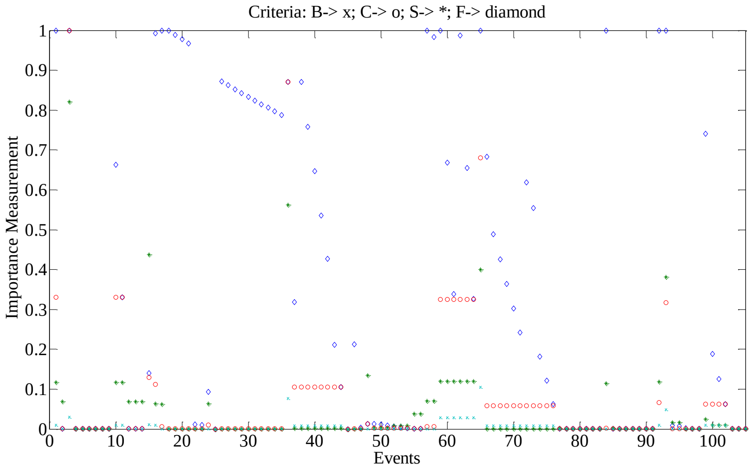


Figure 5.20 Importance measures for the WT.

Importance measures are limited to a specific point of time as Figure 5.20 indicates. For this reason, a novel dynamic simulation has been done in order to extend the analysis to a certain period of time. The literature does not include the values of the failure probabilities of the basic events and the WT operators are reluctant to publish it. Moreover, the nature and conditions of the events considered in the dynamic FT analysis could be very different. Consequently, several probability models are used for this purpose. The time-dependent probability models showed in section 5.1.1 are considered in this paper to describe the behaviour of events throughout time.

The ANNEX IX shows the fault probability functions assumed for each event. The experiences of wind turbine operators involved in the NIMO [79] and OPTIMUS FP7 European projects [80] have been considered in order to set the parameters of the time-dependent probability functions. The main purpose of this study is to show an example as close to reality as possible. This model could be adjusted to the specific wind turbine analysed, or to specific components.

Figure 5.21 shows the failure probability assigned to each event throughout time. This probability has been obtained for 600 samples where each sample represents one day. The events of the FT have different behaviours according to their nature and the values of their parameters.

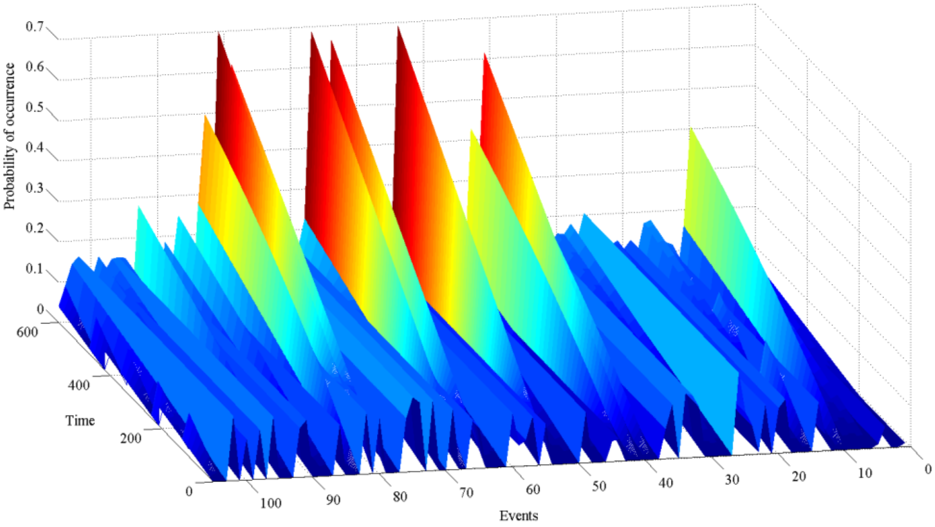


Figure 5.21 Probabilities of occurrence of the events over the time.

Figure 5.22 presents the probability of failure of the wind turbine ($Qsys(t)$) over the time. It is not continuously rising because there are events involved in preventive maintenance tasks, defined in Appendix I as periodic functions.

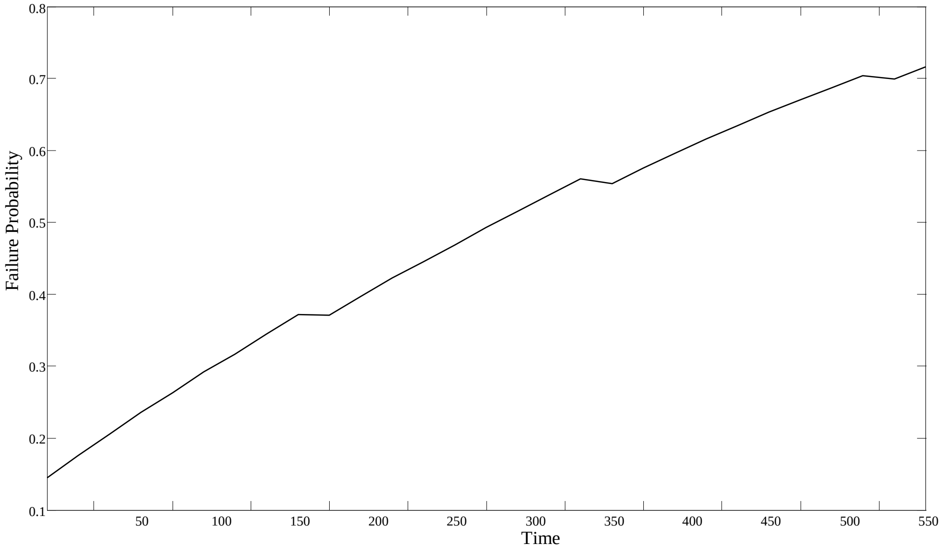


Figure 5.22 Probability of WT failure (Qsys(t))

Figure 5.23 shows the IMs employing the methods Birnbaum (B), described in Section 3.3.2 and applied to the FT in ANNEX IX. The events e_{084}, e_{036}, e_{065} have the highest IM according to the Birnbaum criterion over the time, these events should be studied in detail because the method provide a large IM value. There is a set of events with a significant IM over the time, such as events e_{077}, e_{085}, e_{093}, e_{092} and e_{003}. The rest of the events present lower Birnbaum IMs, i.e. they are usually less involved in the occurrence of the top.

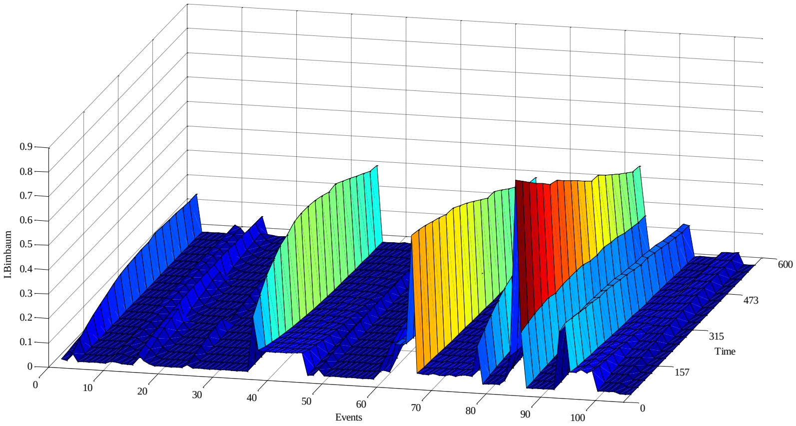


Figure 5.23 Birnbaum importance over the time.

The analysis leads to dynamic decisions from a quantitative point of view, enabling WT diagnostic and prognostic tasks to be carried out efficiently. Therefore, scheduled maintenance strategies can be implemented more effectively. The behaviour of the system over time allows operators to obtain optimal maintenance decisions since identified components can be repaired or replaced based on their effect on the global system.

For example, considering the maximum allowable probability of system failure of 0.5 (Figure 5.22 shows that this value is reached at the 300th sample), it is ensured that the unavailability of the system is under control until the mentioned sampled, and it is recommended the maintenance tasks required before reaching that undesired value. Once the system is in the critical iteration in which the maximum allowable unavailability is reached, it is necessary to act upon the components in order to reduce the failure system probability. Figure 5.23 provides useful information about how to focus the efforts to reduce such probability. Figure 5.24 corresponds to a cross section of Figure 5.23 and it shows the Birnbaum I.M. of the events at the 300th sample.

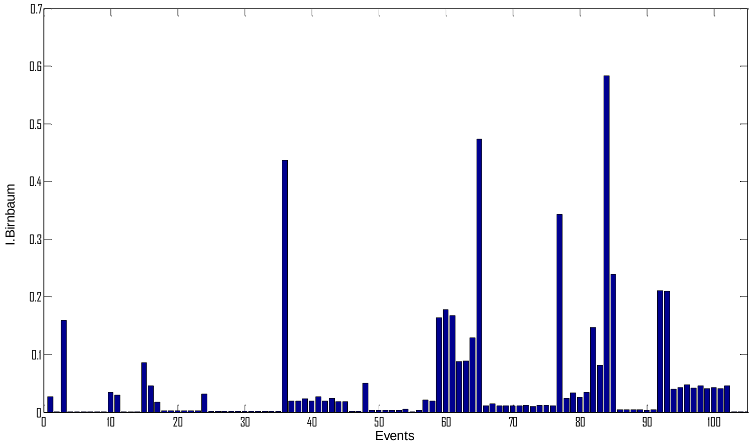


Figure 5.24 Birnbaum importance in a certain time

According to Figure 5.24, the most relevant information is the ranking of events that can be gathered from the Birnbaum I.M. The first three events that should be taken into account to plan a maintenance strategy are the events e_{084}, e_{065}, e_{036}, i.e. corresponding to overheating gearbox, and abnormal vibrations.

5.2.3 Maintenance for Wind Turbines

Due to the importance of the O&M tasks, a lot of studies are being developed in order to optimize them [114]. The early detection of possible failures of different components allows for reducing the losses of energy, the downtimes, the O&M costs and, consequently, the cost of energy (COE) [115]. With this purpose, multitude of sensors and systems are installed in the wind turbines, e.g. CM or SCADA systems [23]. The efficiency of this systems has been proved in several research studies [116],[117].

This research work proposes a maintenance planning that aims to maximise the RAMS of the offshore wind farms optimising the resources such as human or material, conditioned to exogenous variables, e.g., weather conditions [19]. This approach is based on the probability of failure of each WT. The operation of the WT will be focused on a set of components collected by a FT (see Appendix 2). The fault probability of any component is simulated by a statistical function of failure probability over the time (see Appendix 1). Then, the failure probability of a WT is set by the Boolean expression obtained from the BDD. Therefore, according to the resources, the maintenance task will be done in the WTs that present more fault probability over a threshold set. It will lead to predict any preventive/predictive maintenance task over the time. The importance measurements will determine the components that need a maintenance task. A low probability threshold is set to determine if the fault probability of the WT is under control or not. The importance measurement is calculated with the Criticality IM method. The downtime can be defined as the period of time that is required to carry out the corresponding maintenance task. Each event of the FT has associated one maintenance task with a specific downtime. The downtime depends on endogenous and exogenous variables. Figure 5.25 shows the flowchart of the procedure maintenance management.

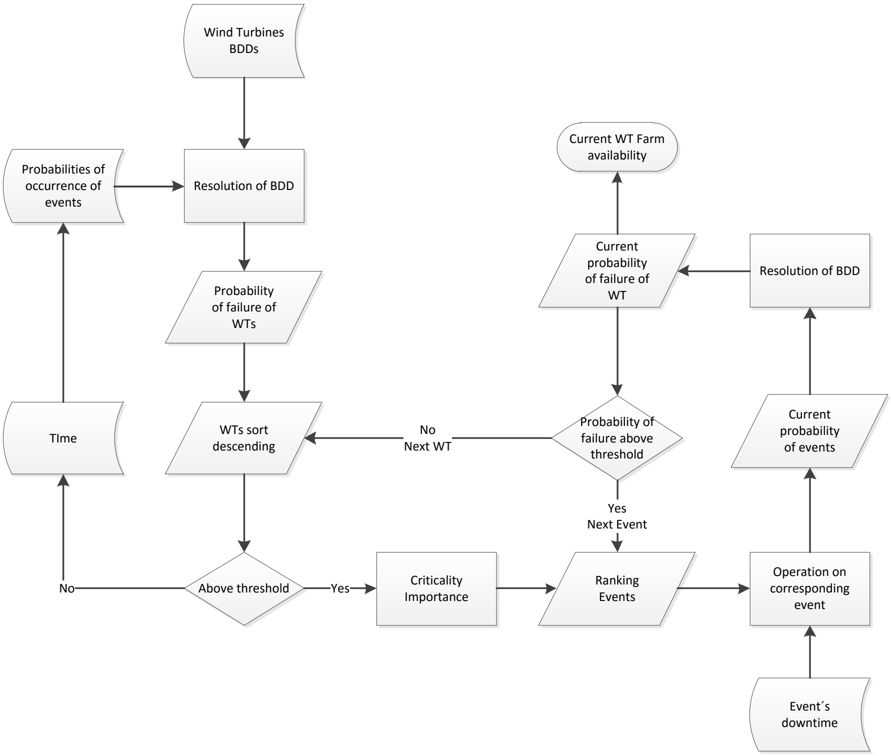


Figure 5.25 The maintenance management procedure.

An offshore wind farm composed by 20 WTs has been taken into account. The offshore wind farm has been designed taking into account considerations from expert of the NIMO and OPTIMUS research projects. It has been designed in order to demonstrate and validate the approach proposed in this paper. The WTs are the same type, with the same FT. Different mathematical models explained in 5.1.1 have been defined for each event (see ANNEX IX). These models have been based on time-dependent probability functions to describe the behavior of events over the time. These probability models are not intended to match exactly the real behavior of the events because there is no dataset to validate it, therefore it they have been set by the aforementioned expert. For example, the event e_{006} corresponds to the corrosion of the foundation or tower, where a linear increasing probability have been assigned to this event, this is due to the salinity that is assumed to be constant over the time. The main novelty lies in the procedure to elaborate qualitatively and quantitatively a preventive maintenance planning process based on the knowledge of the WTs and on statistical data that, for example, could be collected through CMSs [118],[119].

Figure 5.26 shows the probability of the events of one WT over the time taken into account the probability function assigned to each event. The simulation has been carried out for 600 samples, where each sample can be considered as a period of one day.

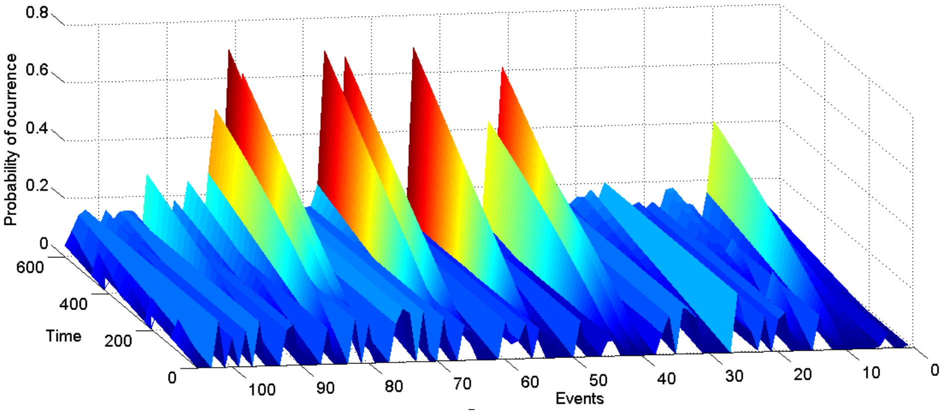


Figure 5.26 Occurrence probabilities of events

The objective is to propose an algorithm able to collect stochastic information of the failure probability of a complex system.

Considering the last probabilities obtained for each event and the analytical expression of the system failure provided by the BDD, the probability of failure for all WTs of the offshore wind farm can be achieved. Figure 5.27 presents the failure probability of each WT over the time. The probability of failure for each WT is different among them and over the time, because the values of the parameters that represent the occurrence function of each event are not exactly the same.

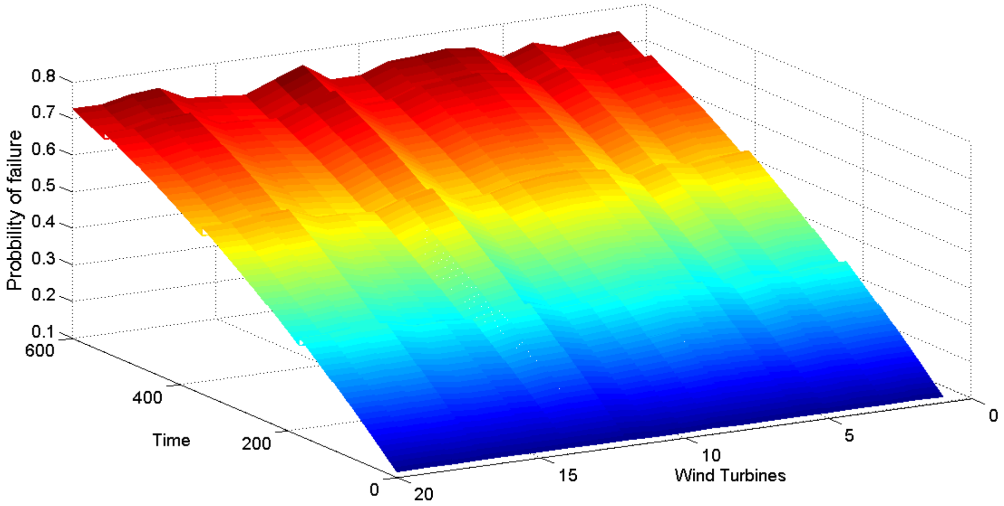


Figure 5.27 Probabilities of failure of each WT over the time.

The components that require any maintenance task have been set by the importance measurements, specifically by the Criticality IM method. Figure 5.28 shows the criticality importance of the events of all WTs considered in this case study in a period of time (in this case the study has been considered for a total of 600 days).

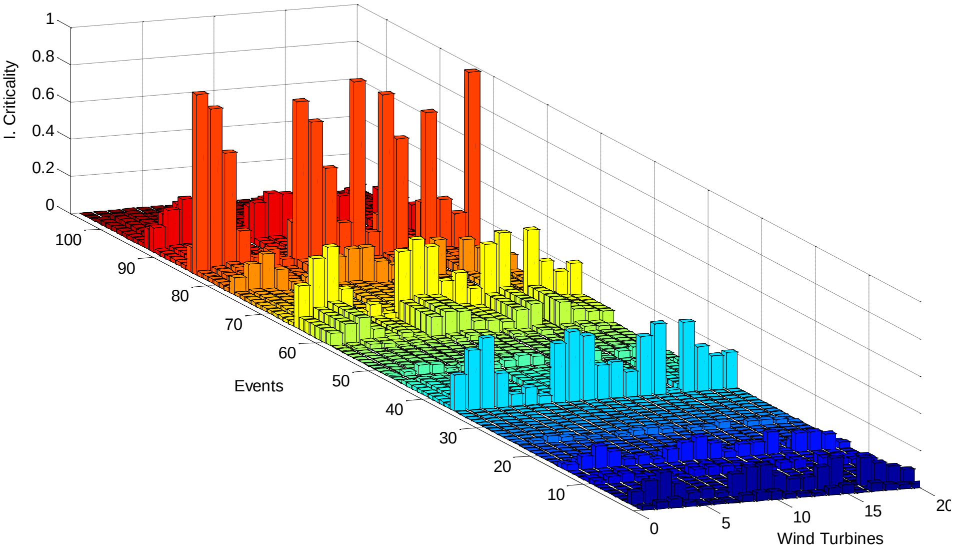


Figure 5.28 Criticality importance of the events in a given time.

The exogenous conditions such as maintenance budget, human and material resources and weather conditions will determine the downtimes, together with the time required to carry out any maintenance task. Figure 5.29 shows the fault probability over the time of a WT considering different maintenance polices. An upper probability threshold of 0.20 has been established to suggest when the maintenance must be started. Moreover, a lower threshold of 0.15 has been set indicating when the maintenance should be finished. The availability of resources will lead to attend to one or several WTs at the same time.

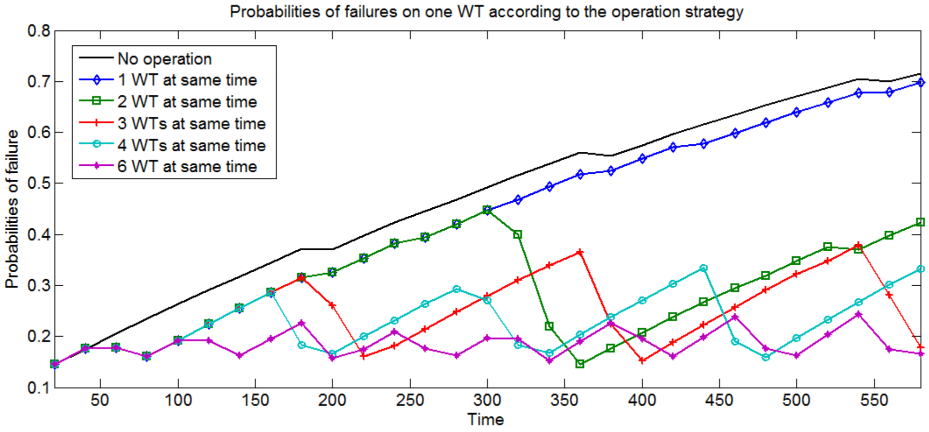


Figure 5.29 Probabilities of failure of a WT.

The average fault probability of the offshore wind farm according to the resource employed is illustrated in Figure 5.30. The probability decreases when the potential of maintenance tasks is bigger. In this case study, the average fault probability of the offshore wind farm decreases faster when it is attended at the same time 2 instead of 1 WT, then 4 instead of 3 WTs. The main conclusion is that a correct resources use could optimize the average fault probability of the offshore wind farm.

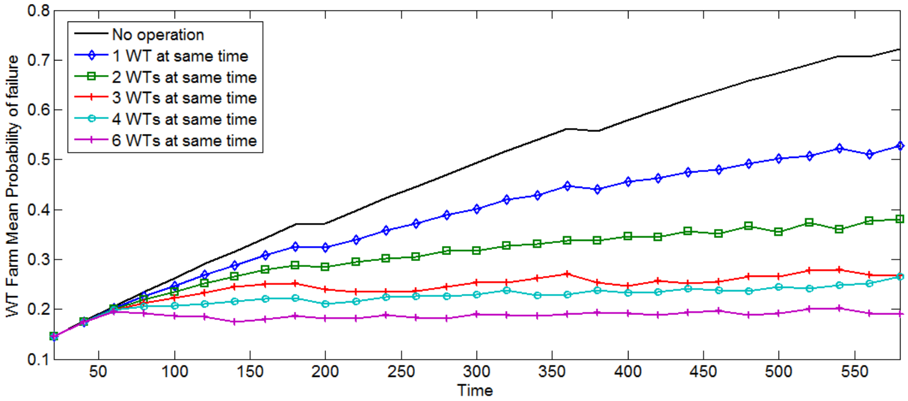


Figure 5.30 Average fault probability of the offshore wind farm.

The boxplots of Figure 5.31 show the behavior of the offshore wind farm for different maintenance management policies. The approach lead to control the average probability of failure by a correct maintenance police, and the boxes to be smaller, *i.e.*, presenting a homogeneous probability distribution in all WTs.

The maintenance management performance for offshore wind farms is subject to several uncertainties related to the randomness of exogenous conditions, e.g., weather conditions [120]. Therefore, the approach presented requires weather forecasting. Weather forecasting depends on the temperature, dew point, wind velocity, pressure, visibility, cloud height and quantity [9]. In addition, the state of the sea, the wind and the wave heights need to be considered. There are some probabilistic models based on historical wave height data that are used to determine the conditions of the sea in a certain moment, e.g., the

Markovian wave height model [121], forecasting of safe sea-state using finite elements method and artificial neural networks [122], short-term predictions based on nonlinear deterministic time series analysis [123], Gaussian processes [124], resampling methods, parametric models, *etc.*

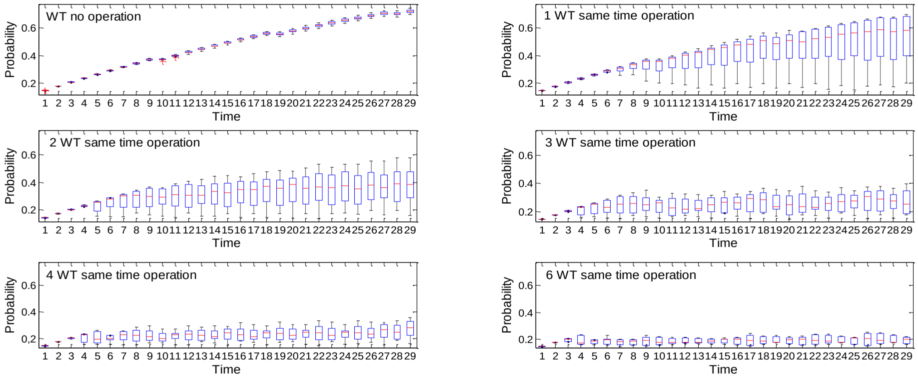


Figure 5.31 Boxplot of the fault probability of the offshore wind farm for WT operated at the same time.

The maintenance task will be carried out when certain permission value is reached. This dimensionless value, which varies from 0 to 1, will be given by a weighting of the weather conditions and external permissions. It has been simulated in this paper and validated by experts. Figure 5.32 shows the maximum allowed value assigned to each event. The maximum allowed valued is randomly generated for this case. It is due to the goal of this study is to clarify how the proposed methodology should be applied, taking into account that the method is close to the reality only from the qualitative point of view. This value is compared with a predicted value given randomly in this paper in order to consider the stochastic of the system. If the value assigned to the task is bigger than the predicted value, the maintenance task must be carried out, in other case, it must be necessary to wait for a suitable value from the forecasting.

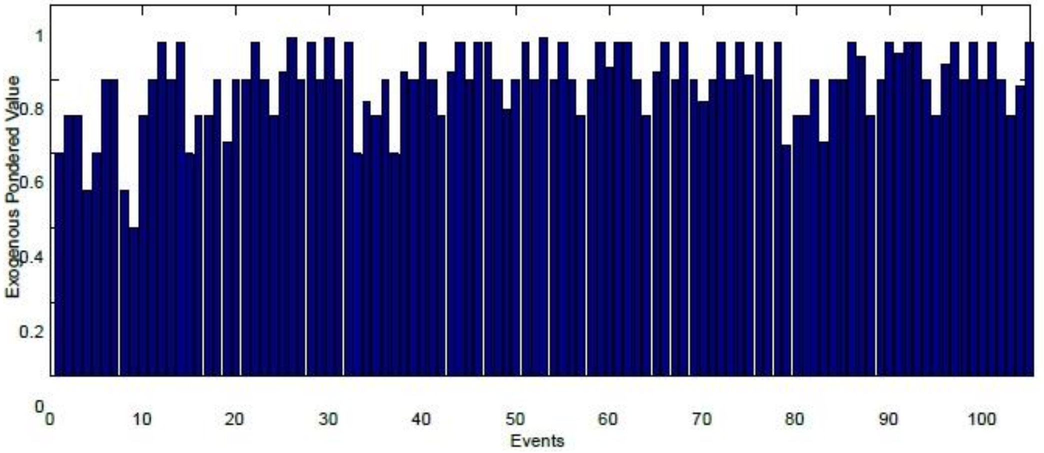


Figure 5.32 Maximum allowed exogenous pondered value for each maintenance tasks.

Figure 5.33 shows a randomized forecasting value of the weather conditions given for each day (sample) evaluated in the example. This figure can be used to determine the tasks that can be performed according to the exogenous variables. For example, in the 100th day (green circle) there is a value of 0.2 (this value is a ponderation between temperature, dew point, wind velocity, pressure, visibility, *etc.*), *i.e.*, any maintenance task can be carried out because this value is lower than all the maximum allowed exogenous pondered values. However, in the 300th day (red circle) none of the tasks can be carried out because the value is higher than the allowable value in all the cases.

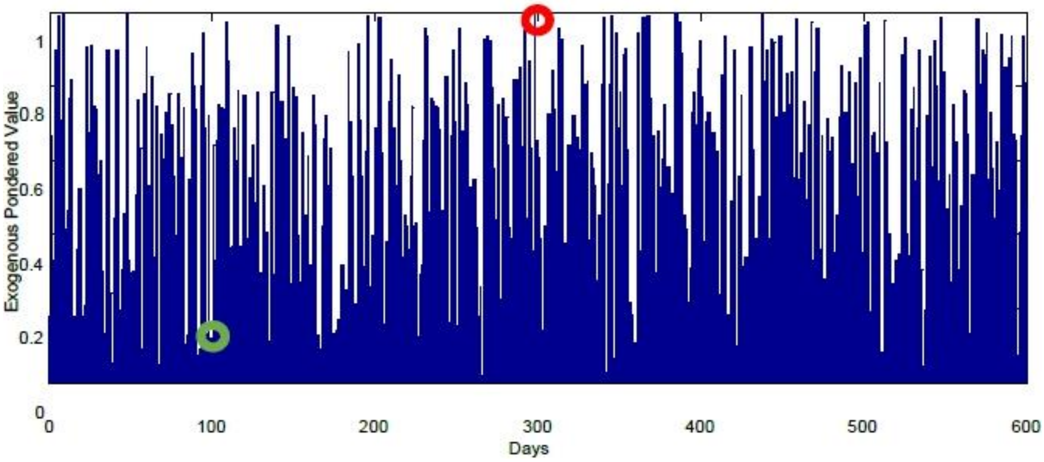


Figure 5.33 Representative exogenous pondered value forecasting per day

Figure 5.34 represents the weather influence on the distribution of the failure probabilities of the WTs over the time. Different weather scenarios have been taken into account randomly in order to evaluate the weather conditions and the influence to the maintenance tasks.

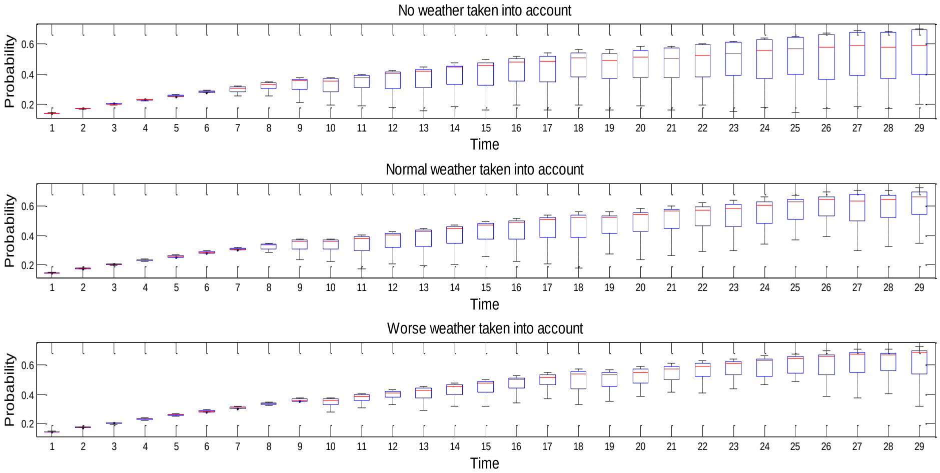


Figure 5.34 Influence of exogenous variables on the state of the offshore wind farm.

In the top boxplot of Figure 5.34, the weather conditions have not been taken into account. In the second one, the weather forecasting presented in Figure 5.33 has been considered. In the last one boxplot, an adverse weather conditions have been established. The presence of adverse weather conditions makes to increase the average fault probability of the offshore wind farm, and the size of the boxes of boxplot decreases because the maintenance tasks that can be done are minimum.

5.3 Case study. Diagnostic and Prognostics in Railways

This section presents a qualitative and a novel quantitative analysis of the point mechanisms, a critical component of the railway networks. It will lead to perform diagnostics and prognostics of these mechanisms based on the health monitoring systems, i.e. it will allow to evaluate the state of the system in a desired moment and to forecast the conditions of the mechanisms. The main objective is to increase the reliability, availability, safety, maintainability

(RAMS) in these mechanisms. A novel approach for maintenance management based in FT analysis is proposed.

A real case study in a point machine (M63) has been considered. The results provide useful information for supporting the operation and maintenance tasks, and also for a correct planning for diagnostics and prognostics. The approach considers a methodology to establish a maintenance planning, being a flexible and simple method and taking into account a non-linear system that leads to an optimal resources allocation.

5.3.1 Introduction

A train can move from one track to another only in certain places employing mechanical devices called "turnout". The turnout has moving parts, called switches (US: blades), and which steer the trains in one of two directions, normal (straight through) or reverse. The locking devices for the switch blades are employed to ensure correct and safe operations of points.

The research works done in point mechanisms are not enough related to the terrible consequences of point mechanisms failures on the railways. The objective of the automatic detection [125], failure analysis and diagnosis [126], or wear assessment [127] and [128], is the use of CM equipment to analyze the various operating profiles some of which, can vary considerably when switching either from normal to reverse (N-R) or vice-versa (R-N) – especially if faulty [16], [126],[128], [129] and [130].

The operating force profiles of such mechanisms under even normal, fault-free operation are not particularly repeatable. Even from hour to hour – and certainly day to day – changes in environmental conditions (such as humidity or temperature) will mean that the various profiles will differ from operation to operation. From month to month, continuously varying amounts of wear in the components – all of which may at particular maintenance intervals be lubricated, reset or replaced – means again that one profile cannot be expected to be the exactly the same as another.

Approximately 55 % of railway infrastructure component failures on high speed lines are due to signaling equipment and turnouts. “Signaling equipment” covers signals, track circuits, interlocking, automatic train protection (ATP) or track loop based ATP (LZB), and the traffic control center. The annual cost of maintaining points is rather high compared to other infrastructure elements, about 3.4 million UKP (United Kingdom Pound) per year for about 1000 km of railway. TC-TCR trade circuits, for example, cost 2.1 million UKP per year for the same area. Of the points expenditure, 1.2 million UKP is for clamp lock type (hydraulic) turnout and 1.4 UPK million for electrically operated turnouts (data provided by a British asset manager).

Most standard point machines (*see Figure 5.35*) contain a switch actuating and a locking mechanism which includes a hand-throw lever and a selector lever to allow operation by power or hand. The mechanism is normally divided into three major subsystems: (i) the motor unit which may include a contactor control arrangement and a terminal area; (ii) a gearbox comprising spur-gears and a worm reduction unit with overload clutch; and (iii) the dual control mechanism as well as a controller subsystem with motor cut-off and detection contacts. Generally, there are also mechanical linkages for the detection and locking of the point. The standard railway point is therefore a complex electro-mechanical device with many potential failure modes.

The circuit controller includes detection switches and a pair of snap-action switches to stop the machine at the end of its stroke and to brake the motor electrically so that the mechanism is not subject to impacts. The detection switches have high pressure wiping contacts made of silver/cadmium oxide or gold and they are operated by both the lockbox and the detection rod. The detection switches have additional contacts to allow mid-stroke short circuiting of the detection relays to avoid wrong indications in the signal box or electronic interlocking.

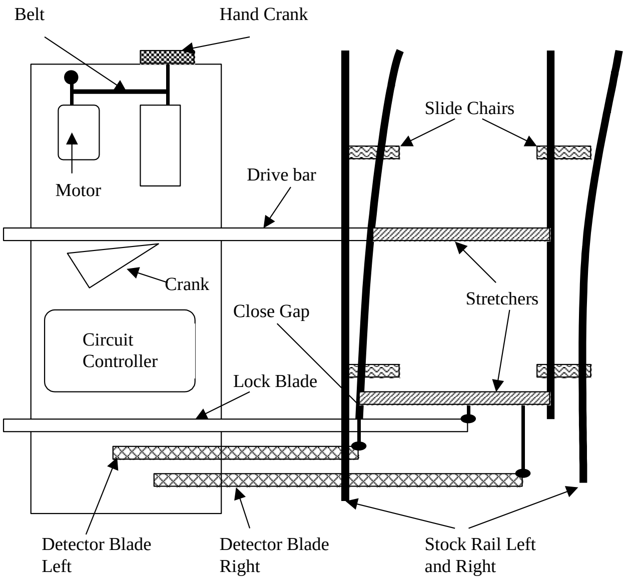


Figure 5.35 Point Mechanism

This paper proposed a novel approach in order to identify the critical components of the point mechanisms online, according to the probability of fault of each component over the time and the FT of the mechanisms. The study is based in a real case study carried out between the Sheffield University (UK), Castilla-La Mancha University (Spain) and Balfour Beatty Rail (UK).

5.3.2 Diagnostic Maintenance

Diagnostic leads to evaluate the state of a system in a certain time. It is done in this paper employing importance measures (IMs) for ranking the events and to show their relative importance over the top event probability. The IM methods used are Birnbaum and Criticality [44],[131] and [51].

Figure 5.36 shows the Birnbaum importance value obtained for each event using the probabilistic values given in ANNEX VIII, together with the FT. The FT is obtained from a research project mentioned in references [132],[133] and [129]. There is not previous context to explain the data-set, that has been calculated by simulations in this paper, because the objective of this paper is to propose a novel methodology to analyse and optimise the prognostics, diagnostics, and maintenance tasks. There is an important difference between the IMs of the events. It is recommended for diagnostics tasks to identify the events with more importance because it will lead to increase the accuracy of the diagnostic and, consequently, to set the facilities assigned to the maintenance tasks, to optimise costs, to increase the reliability, etc.

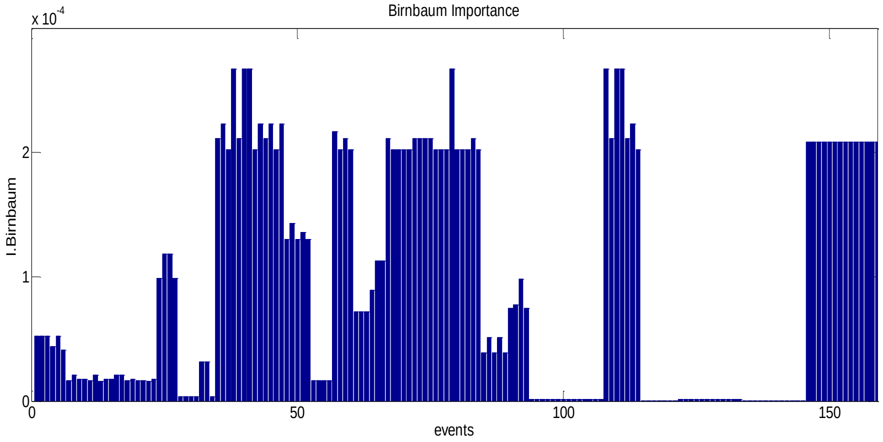


Figure 5.36 Static Birnbaum Importance

Figure 5.37 shows the criticality importance of all the events using the input probabilistic values and the FT showed in ANNEX VIII.

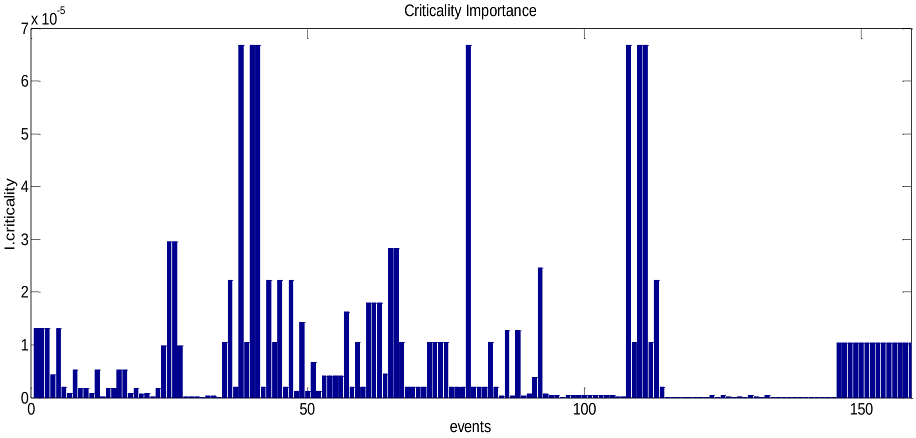


Figure 5.37 Static Criticality Importance

The differences between Figure 5.36 and Figure 5.37 are given by the probabilities of occurrence of the events. Figure 5.37 is avoid the identification of rare events as major events. The rare events can be identified evaluating the difference between Birnbaum and Criticality IMs. The quantitatively result analysis of the IM are only employed in order to classified the events, and to use them for the prognostics and diagnosis of the system.

5.3.3 Prognostic Maintenance

Prognostic maintenance leads to anticipate the occurrence of a possible fault. The objective in this paper is to support the maintenance management providing the necessary information in order to set the optimal maintenance tasks.

Figure 5.38 shows the values assigned to the parameters of the probability functions aforementioned of each event given in ANNEX VIII. Figure 5.38 plots the fault probability distribution of each event over the time. These data has been randomly generated for each event to make simulations of the state of the system over the time, however the probability functions have been set by the authors considering the engineering interpretation of each event. For example, the event 'e_2' is considered a linear increasing probability of occurrence over the time because it is assumed that the wear is constant, and the 'e_{11}' has a constant probability, i.e. it is taken into account that the oversize holes or the undersize bolts have always the same size. These functions allow calculating the probability of the failure system over the time via FTA. It is important to remark that in this paper the probabilities functions for each event would not be optimal. The probability function of each event should be obtained by condition monitoring, a statistical analysis, etc. The main purpose is to present an approach for analysing the Q_{sys} considering a stochastics system.

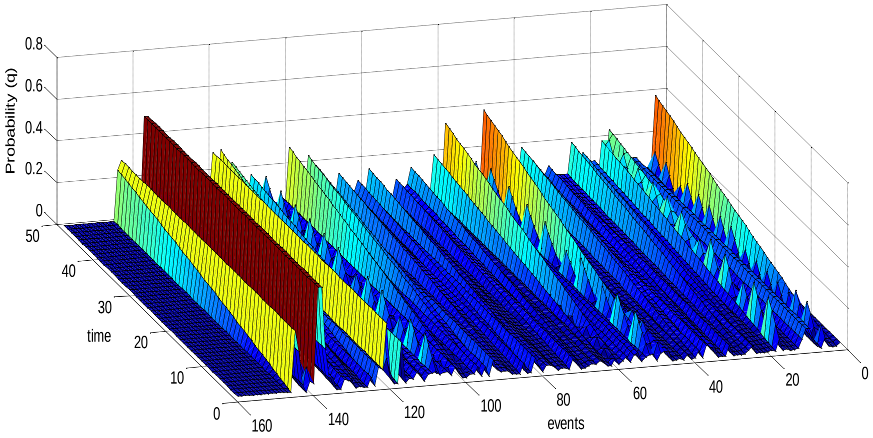


Figure 5.38 . Dynamic Probability for Each Event

The fault probability of the top event will increase over the time if there is not any maintenance task, according to the data showed in Figure 5.38. The event with an occurrence probability assigned as periodic, it will set to 0 in each period. Figure 5.39 plots the probability function of the top event over the time, with a general rising trend. The reason that is not always rising is because there are events with periodic probability functions. This probability has been obtained for 50 samples. The curve in Figure 5.39 can be employed in order to do prognostics, to fit the operations thresholds, etc.

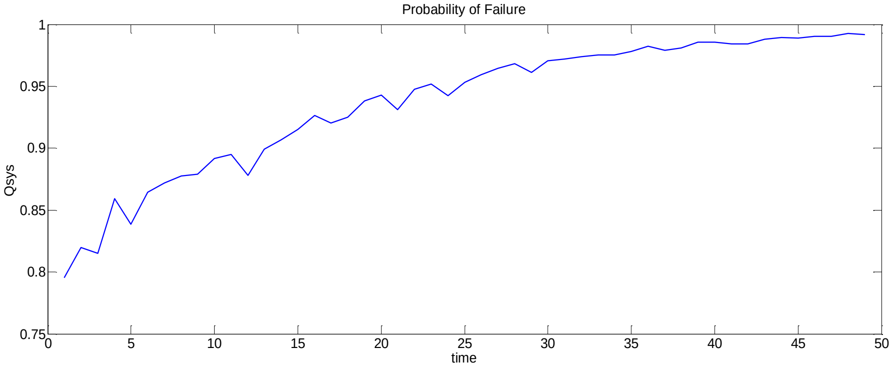


Figure 5.39 Dynamic Probability of System Failure.

The dynamic analysis proposed in this paper can facilitate to establish a maintenance planning because the probability of the system failure is available over the time. It leads to keep the reliability of the system under control.

This study uses the IMs in order to classify the events according to their relevance in the state of the top event in a certain time. Figure 5.40 shows the outcomes of Birnbaum IM calculated for all samples and all the events of the point machine detailed in ANNEX VIII. It leads to identify the critical events for each time in order to be considered in the diagnostic and prognostic tasks. For example, the event 'e_{120}' shows a low Birnbaum Importance compared to the event 'e_{59}'. The difference between them is bigger over the time, i.e. the event 'e_{59}' is more critical for the system in any time.

The most important information from this study is the ranking of the events in any time, because it will lead to set the diagnostic analysis of the state of the top event, e.g. it will help the operators to optimize the maintenance tasks and the cost assigned because they will have information about the critical events in this moment.

The methodology employed in this paper allows establishing a variable time increment over the time. It can be useful when the system needs to be studied in further detail in a specific period of time.

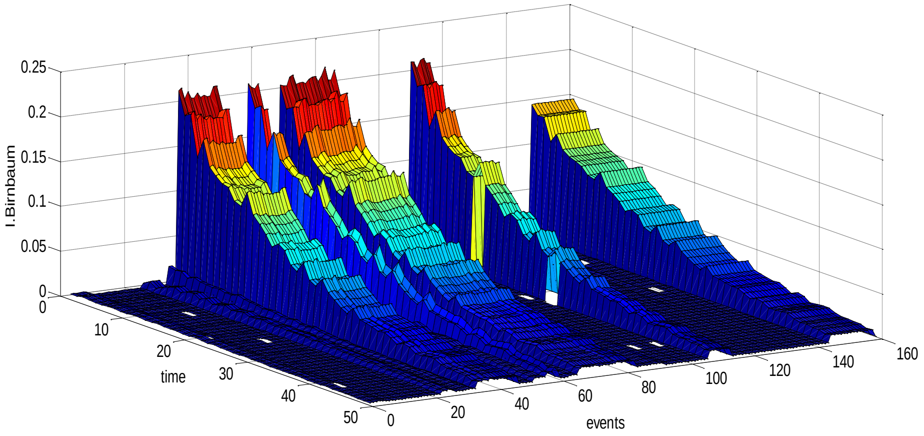


Figure 5.40 Dynamic Birnbaum Importance Measure

Figure 5.41 shows the Criticality IM for all the events, considering the probability of the events in each time. Different trends can be observed, e.g. the event ‘e59’ begins presenting a rising curve and then a descendant one. However, the event ‘e120’ presents a constant trend. When an event is analysed, the probabilities of the rest of the events are less important than in the Birnbaum IM.

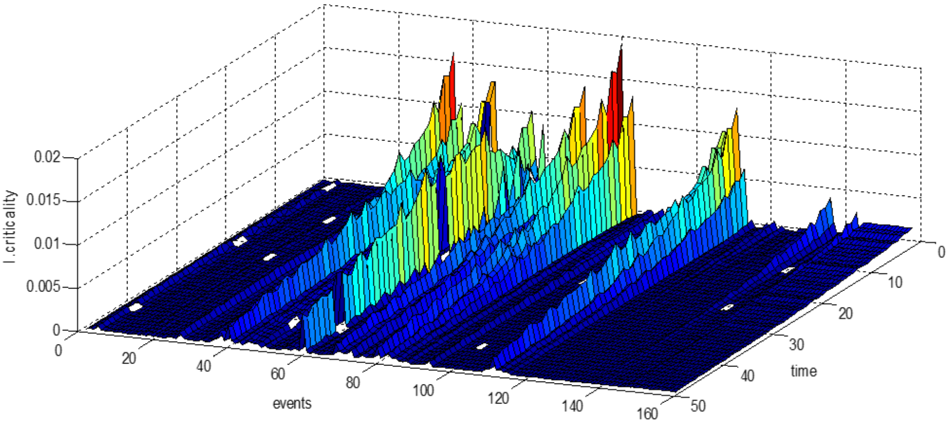


Figure 5.41 Dynamic Criticality Importance Measure

Birnbaum and Criticality IMs are complementary, but there is not any conclusion about what IM is better. It will depend if the event requires to be analysed according to the topology of the tree, where the Birnbaum IM is used. The Criticality IM is recommended in case that the event needs to be study regarding to the probability of the event and the system, together with the topology. In case that rare events require to be analysed, it will be employed the Criticality IM.

5.3.4 Preventive Maintenance Strategy.

The approach proposed in this paper provides quantitative results that facilitate the diagnostic and prognostic tasks. The diagnostic tasks are done setting the condition of the system and identifying the critical components. The objective of prognostics is to determine the state of the system in a certain time.

The information provided by IMs is employed for a diagnostic task. In case of system failure, the events that have a major contribution to the system failure probability can be identified. The data shown in Figure 5.37 allow classifying the components according to their importance and establishing a priority ranking of events.

The approach leads to develop a dynamic preventive maintenance planning. The results provided by the dynamic analysis lead setting the tasks according to the reliability of the system. The approach presented in hereby consists of the following steps:

- To set a threshold that determines the maximum unavailability allowed.
- To determine the cut-off point between the system failure probability and the established threshold.
- To identify when a certain system failure probability is reached.
- To determine the most important events at the critical cut-off point
- To set maintenance tasks in certain events in order to maximise the reliability of the system.

Figure 5.42 shows the system failure probability over the time obtained in section 5.3.3, where:

- The unavailability threshold represented by the horizontal dashed line.
- The cut-off point is marked with a circle.
- The failure probability (horizontal line) given in a certain time by the vertical dashed line. It corresponds to the thirteenth sample in Figure 5.39.

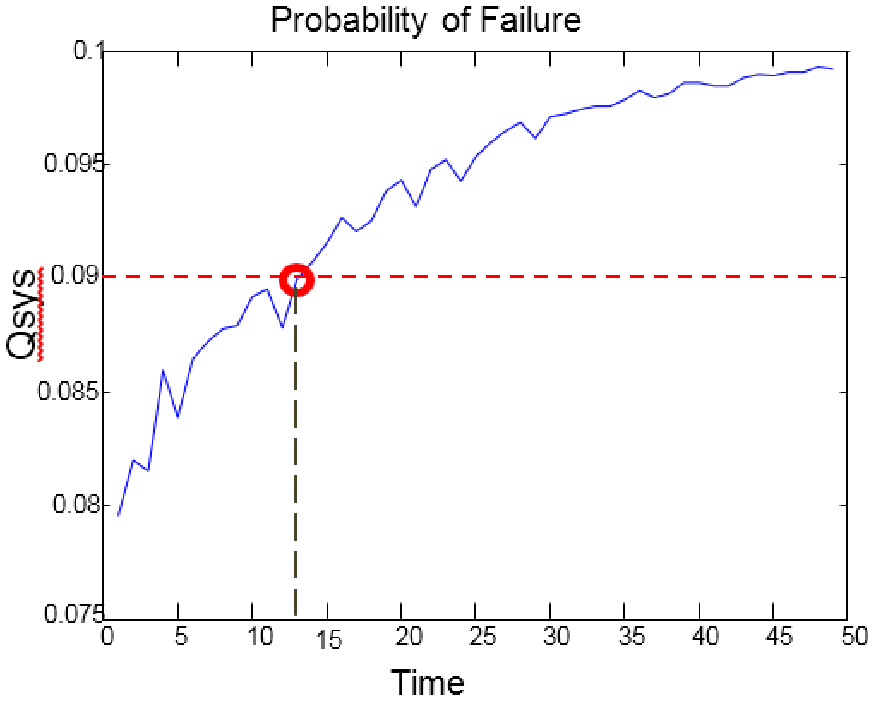


Figure 5.42 System failure probability over the time

Figure 5.43 presents a cross-section of Figure 5.41 at the thirteenth sample, where the most influential events are highlighted.

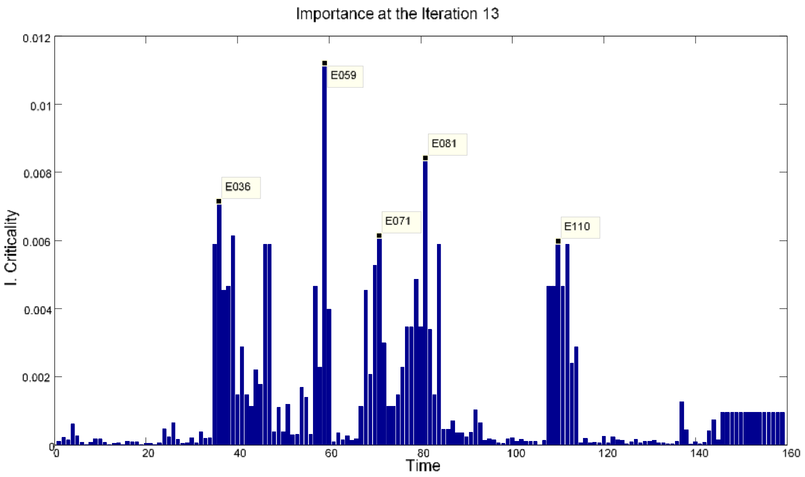


Figure 5.43 Dynamic Criticality Importance Measure

Table 5.9 classifies the events according to their importance into three groups: A, B and C. This classification is not necessarily constant, i.e. the size of the groups depends on the amount of resources to be allocated and the reliability to reach. This flexibility allows developing an optimised online decision making process by choosing a correct number of events automatically. The non-linearity of the system can lead to choose more events than necessary when a specific reduction of the system failure probability is required.

Table 5.9 Classification of Events According to their Importance (see Figure 5.43)

Group	Scope	Number of events	Events
A	3% more important events	5	e_{59}, e_{36}, e_{81}, e_{71} ,e_{110}
B	15% more important events	20	*A&* e_{35}, e_{37}, e_{38}, e_{46}, e_{47}, e_{57}, e_{60}, e_{68}, e_{70}, e_{79}, e_{084}, e_{108}, e_{109}, e_{111}, e_{112}
C	25% more important events	35	*A&B&* e_{41}, e_{44}, e_{45}, e_{54}, e_{55}, e_{58}, e_{69}, e_{72}, e_{76}, e_{77}, e_{78}, e_{82}, e_{83}, e_{113}, e_{114}.

The occurrence probability of the events is reduced to zero when they require maintenance tasks. Figure 5.44 shows the probability of system failure over the time. This probability decreases according to the components repaired. The number of events taken into account determines the period of time that the system will be below the threshold. It leads to solve this decision making problem dynamically because the criteria can be different over the time. Figure 5.44 has been obtained using a variable time increment: from 0 to 10 the time increment is five times bigger than the increment used from 10 to 23. This is an important advantage because the critical zone (dashed square) can be analysed in further detail.

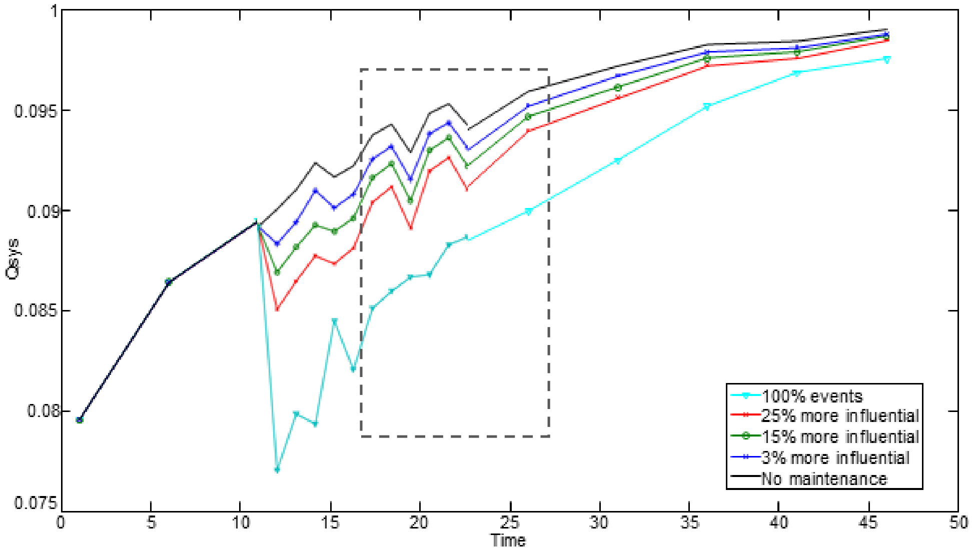


Figure 5.44 Applying Maintenance Strategy

The system failure probability reduction obtained in the fourteenth sample by the event set A, B, and C is 13.38%, 24.06% and 38.15%, respectively.

6 DECISION MAKING OPTIMIZATION

Chapters 3 and 4 presents a method based on the conversion from LTs to BDDs. This method allows obtaining an analytical expression of the occurrence probability of the top event in function of the probabilities of the basic events. This expression defines quantitatively the behavior of a whole system or process but it does not take into account the exogenous variables that can be more influential than those that belong to the LT.

Two novel approach are presented in this Chapter in order to take into account external factors that may be important for optimizing the decision making processes.

It is important to keep in mind that the notation for Decision Making explained in section 3.1 is used in this Chapter.

6.1 Decision making State of the art

Decision Making (DM) is a criteria selection method of the best alternative. It is daily used in a personal and professional context. There are a large number of times where DM is carried out, whereas some other times it takes weeks, even months in order to reach the best alternative.

Nonetheless, what does “to choose the best alternative” mean? In a DM scenario, it is commonly known there is an event is (not) desired to occur, which entail a path among the different alternatives which let to reach the objective

[134]. In any case, it is requested the optimal situation, i.e. the one which will provide the best results [135]. With this purpose, to select different criteria that allow discerning which the most advantageous situation is, emerge as a vital matter [136].

DM can be defined as:

“the research of identifying and choosing alternatives based on the decision-maker weighs/values and preferences. Make a decision entail there are several alternatives to be considered, and not only identify the alternatives is sought but also to choose the one that best fits the aims, restrictions, etc. is desired”[137].

DM consists on the transformation process from data to proceedings [138]. The data collection is a strategy for DM, and consequential proceedings are possible to be carried out. Feedback is obtained with mentioned proceedings, which actually helps to keep improving the problem giving more data to the system. It suggests DM requires a continuous communication process where data obtained from the proceedings carried out leads to an improvement of the available data.

DM must be carried out when a certain problem occurs. It would be desired to be able to discern whether there is a real problem in a business. If a mismatch is found, to try to solve it by focusing on finding the difference between the real and the ideal/desired situation is proposed in reference [139]. The main role of the DM in mentioned scenarios is to reduce that gap.

Decision may be defined as “a thorough selection of proceedings among available alternatives, aiming a desired result, knowing the resources are limited”[140].

There are several alternatives to classify the decisions performed in a business. According to the needs developed in this research study, Figure 6.1 shows the classification most used.

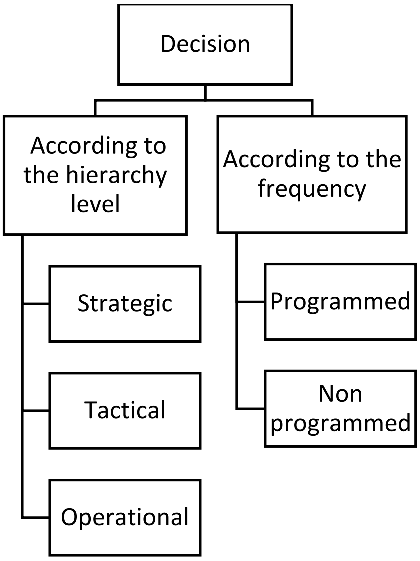


Figure 6.1 Decisions Classification

Strategic decisions are those defined by the objectives and proceedings lines to be followed by the business. There is a shortage in data and its impact on the business is crucial. Wrong strategic decisions may lead to the business's bankruptcy. A high degree of knowledge is required and in consequence senior managers are responsible for this kind of decisions.

Tactical decisions occur with major incidence than the previous ones. A procedure and routines are developed in order to control these decisions. Enough data about how to analyse them is commonly available. Wrong tactical decisions may bring troubles to the business, but a solution could be given without the business bankruptcy. Lower range managers are usually responsible for these decisions.

Finally, the operational objective is to carry out strategic decisions. Junior managers and also workers are generally responsible for them. Wrong operational decisions have not far-reaching implications for the future and may be fixed easily.

According to its frequency, on the one hand, programmed decisions are those repeated frequently in a business, where a procedure is developed to carry out every time it occurs.

On the other hand, non-programmed decisions are those emerged unexpectedly. Decision makers need to take decisions employing their awareness and experience. Usually it involves a high degree of difficulty and they must be done by experts in the field.

Decision Making (DM) processes can involve a large number of variables, increasing the complexity and difficulty of qualitative and quantitative analysis. The main issues considered in this paper for the DM processes are the decision maker, the main problem, the scenario, the constraints and the consequences of the decision.

The decision maker is the person, system or organization that makes a decision. All decisions and assessments will be influenced by the decision makers. Decision maker should possess some essential skills including: experience, good judgment, creativity and quantitative knowledge. The first three skills are personal, and the final skill is supported by existing methods and support systems for DM. These DM support systems are used in order aid decision makers in choosing between several alternatives and, consequently, to help the decision maker to decide what alternative is the best [141],[142],[143]. This paper presents and describes a quantitative method to support the DM process.

The DM process described in this paper is focused on a main problem, which represents an undesired event, called Main Problem (MP), whose occurrence probability (Q_{MP}) needs to be minimized. The logical structure of the MP is approached by a LT that in this case is named Logical Decision Tree (LDT) and is composed by basic events that represent possible causes of the MP, called Basic Causes (BC).

The following main scenarios can be distinguished according to the information available in the DM process:

- *DM under certainty*: The problem is entirely known and all possible states for all the variables are known and any consequences of each decision can be completely achieved.
- *DM under risk*: Implies partial information and some information to the problem is stochastic. This will be the scenario considered in this paper.
- *DM under uncertainty*: Information about the MP and its causes is not complete and part of the information is missing [144].

The scenario developed in this paper corresponds to a DM under risk, where probabilistic values are assigned to the BCs. These probabilistic values are assumed by the following functions:

- *Classic probability*: It can be defined as "a priori" probability, based on a rationalist point of view and calculated deductively without the need to conduct an experiment [145]. For any event A it can be obtained by: $P(A) = \frac{m}{n}$,

  where A is satisfied by exactly m of n possible outcomes. This approach is not recommended to be applied when outcomes are not equally likely or when all possible outcomes cannot be taken into account.
- *Frequentist probability*: It can be considered as "a posteriori" probability and related to an empiricist perspective. While classic probability focuses on deductive reasoning, frequentist probability focuses on inductive reasoning [145]. It is defined for a generic event A as shown in equation 6.1:

  $$P(A) = \lim_{n\to\infty} \frac{m}{n} \tag{6.1}$$

where m is the number of times that A has been satisfied, and n is the number of times that the process has been performed.

- *Subjective probability*: This probability is also called Bayesian probability (degree of belief mapped onto the unit interval [0, 1]) [146], representing a mode of judgment. Therefore, it is closely related to the experience, beliefs, feelings and interests of the person who estimates the probability, i.e. Bayes' theorem constitutes a learning mechanism to approach unknown quantities of interest. This probability allows for the addition of new information to the 'a priori' available probability in order to actualize the probability estimation once new data have been obtained. It is defined by equation 6.2, where A and B are events and $P(B) > 0$:

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)} \tag{6.2}$$

where $P(A|B)$ is the probability for A to be true if B is already true.

In general, DM processes are not completely reliable due to their inability to take into account the total range of events involved in the solution. The goodness of a decision can only be known in an 'a posteriori' evaluation of consequences. Particularly, the evaluation of consequences is essential for improving those DM processes with data from forecasting studies. The results derived from a decision can affect the very structure of the problem or modify some features of the constraints and requirements. Feedback is necessary in order to determine the quality of the decision because the decision maker must know if the system responds as expected. This feedback is the only way for the decision maker to know whether the method brings the problem close to the reality or not. Moreover, there are some decisions that need to be made periodically and feedback is useful in order to improve the quality of the new decision, compared to the preceding decision. Figure 6.2 shows how decision making is carried out using LDTs and BDDs.

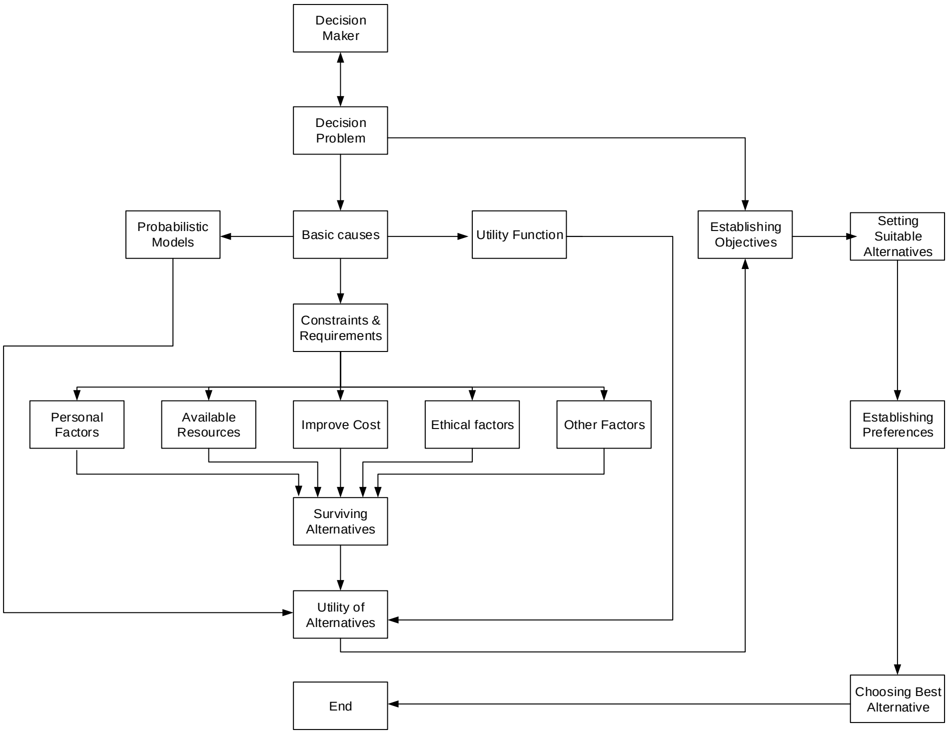


Figure 6.2 DM process

Every DM process has a number of constraints or requirements to consider, e.g. existing resources, available budget, environmental precautions, social issues, legal provisions, etc. Generally, the constraints are exogenous and not subject to mathematical or empirical models. An endogenization of constraints might be carried out in order to consider them. This endogenization involves conceptualizing the constraints as goals of the decision-maker, making it been possible to reformulate a constraint as an objective.

A lot of studies present different models for decision making, e.g. cost-benefit analysis; elementary methods [147] such as pros and cons analysis, maximin and maximax methods, conjunctive and disjunctive methods, lexicographic method; simple multiattribute rating technique, generalized means, the analytic hierarchy process, Outranking methods; ELECTRE and PROMETHEE [147]; the fuzzy preference relations [148]; cognitive decision–making models [149]; large group decision making methods [150]; etc. Moreover, there are a large amount of studies about decision making processes under risk contexts. Some of the most important models are collected in references [151], [152] and [153].

The methods proposed in this paper attend to a new way of solving decision making problems through the linkage of graphical and mathematical tools. The first method (mathematical optimization approach) uses the expression provided by a BDD as an object function for a programming problem. This is a novel use of a Boolean function. Regarding the second method (Birnbaum-Cost Measure Method), the novelty lies in the adaptation of the Birnbaum measure in order to consider possible decision variables such as costs. Both methods are limited by the possibility of building a LDT that represents the problem with a correct accuracy.

It is important to remark that these methods can be applied to other decision problems [72], e.g. at the design stage of products [115], to elaborate preventive or predictive maintenance plans [114]. In general, they are adaptable to those problems that can be logically defined and several alternatives can be considered.

This research work is focused on expected-utility DM under constraints that can be considered as a process that provides a solution with two objectives: to satisfy the constraints and rule out unfeasible solutions and to optimize objective functions among the surviving options [142]. For a quantitative stochastic DM case, the objective function provides a value that determines the goodness of the solution considered. It is formulated as an analytical expression gathered from a BDD. Some thresholds or constraints might be established in order to determine the solutions that are most suitable. Those alternatives remaining outside of those thresholds can be directly ruled out.

6.2 Management Planning and Control

DM is a vital method whenever the optimal solution to a given problem is given. Nevertheless, the DM would not be able to solve the problem by itself due because a planning must not only be considered into account, and taking control over the ongoing proceedings. This is where Management Planning and Control does its main role.

Planning is *"one of the chief administrative roles, which is circumscribed as the rational process of DM to select the right future course among different alternatives. Its fundamental goal is to narrow the gap between the current situation and the desired one"* [154].

In order to have a notion of the problem's magnitude, planning is very important because it shows the feasible solutions and make available a pathway in consequence. With regard to this planning, it will be possible to take part in the DM and act in accordance with it.

Planning seeks to solve the following questions:

- Which is the main objective and which is the process to achieve mentioned objective?
- How is it possible to trace a start-up plan to achieve the objective?
- Which departments will be involved and what resources will be needed?
- Is it worth the money/resources and time investment over this problem?
- What will be the impact on the business?

On the other hand, control also plays such a main role. A precise planning without a subsequent control may do the mistake of a mismatch in the business objectives because it will not be totally clear whether the planning is performed right or wrong. Therefore, even though the best planning is being carrying out, if it is not managed by the control and it will become completely ineffective.

Business's control can be classified depending on the course of action [136]:

- Previous Control
- Constant/Actual Control
- Subsequent Control

Previous Control is fount to prevent the problem. A start-up and revision procedure is needed to carry out the planning precisely in order to perform mentioned prevention.

Actual Control verifies whether the Planning is being achieved in the same manner as it had been formerly established.

Finally, subsequent control evaluates the obtained results and crosscheck whether they are as expected.

Control process is close linked to the existence of the following variables [155]:

- An indicator set which allows orientating and evaluating each department performance.
- A predictive model that make possible to the prior estimate the result of an action which will be carried out by the managers.
- A business strategy.
- Some department information about the performance and result of its operation.
- The appraisal of each DM made by any department in the business.

Furthermore, a procedure must be carried out to get the results as tight as possible to reality. The procedure considers:

- To gather as much data as possible.
- Compare and contrast the data against the established goals.
- Finally, whenever a mismatch is found, execute corrective measures.

Management control is defined as a complete set of techniques used to regulate properly that the Planning is being achieved precisely. *“Management* *control main purpose is to handle and monitor the goals, plans, management programs and decisions, needing continuous information obtained through the specific areas and techniques"*[136].

Management control system is considered a system whereby data and control are close linked to the resource management and business. Its main purpose is to assure that the objectives have been successfully completed and to propose corrective measures whenever it is needed.

Moreover, management control must be understood as a dynamic control, i.e. a system capable to change in time and to be able to respond to continuous changes. A business having a precise management will be more efficient and effective, and it will be also able to deal with the ongoing problems as they appear.

Control systems, particularly those that are intended to analyse companies' situations similar to the ones above explained, must be processed as dynamic systems, i.e. exogenous influences must be taken into account when its influence becomes more important.

Management control will turn into the most important approach. It helps to the quantitative analysis in DM and it is also in charge of controlling the whole process. It helps to understand the simulation' results, and it provides a framework which encompasses this kind of business issues.

6.3 Decision Making procedures proposed

An optimal investment, subject to the limitation of resources, is the main objective of the DM process hereby considered. This paper presents two methods for supporting investments and resource allocation in a constrained risk environment. These methods are based on the application of LDT and BDD as an approach that allows quantitative analysis of a qualitative study. The scenario considered in this paper is a decision making process under risk environment, where stochastic variables are considered. The two novel procedures are introduced to facilitate the resource allocation as the objective of the DM process. The first procedure uses the analytic expression provided by BDD as an objective function of a non-linear programming model. The second procedure introduces an importance measure that takes into account some external constraints, unlike the classical importance measures that only consider the topology of the tree. The first technique, called Mathematical Optimization Approach, will optimize the outcomes and the second, called Birnbaum-Cost Measure Method, will provide a good approximation of the outcomes using simpler calculations. Figure 6.3 shows the three common steps taken before the use of the novel methods presented in this paper.

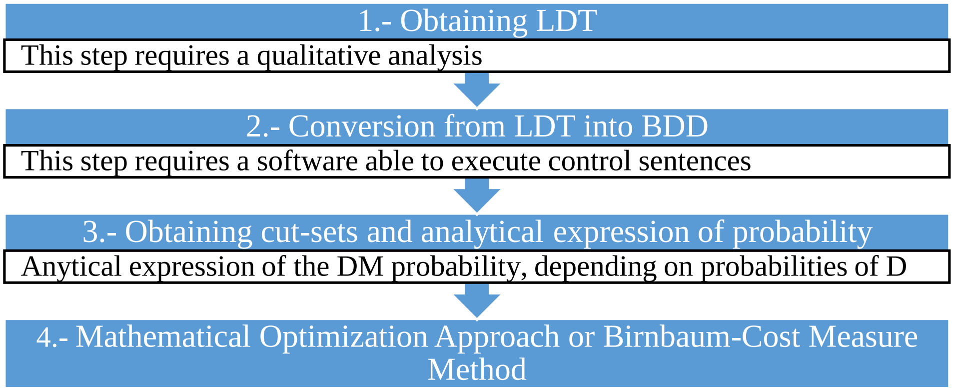


Figure 6.3 Proposed optimization method process

The methods proposed in this paper attend to a new way of solving DM problems through the linkage of graphical and mathematical tools. The first method (mathematical optimization approach) uses the expression provided by a BDD as an object function for a programming problem. This is a novel use of a Boolean function. Regarding the second method (Birnbaum-Cost Measure Method), the novelty lies in the adaptation of the Birnbaum measure in order to consider possible decision variables such as costs. Both methods are limited by the possibility of building a LDT that represents the problem with a correct accuracy.

It is important to remark that these methods can be applied to other DM problems [72], e.g. at the design stage of products [115], to elaborate preventive or predictive maintenance plans [114]. In general, they are adaptable to those problems that can be logically defined and several alternatives can be considered.

6.4 Mathematical Optimization Approach

Once the function Q_{MP} is achieved [72], the goal is to minimize the occurrence probability of the main problem. It will be assumed that LDT will be stable and therefore the reduction of the Q_{MP} will be performed by taking corrective actions (using additional resources) on the different BCs.

The nature of problems that can be examined via DM processes can be very different, but the presented method requires a probability assignment in all cases. This probability assignment, from a frequentist point of view, only depends on the frequency of occurrence of different BCs. The more frequent a BC is, the higher the occurrence probability it has. The position of a BC within the LDT and its own frequency of occurrence are two relevant factors in an optimized DM process.

Given a BC, the goal is to determine the investment on it in order to reduce its probability of occurrence, considering all the probabilities of BCs and the total investment. The objective function seeks to minimize the probability of occurrence of the top event Q_{MP}. A new vector **Imp (BC)** that considers this reduction is defined by equation 6.2:

$$\boldsymbol{Imp}(\boldsymbol{BC}) = [Imp(BC_1), Imp(BC_2), \ldots Imp(BC_i) \ldots Imp(BC_n)] \qquad (6.2)$$

The *i*th component of $\mathbf{Imp}(\mathbf{BC})$ provides the reduction of the probability of occurrence when some resources are allocated on the *i*th BC. Each component of $\mathbf{Imp}(\mathbf{BC})$ corresponds to an optimization variable. In addition, a probability vector $\mathbf{P}(\mathbf{BC})$ is defined by equation 6.3:

$$\boldsymbol{P}(\boldsymbol{BC}) = [P(BC_1), P(BC_2), \ldots P(BC_i) \ldots P(BC_n)] \qquad (6.3)$$

The i^{th} component of $\mathbf{P}(\mathbf{BC})$ provides the probability of occurrence of the i^{th} BC. Once BCs have been improved, the new probability assignment $P*$ is the difference between its probability of occurrence P and its *Imp*. It is defined by equation 6.4:

$$\boldsymbol{P}^*(\boldsymbol{BC}) = [P^*(BC_1), P^*(BC_2), \dots P^*(BC_i) \dots P^*(BC_n)] = [P(BC_1) - Imp(BC_1), P(BC_2) - Imp(BC_2), \dots, P(BC_i) - Imp(BC_i), \dots, P(BC_n) - Imp(BC_n)] \tag{6.4}$$

The BDD evaluated using $\mathbf{P}(\mathbf{BC})$ provides the value of Q_{MP}. If it is being evaluated using $\mathbf{P}^*(\mathbf{BC})$, the data obtained will be defined as Q_{MP}^*. It would be desirable to have $Q_{MP} \geq Q_{MP}^*$, otherwise the optimization procedure generates incorrect results.

The analytic expression provided by BDD becomes an optimization function when it is evaluated employing $\mathbf{P}^*(\mathbf{BC})$. The optimization function will be defined as $Q_{MP}(Imp)$.

This research study assumes that all the BCs can be improved but not necessarily corrigible, i.e. there can be some BCs whose occurrence probabilities cannot be reduced to 0. Therefore, each $\boldsymbol{Imp}$ will be between 0 and a certain threshold, a. This constitutes the first constraint, which is defined by equation 6.5:

$$0 \leq Imp(BC_i) \leq a_i \tag{6.5}$$

where a_i determines the maximum improvement that can be implemented in the i^{th} BC. This parameter is subject to the constraint given by equation 6.6:

$$0 \leq a_i \leq P(BC_i) \tag{6.6}$$

Therefore, $a_i = 0$ indicates that the i^{th} BC is capable of being totally corrected because the BC allows its own probability of occurrence to be 0 (in this case BC_i will not continue contributing to the MP occurrence). If $a_i = P(BC_i)$,then the improvements in the i^{th} BC are not possible.

An improvement cost vector, **IC (BC),** is defined for each BC, where a high IC for a BC means a large amount of resources must be invested to reduce the probability of occurrence of such BC. IC refers to marginal improvement costs, given by equation 6.7:

$$\boldsymbol{IC}(\boldsymbol{BC}) = [IC(BC_1), IC(BC_2) \ldots\ IC(BC_i) \ldots IC(BC_n)] \tag{6.7}$$

where $\boldsymbol{IC}(\boldsymbol{BC_i})$ indicates the amount of resources invested in *BC*$_i$ for reducing the probability of occurrence of *BC*$_i$ from 1 to 0.

The total amount of resources at the time of the investment operation is given by the Budget (Bg), obtaining the following constraint given by equation 6.8:

$$\sum_{i=1}^{N} IC(BC_i) \cdot Imp(BC_i) \leq Bg \tag{6.8}$$

Considering the abovementioned constraints, the optimization problem is defined in its standard form as shown in equation 6.9:

$$\begin{aligned} &minimize\ \ Q_{MP}(Imp) \\ &subject\ to \quad \sum_{i=1}^{N} IC(BC_i) \cdot Imp(BC_i) \leq Bg \\ &\qquad\qquad Imp(BC_i) - a_i \leq 0 \\ &\qquad\qquad -Imp(BC_i)\ \leq 0 \end{aligned} \tag{6.9}$$

This method allows establishing preferences among possible solutions. This could be done by defining a ponderation between the different constraints. The constraints can be associated with a certain value that can give a numerical importance to them. The solution that best meets the more important constraints will be the chosen one.

The resulting non-linear programming problem is an NP-hard problem, where the optimization function is non-linear. The complexity of the problem depends on the number of variables and the structure of the programming problem (objective function and constraints). Therefore, the necessary conditions of optimality are defined by the Karush-Khun-Tucker (KKT) conditions [156].

The procedure employed in this paper allows connecting the LDT analysis to any traditional optimization approach including mathematical optimization algorithms (e.g. Newton's method and Gradient Descent) and direct search methods (e.g. Simplex method and the Nelder-Mead method), but the complexity of the problem could require the use of unconventional optimization algorithms such as heuristics (e.g. Simulated Annealing, Deterministic Annealing, Tabu Search, Genetic Algorithms, Ant Systems or Neural Networks, etc.) [157],[158],[159],[160],[161],[162].

6.4.1 Mathematical NLPP background

The problem to address in this section is an optimization problem shown by equation (6.10)

$$\begin{aligned} &minimize && f(x) \\ &subject\ to && h(x) = 0 \\ & && g(x) \leq 0 \end{aligned} \tag{6.10}$$

where in case that any of the functions involved are nonlinear, the problem will be a Non-Linear Programming Problem (NLPP). NLPP is defined by a function $f: R^n \rightarrow R$ and two vector functions that represent the constraints that must be respected. They are defined by equations 6.11:

$$h: R^n \rightarrow R^m,\ g: R^n \rightarrow R^d \tag{6.11a}$$

$$x \in R^n \tag{6.11b}$$

$$f: R^n \rightarrow R \tag{6.11c}$$

$$h: R^n \rightarrow R^m \tag{6.11d}$$

$$g: R^n \rightarrow R^d \tag{6.11e}$$

Where m is the number of (non-linear) equality constraint and d is the number of (non-linear) inequality constraints. As a result, the set of vectors defined by equation 6.12 will correspond to the feasible set :

$$A = \{x \in R^n \colon h(x) = 0, g(x) \leq 0\} \subset R^n \tag{6.12}$$

When above-lines problem is trying to be solved, some conditions of optimality are desired to be found. The Lagrange multipliers for a NLPP with both equality and inequality constraints are defined by equation 6.13:

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda \cdot h(x) + \mu \cdot g(x) \tag{6.13}$$

In a NLPP, the necessary conditions of optimality are defined by Karush-Khun-Tucker (KKT) conditions [156]. If vector x_0 is optimal for the problem, then there exist two multipliers vectors $\lambda \in R^m$ and $\mu \in R^d$ as shown in equation 6.14:

$$\begin{gathered} \nabla f(x_0) + \lambda \cdot \nabla h(x_o) + \mu \cdot \nabla g(x_o) = 0, \\ h(x_o) = 0 \\ \mu \cdot g(x_o) = 0 \\ \mu \geq 0, g(x_o) \leq 0 \end{gathered} \tag{6.14}$$

Where μ must be $\mu \geq 0$ and $g(x_o) \leq 0$ whenever the minimum is trying to be found. It is important to denote that these conditions are necessary but not sufficient.

There are some important disadvantages about KKT conditions such as:

- The system is in fact a Non Linear System.
- A priori, there is no way to know how many solutions will be found.
- Every single feasible point will have to be evaluated in the main function $f(x)$.

Some sufficient conditions when dealing with NLPP are:

- The feasible set $k = \{x \in R^n : g(x) \leq 0, h(x) = 0\}$ is limited or $\lim_{|x| \to \infty} = +\infty$
- If $f'(x)$ is monotonically increasing, then $f''(x) > 0$, and $f(x)$ is convex.

The sufficient conditions of optimality require the study of convexity for the function f, g constraints as well as linearity for h constraints.

To sum up, every optimal solution must be a solution of the system of necessary conditions of optimality (KKT). These conditions are defined by equations 6.15:

$$\nabla f(x) + \lambda \cdot \nabla h(x) + \mu \cdot \nabla g(x) = 0 \tag{6.15a}$$

$$\mu \cdot g(x) = 0 \tag{6.15b}$$

$$h(x) = 0 \tag{6.15c}$$

$$\mu \geq 0,\ g(x) \leq 0 \tag{6.15d}$$

To find all the solutions for the KKT conditions, counting the equations and unknowns:

$d: inequalities, \quad \mu \in R^d$

$m: equalities, \quad \lambda \in R^m$

$n: variables, \quad x \in R^n$

The solutions of the $d + m + n$ equations with $d + m + n$ unknowns must be found, and due to its structure there are 2^d case studies.

6.4.2 Degree of certainty in the results

The function *fmincon* available in Matlab's toolbox is the responsible for the results given in this research [163]. It represents such a powerful tool due to the fact that it allows to simulate the kind of problems hereby presented regardless the number of BCs involved.

The algorithms used to solve every NLPP hereby presented are “Medium Scale: SQP, Quasi Newton, Line Search”. More information is detailed in Appendix III and references [163] as well as [164].

Exitflag, as well as output, are requested to the function used, as it has been previously stated, considering that it provides extremely important information about the optimization procedure. This parameter provides the way to know whether the function converges or not. An *exitflag*> 0 is always desired. Indeed, *exitflag*=1 means the result is a local minimum. Nevertheless, positive *exitflag* does not necessarily imply that it must be a “good” result. Additional parameters are needed to have into account.

Moreover, *output* parameter is responsible for giving further information such as the measure of the first order optimality or the number of iterations until it found the feasible solution.

For instance, the resolution of the “Tools shortage” MP which downgraded the probability of occurrence of the second BC from 80% to 50%, was obtained with the following exitflag and output outcomes:

“Optimization terminated: first-order optimality measure less than options.TolFun and maximum constraint violation is less than options.TolCon”

Above mentioned clause implies [165].

“First-order optimality measure is less than options.TolFun” means that the equation 6.16 is satisfied.

$$\left\| \nabla f(x) + \sum_{i=1}^{m} \lambda_i \cdot \nabla h_i(x) + \sum_{j=1}^{d} \mu_j \cdot \nabla g_j(x) \right\|_{\infty} \leq TolFun \tag{6.16}$$

As well as equation 6.17:

$$\forall j, \left| \mu_j \cdot g_j(x) \right| \leq TolFun \tag{6.17}$$

"Maximum constraint violation is less than options. TolCon" means that the following equations 6.18 are satisfied

$$[g_j(x)]_+ \leq TolCon \quad \forall j \tag{6.18a}$$

$$|h_i(x)| \leq TolCon \quad \forall i \tag{6.18b}$$

Where TolFun=1·10-6 as well as TolCon=1·10-6 by default.

6.4.3 Software development

The simulations and case studies hereby presented are based on the "fmincon" function of Matlab [166]. It will help to find the optimization results. Several parameters which need to be defined to run fmincon function are requested.

A NLPP fmincon standard form is written as:

$$\begin{aligned} \min_{x} \quad & f(x) \\ under \quad & c(x) \leq 0 \\ & c_{eq}(x) = 0 \\ & A \cdot x \leq b \\ & A_{eq} \cdot x = b_{eq} \\ & l_b \leq x \leq u_b \end{aligned}$$

Where

x, b, beq, lb, ub: are vectors

A, Aeq: are matrices

f(x): is a function that returns a scalar

The function fmincon is an optimization solver from Matlab's toolbox. It finds a minimum of a constrained nonlinear multivariable function. The syntax used in this simulation is as shown in equation 6.19

$$[x, fval, exitflag, output] = fmicom(fun, x_0, A, b, lb, up, options) \quad (6.19)$$

Some algorithms need to be developed in favour of a wider implementation regardless both the number of variables and the number of constraints involved. In this manner a fully development of the problem is able to be achieved with this algorithm.

First of all, to obtain the desired function to be optimized is needed, i.e. to define "fun" function. The CSs and *Qsys* is defined as former sections. How f(Imp(en)) is obtained is detailed in Figure 6.4, where its flowchart is depicted.

Figure 6.4 How to obtain f(Imp(e_n))

Once f(Imp(en)) is achieved, the precise parameters for each simulation must be defined. Initial conditions as well as boundary conditions must be properly defined. In accordance to the standard definition of a NLPP defined by fmincon

Lower (**lb**) and upper (**ub**) bounds and b must be vectors

A must be a matrix.

Both boundaries vectors **lb** and **ub** are written in the main algorithm straightaway. Nonetheless, two algorithms have been developed to automatically define properly the constraints regardless the optimization problem variables involved.

On the one hand, an algorithm creates the matrix related with the inequalities by knowing the number of variables (n) implicated, i.e. the number of events. Thus, Figure 6.5 shows how the algorithm works

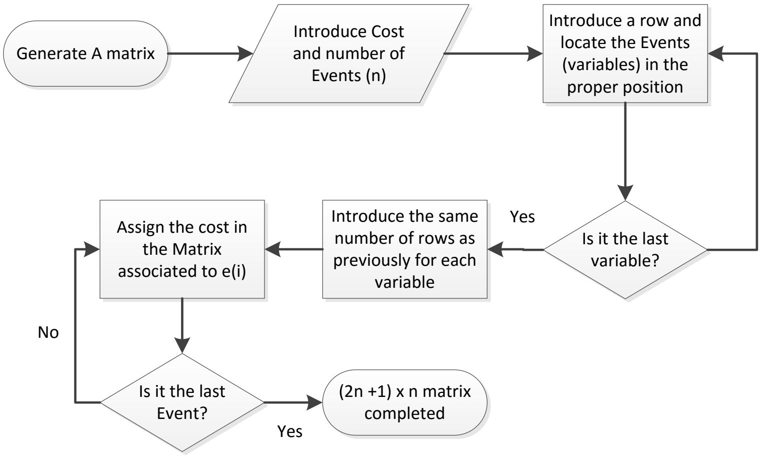


Figure 6.5 How to generate A matrix

In this manner, the constraints on the left of the inequality are defined with the information of each variable as well as its associated cost.

On the other hand, b vector related constraints are defined as follows, see Figure 6.6.

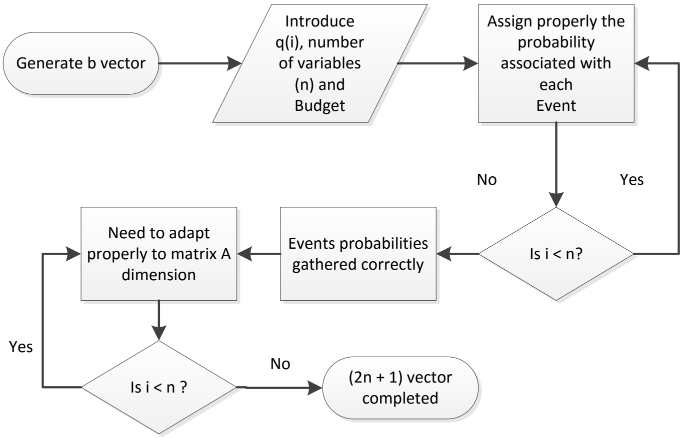


Figure 6.6 How to generate b vector

With this algorithm, vector b is generated. Bear in mind that it needs to fits the size of **A** matrix. Finally, the constraints have been completely defined and so the optimization problem is ready to be run.

With all this information and explanatory flowcharts to simulate a NLPP is achievable regardless the number of events involved.

Figure 6.7 shows the procedure to obtain *x*, *fval*, *exitflag*, and output exit variables. Within this context:

- *x* represents the new probability associated to each event $(Imp(q_n))$
- *fval* is a scalar and is the evaluation of the function *f(Imp(qn))* in the feasible points given.
- *exitflag* will yield the information about the accuracy's solution
- output contains crucial information to know how the optimization is being carried out

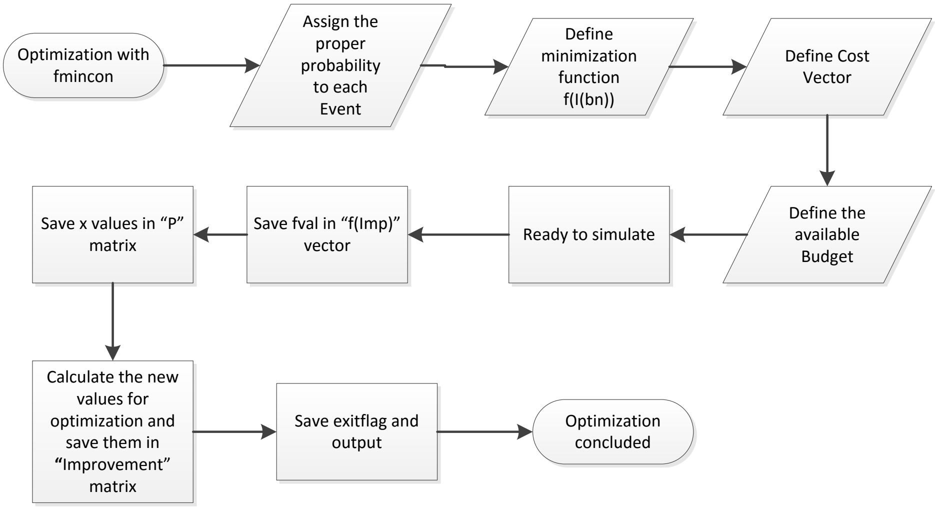


Figure 6.7 Optimization with fmincon

Figure 6.8 presents a flowchart that explains how the algorithm works and how it is viable to simulate over time different scenarios. The dynamic analysis is ready to be run once the initial conditions and both main function and constraints are properly defined. The algorithm consists on evaluating the main function with the initial values. Once the feasible points, where the minimal value for the objective function is achieved, are obtained they must be saved. In order to be able to save those results the following parameters are created:

- **P** matrix is in charge of saving the feasible points after the resolution of the NLPP i.e., **x** values given after the optimization.
- Improvement matrix is in charge of saving the updated probabilities of each BC.
- ***f(Imp)*** vector is responsible for saving the main function evaluated in the feasible points given by solving the NLPP.

Once the constraints and main function are defined, the Matlab function fmincon is responsible for giving the optimal or at least a feasible solution. When the feasible points are obtained and saved in aforementioned variables the algorithm is ready to set the subsequent simulations.

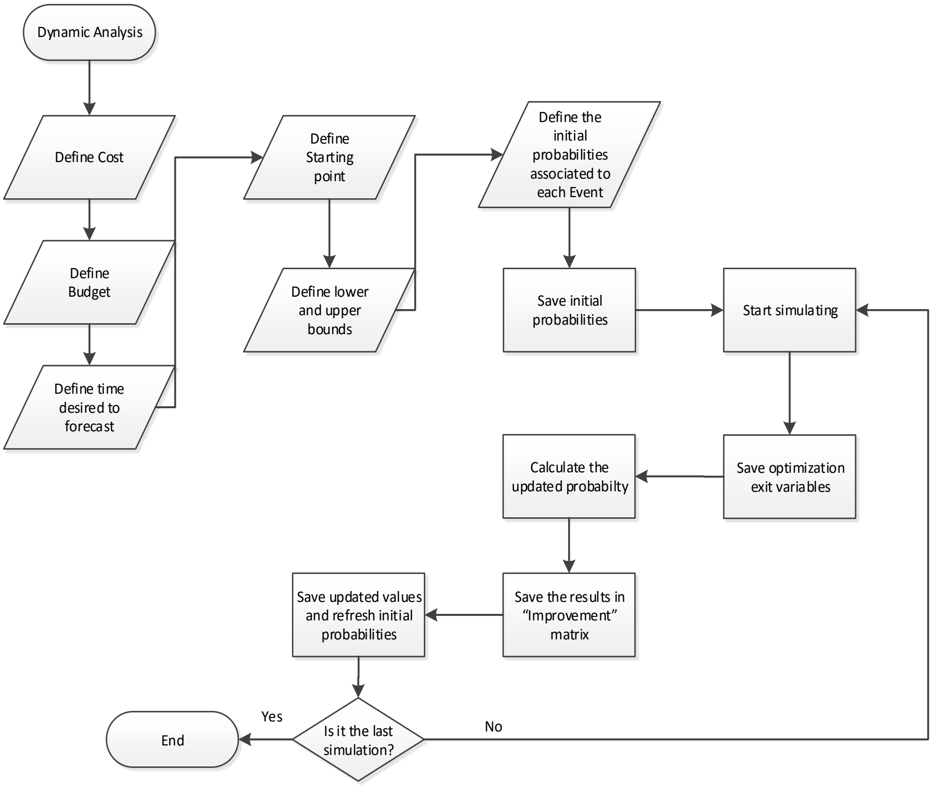


Figure 6.8 Optimization algorithm flowchart

The main purpose with the dynamic analysis is to be able to observe the gradual reduction in the occurrence probability of the top event with a certain degree of confidence. The algorithm is capable of stopping the simulation when a certain threshold limit is achieved. It suggests how much resources should be invested to obtain a certain occurrence probability of the top event in a desired period of time.

Figure 6.9 shows the flowchart including the threshold limit.

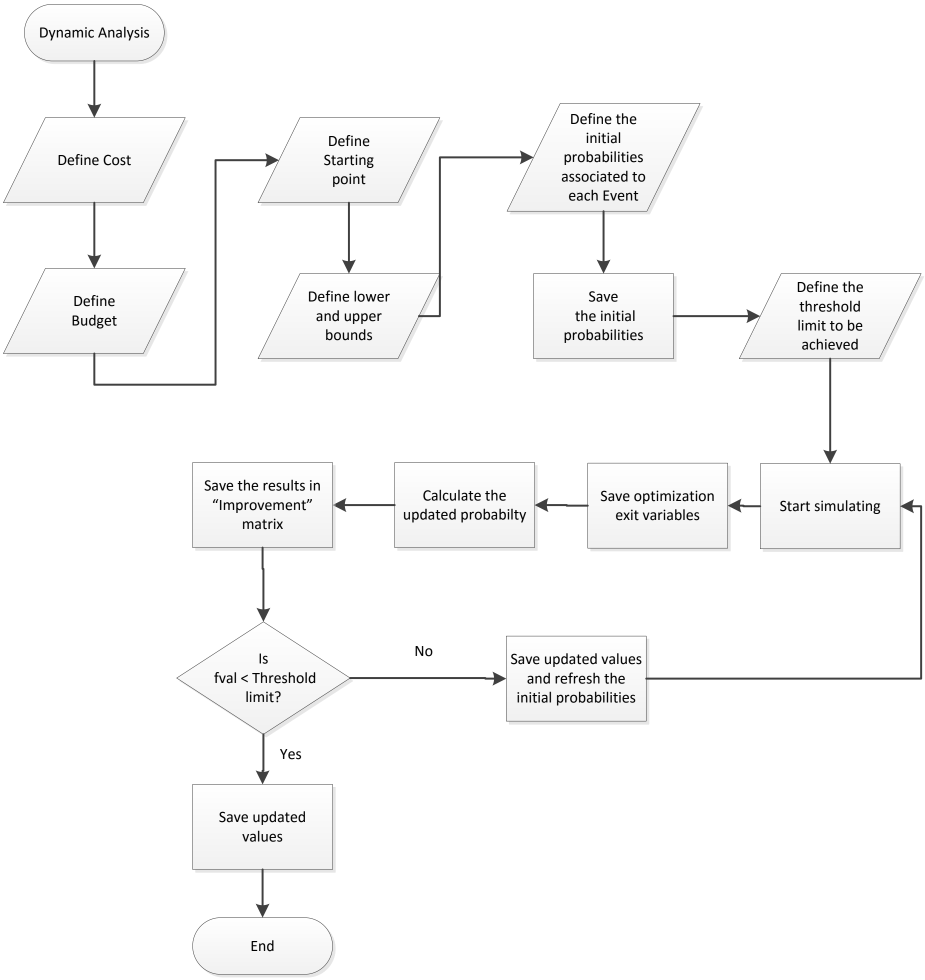


Figure 6.9 Threshold limit simulations flowchart

6.5 Birnbaum-Cost Measure Approach

The importance measures are employed in order to analyse the influence of the BCs in the LDT. Employing importance measures makes it possible to show the events that have more effect on the probability of the MP. These methods are based on the determination of the structural importance of each BC, and quantitatively analyzing the weight of each BC over the MP, i.e. it provides an index of the contribution of each BC with regard to the whole MP. The main limitation of the importance measures is that they only consider the occurrence probability and the location within the tree of the BCs. None of the methods are able to consider some constraints, such as the improvement cost considered in this thesis.

The Birnbaum measure has been chosen to calculate the importance of the BCs due to advantages over the rest of the methods [167].

A new parameter is introduced to obtain the marginal cost by reducing the probability defined as Birnbaum-Cost Measure importance (BCM), given by the equation 6.20:

$$BCM_i = \frac{I_i^{Birm}}{IC} = \frac{1}{IC} \cdot \frac{\partial Q_{MP}}{\partial P(BC_i)} \tag{6.20}$$

The DM based on Birnbaum-Cost Measure would allocate resources on the BC with the highest Birnbaum-Cost Measure value until it reaches its improvement threshold, then on the second one and so on, until all resources have been used.

6.6 Case study

Figure 6.10 presents a zoomed branch of the LDT in **¡Error! No se encuentra el origen de la referencia.**. This LDT could correspond to the analysis of a delivery business that desires to improve the timeliness of the service. The business is seeking to reduce the “delay in the orders”. The LDT is composed by 32 BCs and 26 non basic causes.

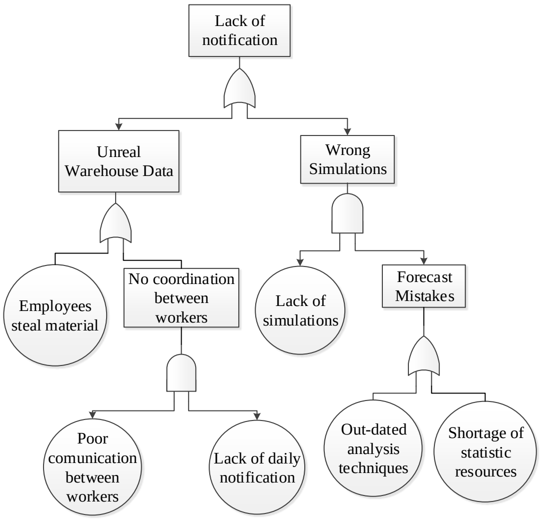


Figure 6.10 Subtree of LDT in ANNEX X

The Figure 6.11 present a simplification of the notation in order to facilitate the case study.

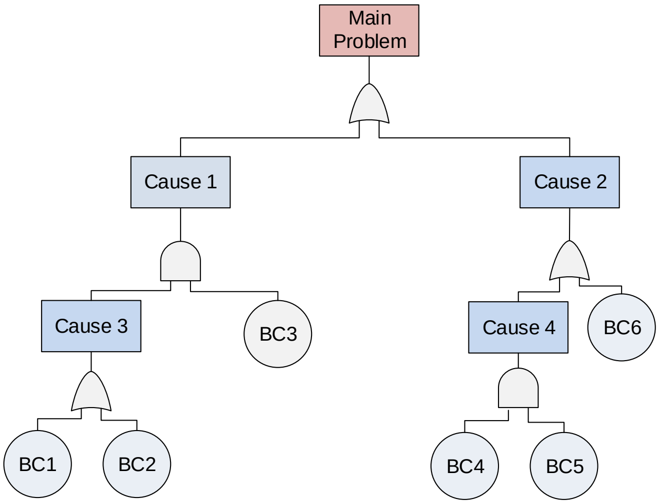


Figure 6.11 Subtree of LDT in ANNEX X(Notation Reduced)

The following Q_{MP} has been obtained by converting the LDT to BDD.

$$
\begin{aligned}
Q_{MP} = {} & P(BC_6) + \big(1 - P(BC_6)\big) \cdot P(BC_3) \cdot P(BC_1) + \big(1 - P(BC_6)\big) \cdot P(BC_3) \\
& \cdot \big(1 - P(BC_1)\big) \cdot P(BC_2) + \big(1 - P(BC_6)\big) \cdot P(BC_3) \\
& \cdot \big(1 - P(BC_1)\big) \cdot \big(1 - P(BC_2)\big) \cdot P(BC_4) \cdot P(BC_5) \\
& + \big(1 - P(BC_6)\big) \cdot \big(1 - P(BC_3)\big) \cdot P(BC_4) \cdot P(BC_5)
\end{aligned}
$$

Considering the following probabilities $P(BC) = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6]$, the Q_{MP} will be:

$$Q_{MP}(P(BC)) = 0.890$$

The initial parameters are given in Table 6.1 .

Table 6.1 Optimization Example Parameters

Basic Cause	P	a	IC **(€)**	Bg **(€)**
BC$_1$	0.1	0.05	1000	1800
BC$_2$	0.2	0.05	2000	
BC$_3$	0.3	0.10	3000	
BC$_4$	0.4	0.15	4000	
BC$_5$	0.5	0.05	5000	
BC$_6$	0.6	0.20	6000	

Once all the parameters have been defined and the analytical expression of Q_{MP} has been provided, it is possible to use the introduced methods.

6.6.1 *Resolution with Mathematical Optimization Approach*

The objective function would be:

$$
\begin{aligned}
&Q_{MP}(Imp(BC)) \\
&\qquad = \big(0.6 - Imp(BC_6)\big) + \Big(1 - \big(0.6 - Imp(BC_6)\big)\Big) \\
&\qquad \cdot \big(0.3 - Imp(BC_3)\big) \cdot \big(0.1 - Imp(BC_1)\big) \\
&\qquad + \Big(1 - \big(0.6 - Imp(BC_6)\big)\Big) \cdot \big(0.3 - Imp(BC_3)\big) \\
&\qquad \cdot \Big(1 - \big(0.1 - Imp(BC_1)\big)\Big) \cdot \big(0.2 - Imp(BC_2)\big) \\
&\qquad + \Big(1 - \big(0.6 - Imp(BC_6)\big)\Big) \cdot \big(0.3 - Imp(BC_3)\big) \\
&\qquad \cdot \Big(1 - \big(0.1 - Imp(BC_1)\big)\Big) \cdot \Big(1 - \big(0.2 - Imp(BC_2)\big)\Big) \\
&\qquad \cdot \big(0.4 - Imp(BC_4)\big) \cdot \big(0.5 - Imp(BC_5)\big) \\
&\qquad + \Big(1 - \big(0.6 - Imp(BC_6)\big)\Big) \cdot \Big(1 - \big(0.3 - Imp(BC_3)\big)\Big) \\
&\qquad \cdot \big(0.4 - Imp(BC_4)\big) \cdot \big(0.5 - Imp(BC_5)\big)
\end{aligned}
$$

The approach to the optimization problem will be:

$$minimize\ \ QMP(Imp(BC))$$

$$
\begin{aligned}
subject\ to\ \ &\big(100 \cdot\ Imp(BC_1) + 200 \cdot\ Imp(BC_2) + 300 \cdot\ Imp(BC_3) + 400 \\
&\cdot\ Imp(BC_4) + 500 \cdot\ Imp(BC_5) + 600 \cdot\ Imp(BC_6)\big) \cdot 10 \\
&\leq 1800
\end{aligned}
$$

$$Imp(BC_1) - 0.05 \leq 0; \quad -Imp(BC_1)\ \leq 0$$

$$Imp(BC_2) - 0.05 \leq 0; \quad -Imp(BC_2)\ \leq 0$$

$$Imp(BC_3) - 0.1 \leq 0; \quad -Imp(BC_3)\ \leq 0$$

$$Imp(BC_4) - 0.15 \leq 0; \quad -Imp(BC_4)\ \leq 0$$

$$Imp(BC_5) - 0.05 \leq 0; \quad -Imp(BC_5)\ \leq 0$$

$$Imp(BC_6) - 0.2 \leq 0; \quad -Imp(BC_6)\ \leq 0$$

Table 6.2 shows all the results for all the optimization parameters once the neural network is applied.

Table 6.2 Optimization Example Results

Basic Cause	**P**	**Imp**	P^*	**Investment (€)**	Q_{MP}	${Q_{MP}}^*$	**%** Q_{MP} **Reduction**
BC1	0.10	0	0.10	0	0.89	0.79	12%
BC2	0.20	0	0.20	0			
BC3	0.30	0	0.30	0			
BC4	0.40	0.16	0.24	634			
BC5	0.50	0.11	0.39	534			
BC6	0.60	0.11	0.50	632			

6.6.2 ***Resolution with Birnbaum Cost Measure Approach***

Figure 6.12 shows a comparison between Birnbaum and Birnbaum-Cost measures based on the case study presented in Figure 6.11. The Birnbaum Measure and the Birnbaum-Cost Measure have been calculated according to the expressions in section 2.5.

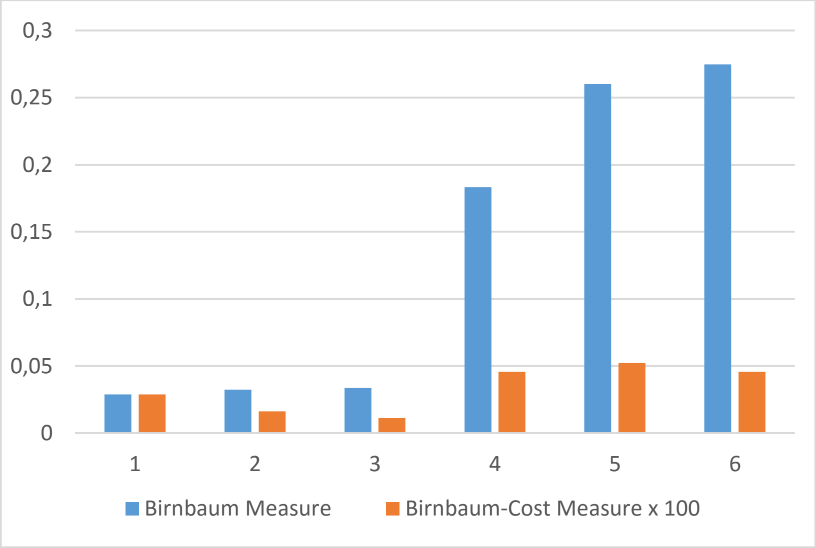


Figure 6.12 Birnbaum Measure vs. Birnbaum-Cost Measure

The results provided by the Birnbaum Measure method show that the investment order should be $BC_6, BC_5, BC_4, BC_3, BC_2, BC_{1,}$ but using the Birnbaum-Cost Measure method the order obtained is $BC_5, BC_4, BC_6, BC_1, BC_2, BC_3$.

Table 6.3 shows where some monetary resources must be allocated according to the proposed analysis. It shows the resource allocation, given the same conditions as the previous section example, i.e. the budget is 1800.

Table 6.3 Birnbaum-Cost Measure Method. Case Study Results

Basic Cause	**P**	**Imp**	P^*	**Investment (€)**	Q_{MP}	${Q_{MP}}^*$	**%** Q_{MP} **Reduction**
BC1	0.10	0	0.10	0	0.90	0.81	9%
BC2	0.20	0	0.20	0			
BC3	0.30	0	0.30	0			
BC4	0.40	0	0.40	0			
BC5	0.50	0.36	0.14	1800			
BC6	0.60	0	0.50	0			

6.6.3 Comparison between the methods, considering different availabilities of resources

A comparison between the aforementioned methods is carried out in this section. The aim of the following analysis is to show the results of each method when different budgets are considered for the investments. The first step is to calculate the maximum investment according to the constraints established by the parameter 'a'. The maximum investment corresponds to equation 6.21:

$$Invest_{max} = \sum_{i=1}^{N} (P(BC_i) - a_i) * IC(BC_i) \tag{6.21}$$

In this case, taking into account the data in Table I, the maximum investment is:

$$\begin{aligned} Invest_{max} = {} & (0.1 - 0.05) * 1000 + (0.2 - 0.05) * 2000 + (0.3 - 0.1) \\ & * 3000 + (0.4 - 0.15) * 4000 + (0.5 - 0.05) * 5000 \\ & + (0.6 - 0.2) * 6000 = 6600 \end{aligned}$$

Table 6.4 shows the investments suggested by the optimization method for different budgets from 600 € to the maximum investment of 6600 €, with an increment of 600 € for each step.

Table 6.4 Optimization Approach. Results with Different Budgets

	BC 1	BC 2	BC 3	BC 4	BC 5	BC 6
Budget 600 €	0	0	0	232	137	231
Budget 1200€	0	0	0	434	333	433
Budget 1800 €	0	0	0	634	534	632
Budget 2400 €	0	0	0	834	733	833
Budget 3000 €	0	0	0	1000	950	1050
Budget 3600 €	0	0	0	1000	1250	1350
Budget 4200 €	50	0	0	1000	1525	1625
Budget 4800 €	50	0	0	1000	1825	1925
Budget 5400 €	50	61	0	1000	1889	2400
Budget 6000 €	50	300	0	1000	2250	2400
Budget 6600 €	50	300	600	1000	2250	2400

Table 6.5 shows the outcomes of the method based on Birnbaum-Cost Measure considering the same budgets as in Table 6.4.

Table 6.5 Birnbaum-Cost Measure Method. Results with Different Budgets

	BC 1	BC 2	BC 3	BC 4	BC 5	BC 6
Budget 600 €	0	0	0	0	600	0
Budget 1200€	0	0	0	0	1200	0
Budget 1800 €	0	0	0	0	1800	0
Budget 2400 €	0	0	0	150	2250	0
Budget 3000 €	0	0	0	750	2250	0
Budget 3600 €	0	0	0	1000	2250	350
Budget 4200 €	0	0	0	1000	2250	950
Budget 4800 €	0	0	0	1000	2250	1550
Budget 5400 €	0	0	0	1000	2250	2150
Budget 6000 €	50	300	0	1000	2250	2400
Budget 6600 €	50	300	600	1000	2250	2400

Finally, the probability of occurrence of the MP (Q_{MP}) is evaluated for each considered budget. Figure 6.13 shows the comparison between the Q_{MP} using each of the methods.

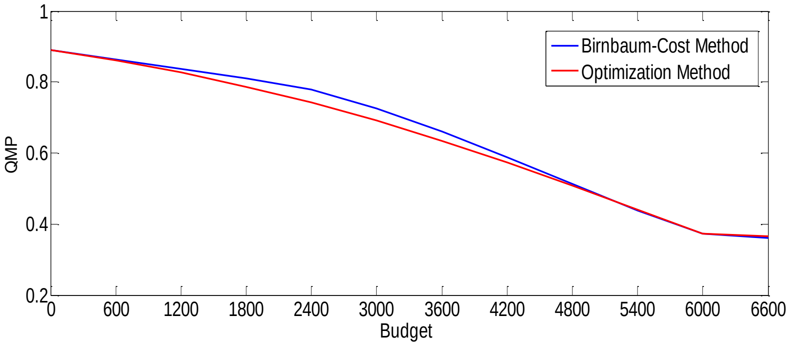


Figure 6.13 . Optimization vs. Birnbaum-Cost Measure Method. Results with Different Budgets

Figure 6.13 demonstrates that the optimization approach provides better results than the Birnbaum-Cost one; however, the optimization approach could require a higher computational cost. Therefore, both methods could be useful according to the desired accuracy.

7 CONCLUSIONS

SCADA or Condition Monitoring systems are critical to determine the state of complex systems. These systems are generating a large amount of information, called Big Data, that requires the new robust approaches in order to be processed.

Three procedures are proposed in this research work with the objective to obtain useful information from Big Data.

- The Pearson correlation coefficient to detect possible redundancies and false alarms. The main correlations between different measures have been represented in ANNEX VII.
- The second method analyses a continuous signal from a CMS by extracting feature parameters, using Neural Networks: This technique can reduce the 99.2% of the data with 71% of accuracy.
- The last one provides a reduction for SCADA systems by filtering the unnecessary data. It can be seen a reduction of the initial 52560 to an 841 samples, representing a decrease of the processed data up to 80% of the total.

The Logical Tree (LT) analysis is a technique based on symbolic logic that is applied in the study of complex systems. It is a deductive method that considers a set of events that forms the structure function of the system. The conversion from LT to Binary Decision Diagrams (BDD) provides some advantages in terms of efficiency and accuracy for quantitative analysis. When the LT has a large amount of basic causes, the direct analysis is often impossible. In these cases, it is necessary to use some truncation techniques and, consequently, a loss of accuracy is produced. BDDs provide an exact analytical expression of the occurrence probability of the main problem by the Boolean algebra. Three novel advances are made in the conversion from LT to BDD in order to reduce the computational cost:

- To study on the topology of the LTs that outcome these conclusions:
  - The probability is indirectly proportional to the number of AND gates, and proportional to the level, which is expected.
  - The effect on the probability of adding a new AND gate is indirectly proportional to the level.
  - The number of Cut Sets (CS) is larger in each level when the number of AND gates increase.
  - The number of CSs is smaller when the level is larger taking into account the same number of AND gates.
- A novel ranking method that improves the results in most of the cases. This approach provides the minimal cut-sets in most of the cases, being the number of cut-sets close to the minimal cut-sets found.
- A new method to optimise the replacement, repair, maintenance, etc. of a group of events in order to ensure a certain reliability to the system, and doing an appropriate use of the available resources.

A comparison between LT and BDD have been made, and some advantages of the use of BDD can be listed as:

- The computational cost is independent of the number of CSs and the way in which the LT is built.
- All the CSs are taken into account.
- They provide exact qualitative and quantitative information.
- The computational cost is between 100-1000 times lower than using classic methods.
- Typical operators of Boolean algebra can be evaluated with quadratic complexity.

The use of LT and BDD has been extended to the System Reliability Analysis and the Decision Making process. Two real case studies have been presented:

- *Wind Energy*. A fault tree has been built for a wind turbine. A maintenance strategy has been designed for a wind farm considering exogenous variables such as weather conditions
- *Point machines*. A fault tree has been built for a point machine. A dynamic analysis has been carried out using the importance measures.

Two novel procedures have been proposed in order to facilitate an optimal allocation of resources for Decision Making.

- The first procedure is to address the problem solving a non-linear programming approach.
- The second one uses a new version of the Birnbaum measure.

Both procedures have been applied in real case studies. They have been compared to each other concluding that the optimization approach provides better results than the Birnbaum-Cost one. However, the optimization approach could require a higher computational cost. Therefore, these methods could be useful according to the desired accuracy.

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ANNEXES

ANNEX I PROBABILITY THEORY

1. Previous concepts

Random experiment: It is that process whose result is not known in advance but there is a set of possible results.

Sample space: It is the set of possible results of a random experiment.

An event is the result or set of result of an experiment, therefore, it can be defined as a specific subset of a sample space. The following types of events can be defined:

- True event: It is an event that always happens. The associated subset is the entire sample space.
- Impossible event: it never occurs as a result of an experiment. The associated subset is the empty set.
- Identical events: They are events that can occur simultaneously for each observation or experiment.
- Complementary even: It is an event that occurs if its complementary event does not occur and vice versa.
- Incompatible events: They are events that cannot occur simultaneously.
- Dependent events: The ocurrence of an event A is conditioned by the ocurrence of other event B.

Complementary event. The complementary event of an event A occurs when A does not occur.

$\bar{A}: [x: x \notin A]$

$P(\bar{A}) = 1 - P(A)$

Intersection. The intersection of two events A, B is other event, or events, belong to A and B simultaneously.

$A \cap B: [(x \in A)\ and\ (x \in B)]$

*Union.*The union of A and B is other event (or events) belong to A, B, or both of them:

$A \cup B: [(x \in A)\ or\ \ (x \in B)]$

2. Useful relations

In the evaluation of LTs, the probability of occurrence of the top event can be achieved as the probability of the union of all the CSs

Inclusion - exclusion principle

The probability of the union of a set "N" with n elements can be calculated as follows:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1\le i\le n} P(A_i) - \sum_{1\le i<j\le n} P\left(A_i \cap A_j\right) + \sum_{1\le i<j<k\le n} P\left(A_i \cap A_j \cap A_k\right) + (-1)^{n+1} P(A_1 \cap A_2 \dots \cap A_n)$$

(AIV.1)

Equation AIV.1 can be expressed as:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{k=1}^{n} (-1)^{n+1} \cdot S_k$$

(AIV.2)

Where each factor Sk is a sum of probabilities of intersections of k-tuples, subsets of N forming all the possible combinations of k elements:

$$S_k = \sum_{J:J\subseteq N,|J|=k} P\left(\bigcap_{j\in J} A_j\right)$$

Each Sk contains the following terms:

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

This formula can be truncated in a summation of terms of even-order or odd-order terms. Thus, Bonferroni's inequalities are obtained:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \geq S_1 - s_2 + \cdots + (-1)^{n+1} S_k, for\ k\ even$$

(AIV.2.a)

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq S_1 - s_2 + \cdots + (-1)^{n+1}, for\ k\ odd$$

(AIV.2.b)

Rare event approximation

If the probabilities of the events are very low, the inclusion-exclusion formula can be truncated avoiding the high order terms. This is the Rare-Event approximation that is reduced to the sum of the probabilities.

$$P\left(\bigcup_{i=1}^{n} C_i\right) \cong P_{RareEvent} = \sum_{1<i<n} P(C_i)$$

(AIV.3)

Upper Bond approximation

The De Morgan's theorem is a tool used to obtain the probability of the union of events. The function that expresses the probability of occurrence of the top can be expressed as:

$$f = \bigcup_{i=1}^{n} C_i = C_1 + C_2 + \cdots + C_n$$

(AIV.4)

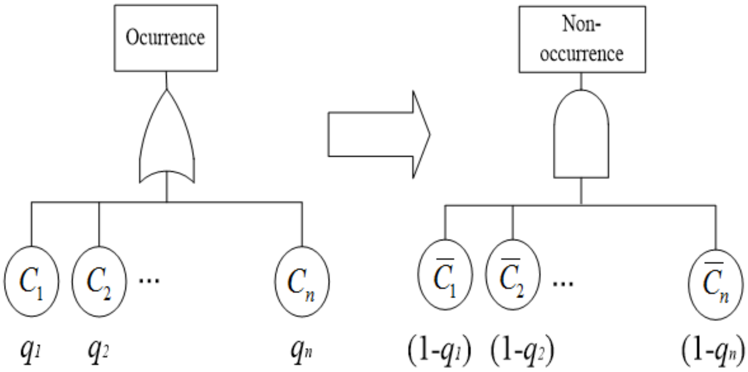
The probability that the top event does not occur can be calculated doing the product of the complementary probabilities of occurrence of the events.

$$Q_{NonOcurrence} = \prod_{i=1}^{n} (1 - q_i)$$

Then, the probability of occurrence is obtained as:

$$Q_{sys} = 1 - Q_{NonOcurrence} = 1 - \prod_{i=1}^{n} (1 - q_i) = \coprod_{i=1}^{n} q_i$$

(AIV.5)



The expression for the Upper-Bond approximation is:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \cong P_{UpperBond} = \coprod_{i=1}^{n} P(A_i) = 1 - \prod_{i=1}^{n} (1 - P(A_i))$$

It is true that:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P_{Inc-Excl} \leq P_{UpperBond} \leq P_{RareEvent}$$

Probability for a k-out of-n

A useful value is the probability in the output of a VOTE gate (k out of n). This probability is:

$$P_{k\ out\ of\ n} = \sum_{j=k}^{n} (-1)^{j-n} \cdot \binom{j-1}{k-1} \cdot S_j$$

Independent events

If the information provided by an event A does not modify the occurrence of the event B, then these events are independents.

Thus, A and B are independent if:

$$P(A \cap B) = P(A) \cdot P(B)$$

Conditioned probability

When a random experiment is carried out it is important to know if the occurrence of an event A provides information about the occurrence of B. The conditional probability aims to solve this problem.

Given a sample space and an event A with P(A)>0, the probability of B conditioned by A P(B/A) is defined as:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Composed probabilities theorem

Given a simple space and the events A and B with P(A)>0 and P(B)>0, then:

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$$

This theorem allows calculating the probability of conditioned events.

Total probability theorem

If $\{A_i\}_{i\in N}$ is a complete system and *P(Ai)>0* for each i∈N, then if an event B is given:

$$P(B) = \sum_{i=1}^{\infty} P(A_i) \cdot P(B/A_i)$$

Bayes's theorem

If $\{A_i\}_{i\in N}$ is a complete system and *P(Ai)>0* for each *i∈N* and B is *P(B)>0,* therefore:

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{\infty} P(A_i) \cdot P(B/A_i)}$$

The last two theorems are valid if the family of events (A_i) is finite. The probabilities of A are called "a priori" probabilities, the A_i / *B* probabilities are called "a posteriori", and *B* / A_i are called "verisimilitude".

ANNEX II BOOLEAN ALGEBRA

Idempotent:

$$A + A = A$$

$$A \cdot A = A$$

Involution:

$$\bar{\bar{A}} = A$$

Commutative addition:

$$A + B = B + A$$

Commutative product:

$$A \cdot B = B \cdot A$$

Associative addition:

$$A + (B + C) = (A + B) + C$$

Associative product:

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Distributive addition:

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Distributive product

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

Absorption rules:

$$A + A \cdot B = A$$

$$A \cdot (A + B) = A$$

De Morgan’s rules:

$\overline{A+B} = \bar{A} \cdot \bar{B}$

$\overline{A \cdot B} = \bar{A} + \bar{B}$

Axioms:

$0 + A = A$

$1 + A = 1$

$0 \cdot A = 0$

$1 \cdot A = A$

Principle of duality:

All logical relation will have a dual one. The dual expression is to change unions for intersections and 1 for 0.

Theorem of functions

Any function can be decomposed depending on its variables and according to the following relation :

$f(A, B, C, \dots) = A \cdot f(1, B, C, \dots) + \bar{A} \cdot f(0, B, C, \dots)$

where $f(1, B, C, \dots)$ is the resulting function when A is valued as 1 and $\overline{A}$ is values as 0. The second term $f(0, B, C, \dots)$ is the resulting function when A is valued as 0 and $\overline{A}$ is valued as 1.

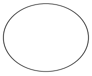
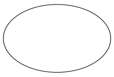
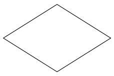
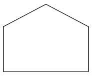
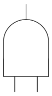
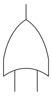
De Morgan’s generalized laws:

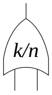
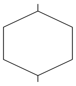
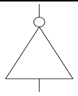
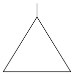
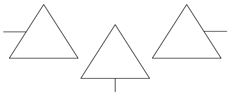
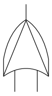
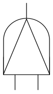
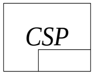
The complementary function is obtained using the complementary variables and exchanging sums and products. This can be expressed as:

$\overline{f(A, B, C, \dots)} = f(\bar{A}, \bar{B}, \bar{C}, \dots)$

ANNEX III SYMBOLOGY OF THE LOGICAL DECISION TREE

Table AI Symbology of LTds

Symbol	Name	Description
	BASIC EVENT	Event that cannot be broken down into more elementary events.
	INTERMEDIATE EVENT	Event that occurs due to previous occurrence of other causes. It can be broken down into more elementary events.
	CONDITIONAL EVENT	Conditions or constraints that are applied to an Inhibition gate or a Priority AND.
	NON-DEVELOPED EVENT	Event that is not developed into more complex events because its consequences are despicable or because there is not enough information.
	EXTERNAL EVENT	Event that usually occurs. It is usually constant and can adopt only one logical value (1 or 0).
	AND	Logical product operator: The output event only occurs when all the inputs events occur.
	OR	Logical sum operator: The output event occurs if, at least, one of the input events occurs.

	VOTE (k out of n)	The output event occurs if, at least, k out of n input events occur.
	INHIBITION GATE	The output event occurs if a specific condition occurs.
	NOT	The output event is the negation of the input event.
	INPUT TRANSFER	The LT continues where this symbol appears again.
	OUTPUT TRANSFER	This connection indicates that an event goes out the LT. This position must be related to the input transfer.
	EXCLUSIVE OR	The output event occurs if the inputs occurs in a specific order activated by a certain condition
	PRIORITY AND	The output event occurs if only one of the input events occurs.
	SEQ GATE	The output event occurs if the inputs occurs in a specific sequence
	SPARE GATE	If the event C occurs, it will be replaced by reserve events
	FUNCTIONAL DEPENDENCE	When the shot event occurs, the basic dependent events are forced to happen

ANNEX IV IMPORTANCE ANALYSIS PRACTICAL CASES

Case 1

This case presents the analysis of the LT in Figure I

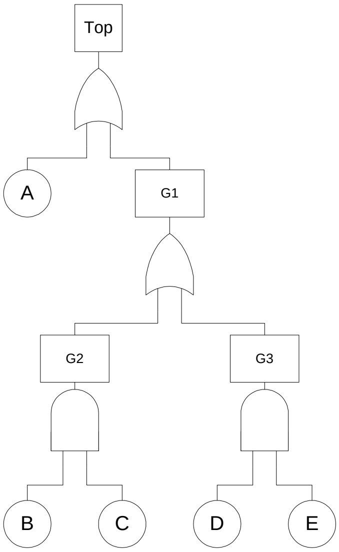


Figure I. LT without MOEs

The Top Event of this LDT is given by the following logical function:

$$f = A + BC + DE$$

The MCSs are:

$$CMS_1 = A$$

$$CMS_2 = BC$$

$$CMS_3 = DE$$

The probability of occurrence of the Top Event of the union of the cut sets. This probability is calculated using the Inclusion-Exclusion Principle:

$$Q_{Top} = P\left(A \bigcup BC \bigcup DE\right)$$

$$= P(A) + P(BC) + P(DE) - P\left(A \bigcap BC\right) - P\left(A \bigcap DE\right)$$

$$- P\left(BC \bigcap DE\right) + P(\mathrm{A} \bigcap BC \bigcap DE)$$

$$Q_{Top} = q_A + q_B q_C + q_D q_E - q_A q_B q_C - q_A q_D q_E - q_B q_C q_D q_E + q_A q_B q_C q_D q_E$$

Assuming a constant probability of occurrence for all the events $q_A = q_B = q_C = q_D = q_E = 0.01$, the probability of occurrence of the Top Event is:

$$Q_{Top} = 0.01 + 2(0.01)^2 - 2(0.01)^3 - (0.01)^4 + 0.01^5 = 0.010198$$

Fussell-Vesely

$$I_A^{FV} = \frac{P(A)}{Q_{Top}} = \frac{q_A}{Q_{Top}} = \frac{0.01}{0.010198} = 0.980584$$

$$I_B^{FV} = I_C^{FV} = \frac{P(BC)}{Q_{Top}} = \frac{q_B q_C}{Q_{Top}} = \frac{0.01^2}{0.010198} = 0.009806$$

$$I_D^{FV} = I_E^{FV} = \frac{P(DE)}{Q_{Top}} = \frac{q_D q_E}{Q_{Top}} = \frac{0.01^2}{0.010198} = 0.009806$$

Birnbaum

Calculating the derivative:

$$Q_{Top} = q_A + q_B q_C - q_D q_E + q_A q_B q_C + q_A q_D q_E + q_B q_C q_D q_E + q_A q_B q_C q_D q_E$$

Then:

$$I_A^{Birn} = \frac{\partial \mathrm{Q_{Sys}}}{\partial \mathrm{q_A}} = 1 - q_B q_C - q_D q_E + q_B q_C q_D q_E = 1 - 2(0.01)^2 + 0.01^4$$

$$= 0.9998$$

$$I_B^{Birn} = \frac{\partial Q_{Top}}{\partial q_B} = q_C - q_A q_C - q_C q_D q_E + q_A q_C q_D q_E$$

$$= 1 - (0.01)^2 - (0.01)^3 + 0.01^4 = 0.0099$$

$I_C^{Birn} = I_D^{Birn} = I_E^{Birn} = 0.0099$

AND criterion

The following ranking is obtained considering the number of the AND gates until reach the top for each event.

Rank	Events	AND gates
1.	A	0
2.	B	1
2.	C	1
2.	D	1
2.	E	1

Heuristic structural criterion

The list of variables according to the TDLR method is: A, B, C, D, E. It is convenient to follow the LT of Figure I to understand the following numerical operations.

<u>Event A:</u>

1º) Being $q_A = 1, q_i = \frac{1}{2}\ \forall i\ \neq A$

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{7}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1-1)\left(1 - \frac{7}{16}\right)\right] = 1$

2º) Considering $q_A = 0, q_i = \frac{1}{2} \forall i \neq A$

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{7}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1-0)\left(1 - \frac{7}{16}\right)\right] = \frac{7}{16}$

being: $I_A^{Struc} = 1 - \frac{7}{16} = \frac{9}{16}$

Event B:

1º) Taking into account $q_B = 1, q_i = \frac{1}{2}\ \forall i \neq B$

Level 3 (AND gates): $G_2 = 1 \cdot \frac{1}{2} = \frac{1}{2}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{5}{8}\right)\right] = \frac{13}{16}$

2º) Being $q_B = 0, q_i = \frac{1}{2}\ \forall i \neq B$

Level 3 (AND gates): $G_2 = 0 \cdot \frac{1}{2} = 0$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[(1 - 0)\left(1 - \frac{1}{4}\right)\right] = \frac{1}{4}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

where $I_B^{struc} = \frac{13}{16} - \frac{5}{8} = \frac{3}{16}$

Fort the events C, D y E, then $I_C^{struc} = I_D^{struc} = I_E^{struc} = 3/16$

Case 2

This case presents the analysis of the LT in Figure II.

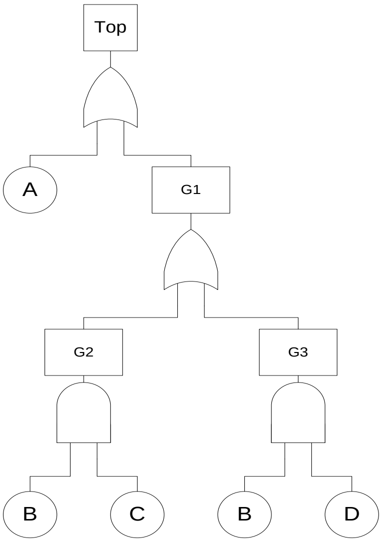


Figure II. LT with one MOE

The Top Event of this LDT is given by the following logical function:

$f = A + BC + BD$

The MCSs are:

$CMS1 = A$

$CMS2 = BC$

$CMS3 = BD$

The probability of occurrence of the Top Event:

$$Q_{Top} = P\left(A \bigcup BC \bigcup BD\right)$$
$$= P(A) + P(BC) + P(BD) - P\left(A \bigcap BC\right) - P\left(A \bigcap BD\right)$$
$$- P\left(BC \bigcap BD\right) + P(\mathrm{A} \bigcap BC \bigcap BD)$$

Assuming that all the events are independent:

$$Q_{Top} = q_A + q_B q_C + q_B q_D - q_A q_B q_C - q_A q_B q_D - q_B q_C q_D + q_A q_B q_C q_D$$

Assuming that $q_A = q_B = q_C = q_D = 0.01$, the probability of occurrence of the Top Event is:

$$Q_{Top} = 0.01 + 2(0.01)^2 - 3(0.01)^3 + 0.01^4 = 0.010197$$

Fussell-Vesely

$$I_A^{FV} = \frac{P(A)}{Q_{Top}} = \frac{q_A}{Q_{Top}} = \frac{0.01}{0.010197} = 0.980681$$

$$I_B^{FV} = \frac{P(BC \cup BD)}{Q_{Top}} = \frac{P(BC) + P(BD) - P(BC \cup BD)}{Q_{sys}}$$
$$= \frac{q_B q_C + q_B q_C - q_B q_C q_D}{Q_{Top}} = \frac{2(0.01)^2 - 0.01^3}{0.010197} = 0.019516$$

$$I_C^{FV} = \frac{P(BC)}{Q_{Top}} = \frac{q_B q_C}{Q_{Top}} = \frac{0.01^2}{0.010197} = 0.009807$$

$$I_D^{FV} = \frac{P(BD)}{Q_{Top}} = \frac{q_B q_D}{Q_{Top}} = \frac{0.01^2}{0.010197} = 0.009807$$

Birnbaum

Calculating the derivative:

$$Q_{Top} = q_A + q_B q_C + q_B q_D - q_A q_B q_C - q_A q_B q_D - q_B q_C q_D + q_A q_B q_C q_D$$

Then:

$$I_A^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_\mathrm{A}} = 1 - q_B q_C - q_B q_D + q_B q_C q_D = 1 - 2(0.01)^2 + 0.01^3$$
$$= 0.9998$$

$$I_B^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_B} = q_C + q_D - q_A q_C - q_A q_D - q_C q_D + q_A q_C q_D$$
$$= 2(0.01) - 3(0.01)^2 + 0.1^3 = 0.019$$

$$I_C^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_C} = q_B - q_A q_B - q_B q_D + q_A q_B q_D = 0.01 - 2(0.01)^2 + 0.01^3$$
$$= 0.0098$$

$$I_D^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_D} = q_B - q_A q_B - q_B q_C + q_A q_B q_C = 0.01 - 2(0.01)^2 + 0.01^3$$
$$= 0.0098$$

AND criterion

This LDT has a MOE (event B) placed in two different nodes of the tree. The number of AND gates must be counted for these two locations and the node with less AND gates must be considered. Therefore, the ranking of events will be:

Rank	Sucesos	Puertas AND
1.	A	0
2.	B	1
2.	C	1
2.	D	1

Heuristic structural criterion

The variable ranking according to the TDLR method is: A, B, C, D and E.

To understand the following numerical calculations, it is convenient to follow the LT in Figure II

Event A:

1º) Considering $q_A = 1, q_i = \frac{1}{2}\ \forall i \neq A$

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{7}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1 - 1)\left(1 - \frac{7}{16}\right)\right] = 1$

2º) Taking into account $q_A = 0, q_i = \frac{1}{2} \forall i \neq A$

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{7}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1 - 0)\left(1 - \frac{7}{16}\right)\right] = \frac{7}{16}$

being: $I_A^{Struc} = 1 - \frac{7}{16} = \frac{9}{16}$

Event B:

1º) Being $q_B = 1, q_i = \frac{1}{2}\ \forall i \neq B$

Level 3 (AND gates): $G_2 = 1 \cdot \frac{1}{2} = \frac{1}{2}$; $G_3 = 1 \cdot \frac{1}{2} = \frac{1}{2}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\right] = \frac{3}{4}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\right] = \frac{7}{8}$

2º) Taking into account $q_B = 0, q_i = \frac{1}{2}\ \forall i \neq B$

Level 3 (AND gates): $G_2 = 0 \cdot \frac{1}{2} = 0$; $G_3 = 0 \cdot \frac{1}{2} = 0$

Level 2 (OR gate): $G_1 = 1 - [(1 - 0)(1 - 0)] = 0$

Level 1 (OR gate): $Top = 1 - \left[\left(1-\frac{1}{2}\right)(1-0)\right] = \frac{1}{2}$

Substracting the final values obtained in each step: $I_B^{struc} = \frac{7}{8} - \frac{1}{2} = \frac{3}{8}$

<u>Event C:</u>

1º) Considering $q_C = 1, q_i = \frac{1}{2}\ \forall i \neq C$

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot 1 = \frac{1}{2}$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\right] = \frac{5}{8}$

Level 1 (OR gate): $Top = 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{5}{8}\right)\right] = \frac{13}{16}$

2º) When $q_C = 0, q_i = \frac{1}{2}\ \forall i \neq C$, then

Level 3 (AND gates): $G_2 = \frac{1}{2} \cdot 0 = 0$; $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 2 (OR gate): $G_1 = 1 - \left[(1-0)\left(1-\frac{1}{4}\right)\right] = \frac{1}{4}$

Level 1 (OR gate): $Top = 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\right] = \frac{5}{8}$

being $I_C^{struc} = \frac{13}{16} - \frac{5}{8} = \frac{3}{16}$

<u>Event D:</u>

Analogously, the importance of this event is: $I_D^{struc} = \frac{13}{16} - \frac{5}{8} = \frac{3}{16}$

Case 3

This case presents the analysis of the LT in Figure III.

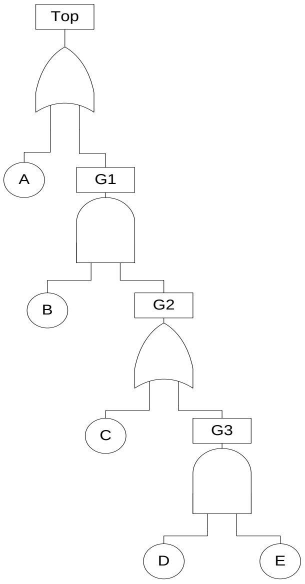


Figure III LDT with two AND gates at different levels

The Top Event is given by the following logical function:

$$f = A + B(C + DE) = A + BC + BDE$$

The MCSs are:

$$MCS1 = A$$

$$MCS2 = BC$$

$$MCS3 = BDE$$

The probability of occurrence of the Top Event:

$$Q_{Top} = P\left(A\bigcup BC\bigcup BDE\right)$$

$$= P(A) + P(BC) + P(BDE) + P(BDE) - P\left(A\bigcap BC\right)$$

$$- P\left(A\bigcap BDE\right) - P\left(BC\bigcap BDE\right) + P(\mathrm{A}\bigcap BC\bigcap BDE)$$

$$Q_{Top} = q_A + q_B q_C + q_B q_D q_E - q_A q_B q_C - q_A q_B q_D q_E - q_B q_C q_D q_E$$

$$+ q_A q_B q_C q_D q_E$$

For $q_A = q_B = q_C = q_D = q_e = 0.01$. It is obtained that:

$$Q_{Top} = 0.01 + 0.01^2 - 0.01^3 - 0.01^3 + 2(0.01)^4 + 0.01^5 = 0.0100998$$

$$\approx 0.0101$$

Fussell-Vesely

$$I_A^{FV} = \frac{P(A)}{Q_{Top}} = \frac{q_A}{Q_{Top}} = \frac{0.01}{0.0101} = 0.990099$$

$$I_B^{FV} = \frac{P(BC \cup BD\,E)}{Q_{Top}} = \frac{P(BC) + P(BDE) - P(BC \cap BDE)}{Q_{Top}}$$

$$= \frac{q_B q_C + q_B q_D q_E - q_B q_C q_D q_E}{Q_{Sys}} = \frac{(0.01)^2 + 0.01^3 - 0.01^4}{0.0101}$$

$$= 0.009999$$

$$I_C^{FV} = \frac{P(BC)}{Q_{Top}} = \frac{q_B q_C}{Q_{Top}} = \frac{0.01^2}{0.0101} = 0.009901$$

$$I_D^{FV} = I_E^{FV} = \frac{P(BDE)}{Q_{Top}} = \frac{q_B q_D q_E}{Q_{Top}} = \frac{0.01^3}{0.0101} = 0.000099$$

Birnbaum

To obtain the Birnbaum IM it is necessary to calculate the derivative with respect to each event:

$$Q_{Top} = q_A + q_B q_C + q_B q_D q_E - q_A q_B q_C - q_A q_B q_D q_E - q_B q_C q_D q_E + q_A q_B q_C q_D q_E$$

Thus:

$$I_A^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial q_A} = 1 - q_B q_C - q_B q_D q_E + q_B q_C q_D q_E$$
$$= 1 - (0.01)^2 + 0.01^3 + 0.01^4 = 0.9998$$

$$I_B^{Birn} = \frac{\partial Q_{Top}}{\partial q_B} = q_C + q_D q_E - q_A q_C - q_A q_D q_E - q_C q_D q_E + q_A q_C q_D q_E$$
$$= 1 - 2(0.01)^3 + 0.1^4 = 0.009998$$

$$I_C^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_C} = q_B - q_A q_B - q_B q_D q_E + q_A q_B q_D q_E$$
$$= 0.01 - 0.01^2 - 0.01^3 + 0.01^4 = 0.009899$$

$$I_D^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_D} = q_B q_E - q_A q_B q_E - q_B q_C q_E + q_A q_B q_C q_E$$
$$= 0.01^2 - 2(0.01)^3 + 0.01^4 = 0.000098$$

$$I_E^{Birn} = \frac{\partial \mathrm{Q}_{Top}}{\partial \mathrm{q}_E} = q_B q_D - q_A q_B q_D - q_B q_C q_D + q_A q_B q_C q_D$$
$$= 0.01^2 - 2(0.01)^3 + 0.01^4 = 0.000098$$

AND criterion

For each event, the number of AND gates is counted from the event to the Top.

Rank	**Events**	**AND gates**
1.	A	0
2.	B	1
2.	C	1
3.	D	2
3.	E	2

Heuristic structural criterion

The variable ranking given by the TDLR method is: A, B, C, D, E.

It is convenient to follow the LDT of Figure II in order to understand the following operations.

Event A:

1º) Taking $q_A = 1, q_i = \frac{1}{2}\ \forall i \neq A$

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1 - 1)\left(1 - \frac{5}{16}\right)\right] = 1$

2º) considering $q_A = 0, q_i = \frac{1}{2} \forall i \neq A$

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

Level 1 (OR gate): $Top = 1 - \left[(1 - 0)\left(1 - \frac{5}{16}\right)\right] = \frac{5}{16}$

Being $I_A^{Struc} = 1 - \frac{5}{16} = \frac{11}{16}$

Event B:

1º) Taking $q_B = 1, q_i = \frac{1}{2}\ \forall i \neq B$

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{5}{8}\right)\right] = \frac{13}{16}$

2º) When $q_B = 0, q_i = \frac{1}{2}\ \forall i \neq B$, then

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

evel 2 (AND gate): $G_1 = 0 \cdot \frac{5}{8} = 0$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)(1 - 0)\right] = \frac{1}{2}$

being $I_B^{struc} = \frac{13}{16} - \frac{1}{2} = \frac{5}{16}$

Event C:

1º) Considering $q_C = 1, q_i = \frac{1}{2}\ \forall i \neq C$

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[(1 - 1)\left(1 - \frac{1}{4}\right)\right] = 1$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot 1 = \frac{1}{2}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\right] = \frac{3}{4}$

2º) Taking into account $q_C = 0, q_i = \frac{1}{2}\ \forall i \neq C$

Level 4 (AND gate): $G_3 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 3 (OR gate): $G_2 = 1 - \left[(1 - 0)\left(1 - \frac{1}{4}\right)\right] = \frac{1}{4}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{8}\right)\right] = \frac{9}{16}$

Therefore: $I_C^{Struc} = \frac{3}{4} - \frac{9}{16} = \frac{3}{16}$

.Event D:

1º) considering $q_D = 1, q_i = \frac{1}{2}\ \forall i \neq D$

Level 4 (AND gate): $G_3 = 1 \cdot \frac{1}{2} = \frac{1}{2}$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{3}{4}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{8}\right)\right] = \frac{11}{16}$

2º) Taking into account $q_D = 0, q_i = \frac{1}{2}\ \forall i \neq D$

Level 4 (AND gate): $G_3 = 0 \cdot \frac{1}{2} = 0$

Level 3 (OR gate): $G_2 = 1 - \left[\left(1 - \frac{1}{2}\right)(1 - 0)\right] = \frac{1}{2}$

Level 2 (AND gate): $G_1 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Level 1 (OR gate): $Top = 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\right] = \frac{5}{8}$

Therefore: $I_D^{Struc} = \frac{11}{16} - \frac{5}{8} = \frac{1}{16}$

Event E:

This case is the same to the event D (it is under the same gate) :

$$I_E^{Struc} = \frac{11}{16} - \frac{5}{8} = \frac{1}{16}$$

In summary, the following tables allow making a comparison of the different methods used to generate the rankings of events in each LDT.

Table AII shows that the same rankings are obtained for every method. A simple example has been chosen to illustrate clearly that in some cases can be get the same results. The LT in Figure III is very simple and no calculations are required. In this case, the AND criterion saves time and provides the same results. The ranking of events at the same level is performed randomly. Any of the 16 possible rankings is valid (permutations of 4 elements, 4! = 16).

Table AII . Rankings considering to the different methods for the example 1

Basic Events	**Fussell-Vesley**	**Birnbaum**	**AND Criterion**	**Heuristic Structural Criterion**
	I_i^{FV} **(rank)**	I_i^{Birn} **(rank)**	**Nº gates (rank)**	I_i^{Struc} **(rank)**
A	0. 980584 (1)	0.9998 (1)	0 (1)	$\frac{9}{16}$ (1)
B	0. 009806 (2)	0.0099 (2)	1 (2)	$\frac{3}{16}$ (2)
C	0. 009806 (2)	0.0099 (2)	1 (2)	$\frac{3}{16}$ (2)
D	0. 009806 (2)	0.0099 (2)	1 (2)	$\frac{3}{16}$ (2)
E	0. 009806 (2)	0.0099 (2)	1 (2)	$\frac{3}{16}$ (2)

Table AIII shows the results for the example given in Figure III. This is a modification of the LT in Case 1, where some events have been replaced. Table AIII shows that Fussell-Vesely, Birnbaum and heuristic criterion provide the same ranking, being the last method is the most efficient because it needs a lower computational cost. In this case, the AND criterion is not sufficient to obtain a good ranking because it places the event B at the same level than events C and D, and it should appear above them, because it is a MOE and, therefore, it has more influence in the LT.

Table AIII Resulting rankings according to the different methods for example 2

Basic Events	**Fussell-Vesley**	**Birnbaum**	**AND Criterion**	**Heuriustic Structural Criterion**
	I_i^{FV} **(rank)**	I_i^{Birn} **(rank)**	**Nº gates (rank)**	I_i^{Heur} **(rank)**
A	0.980681 (1)	0.9998 (1)	0 (1)	$\frac{9}{16}$ (1)
B	0.019516 (2)	0.0197 (2)	1 (2)	$\frac{3}{8}$ (2)
C	0.009807 (3)	0.0098 (3)	1 (2)	$\frac{3}{16}$ (3)
D	0.009807 (3)	0.0098 (3)	1 (2)	$\frac{3}{16}$ (3)

Table AIV provides similar conclusions. It can be observed that the same ranking is provided by Fussell Vesely, Birnbaum and the proposed heuristic. The AND criterion differs to the other methods because it places events B and C at the same level of importance, although they are in different levels of the LT.

Table AIV Resulting rankings according to the different methods for example 3

Basic Events	**Fussell-Vesely**	**Birnbaum**	**AND Criterion**	**Heuriustic Structural Criterion**
	I_i^{FV} **(rank)**	I_i^{Birn} **(rank)**	**Nº gates (rank)**	I_i^{Heur} **(rank)**
A	0.990099 (1)	0.999899 (1)	0 (1)	$\frac{11}{16}$ (1)
B	0.00999 (2)	0.009998 (2)	1 (2)	$\frac{5}{16}$ (2)
C	0.00989 (3)	0.009899 (3)	1 (2)	$\frac{3}{16}$ (3)
D	0.00009 (4)	0.000098 (4)	2 (3)	$\frac{1}{16}$ (4)
E	0.00009 (4)	0.000098 (4)	2 (3)	$\frac{1}{16}$ (4)

ANNEX V RULES OF CONVERSION FROM LT TO BDD

The rules that will be used in the conversion from LTto BDD and to simplify the BDDs are presented in this section.

a) Operation rules

Assuming that F=ite(x, F1 , F2) and G=ite(y, G1 ,G2). The following operation rules are met:

a) In the case x< y:

F<op>G= ite(x, F1<op> G , F2<op> G)

b) In the case x= y:

F<op>G= ite(x, F1<op>G1 ,F2<op>G2)

c) In the case x> y:

F<op>G= ite(y, F <op>G1 ,F <op>G2)

The operator <op> indicates a generic logical operation (OR, AND ...). This is an advantage over other types of structures that require different algorithms to implement each type of operation.

If one of the expressions was a logical value:

F<op>1=1, if <op>=OR

F<op>1=F, *if* <op>=AND

F<op>0=F, *if* <op>=OR

F<op>0=0, *if* <op>=AND

b) *Reduction rules*

The next conversion rules are used to reduce the BDD graphs:

- Rule 1: Elimination of duplicate terminal vertices. Duplicate terminal vertices are eliminated leaving only two terminal nodes (with value 1 and with value 0). The terminal branches are redirected to the undeleted terminal vertices.
- Rule 2: Elimination of duplicate non-terminal vertices. If there are two non-terminal vertices u and v that belong to the same variable and if flow(u)= flow(v) and fupp(u)= fupp (v), one vertex is eliminated and its branches are redirected to the other vertex.
- Rule 3: Elimination of redundant vertices. If there is a non-terminal vertex whose "sons" are flow(v)= fupp(u), it is eliminated and its branches are redirected to the "son" vertex.

A example is proposed in order to clarify the use of this conversion rules. The logical function $f = e_2e_3 + e_1e_3 + e_1e_2e_3$ is given. This logical function is represented by the following Table AV and Figure IV

Table AV Truth table for f = e2 e3 + e1 e3 + e1 e2 e3

e₁	***e₂***	***e₃***	***f***
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

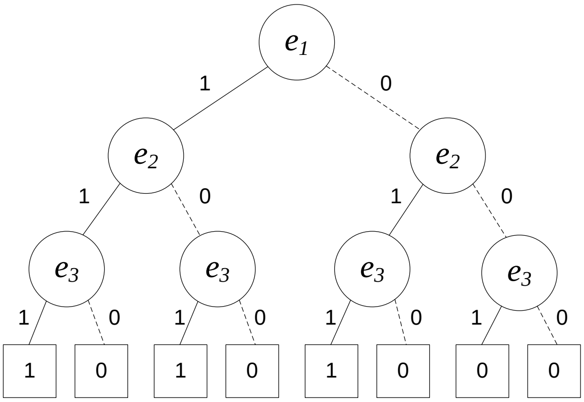


Figure IV . BDD associated with Table AV

The duplicate terminal vertices are eliminated in order to reduce the graph given in Figure IV, obtaining the Figure V

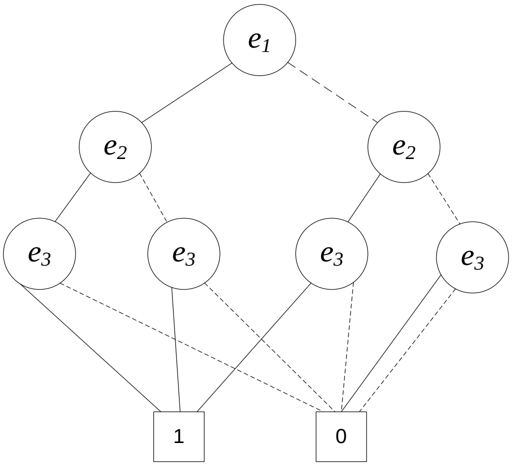


Figure V . Elimination of duplicate terminal vertices

The next step to a further reduction of the graph in Figure V is to eliminate the non-terminal vertices, obtaining the graph given in Figure VI

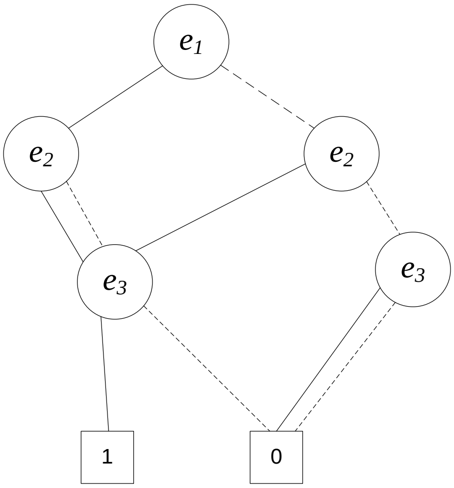


Figure VI Elimination of duplicate non-terminal vertices.

Finally, the last reduction step will be associated with the elimination of redundant vertices, where the Figure VII is obtained.

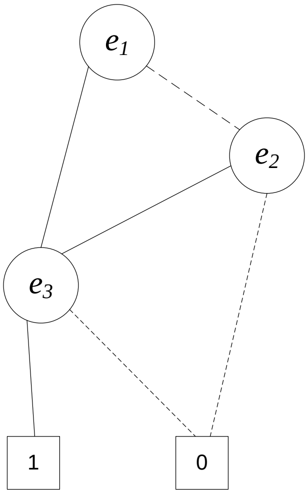


Figure VII Elimination of redundant vertices

Thus, the function $f = e_2 e_3 + e_1 e_3 + e_1 e_2 e_3$ can be expressed as $f = e_1 e_3 + e_2 e_3$

c) *Gate transformation rules*

It is described as follow rules for the logical sum and the logical product.

a) OR gates

Firstly, the ITE format for a logical sum will be generated. The logical variables can be expressed as:

$e_1 = ite(e_1, 1\ ,0)$

$e_2 = ite(e_2, 1\ ,0)$

For instance, taking the order $e_1 < e_2$. The logical sum of these variables according to the equation (3.5) is:

$$\begin{aligned} e_1 + e_2 &= ite(e_1, 1, 0) + ite(e_2, 1, 0) \\ &= ite\big(e_1, 1 + ite(e_2, 1,0), 0 + ite(e_2, 1, 0)\big) \\ &= ite(e_1, 1, ite(e_2, 1, 0)) \end{aligned}$$

It is represented graphically as shown in Figure VIII:

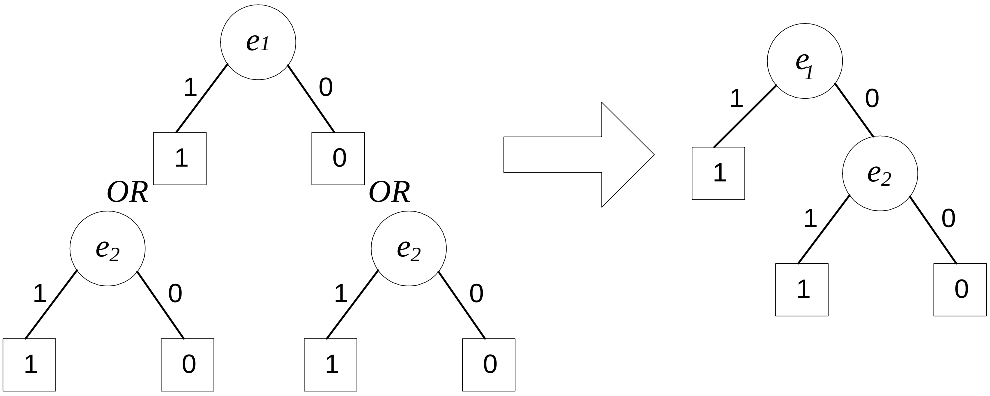


Figure VIII Logical sum of two variables.

This means that if at least one of the inputs is true then there will be a “1” in the output. Therefore, the logical sum of two variables is noted as:

$e_1 + e_2 = ite(e_1, 1, e_2)$

(3.12)

The logical sum of two logical functions expressed in the ITE format is given by equation 3.13 (see Figure IX).

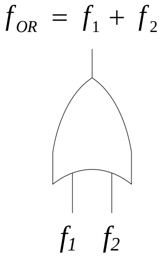


Figure IX Graph associated to Equation 3.13.

$for = ite(f_1, 1, f_2)$ *(3.13)*

This is equivalent to the BDD showed in Figure X.

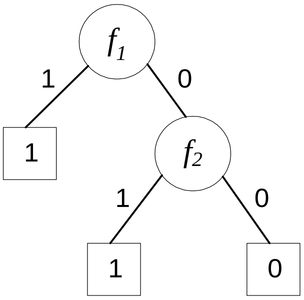


Figure X BDD for the logical sum (OR) of two logical functions

b) AND gates

The ITE format for a logical product is generated according to equation (3.5). Assuming the order $e_1 < e_2$:

$$\begin{aligned} e_1 \cdot e_2 &= ite(e_1, 1, 0) \cdot ite(e_2, 1, 0) \\ &= ite\big(e_1, 1 \cdot ite(e_2, 1, 0), 0 \cdot ite(e_2, 1, 0)\big) \\ &= ite(e_1, ite(e_2, 1, 0), 0) \end{aligned}$$

This can be represented graphically by Figure XI

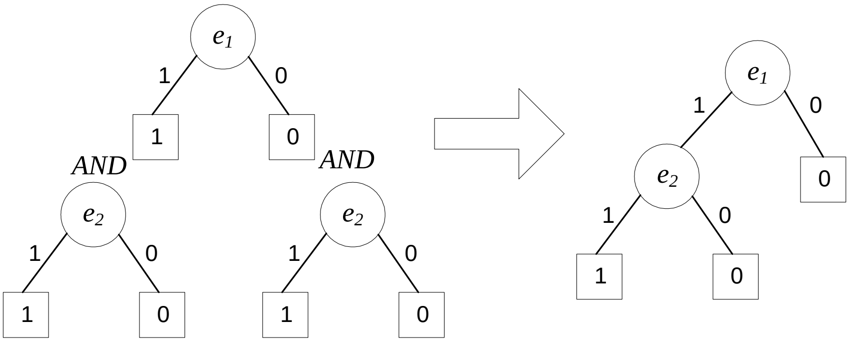


Figure XI BDD for the logical product (AND) of two variables

being

$$e_1 \cdot e_2 = ite(e_1, e_2, 0)$$

The logical product of two logical functions expressed in the ITE format is given by Figure XII:

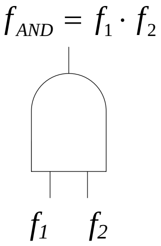


Figure XII Graph associated to Equation 3.15

$$f_{AND} = ite(f_1, f_2, 0)$$

This is equivalent to the BDD presented in Figure XIII.

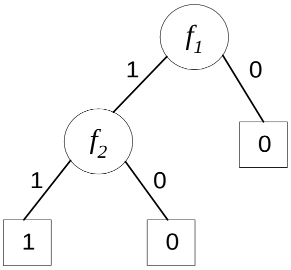


Figure XIII BDD for the logical product (AND) of two logical functions

d) *Simplification rules*

If M and N are logical variables, then:

$ite(1, M, N) = M$

$ite(0, M, N) = N$

$ite(M, M, N) = M$

$ite(M, 1,0) = M$

$ite(M, N, N) = N$

e) *Expansion rules*

$ite(ite(M, N_1, N_2), N_3, N_4) = ite(M, ite(N_1, N_3, N_4, ite(N_2, N_3, N_4)))$

f) *Absorption rules*

$ite(M, ite(M, N_1, N_2), N_3) = ite(M, N_1, N_3)$

$ite\big(M, N_1, ite(M, N_2, N_3)\big) = ite(M, N_1, N_3)$

g) *Order relations*

Each vertex of a LT has an associated index. The following relationships have to be applied when a new variable order is set in the reordering phase of the BDD:

If *index*(M_1)< *index*(M) ≤*index*(M_2):

$ite(M, M_1, M_2) = ite(M_1, ite(M, 1, M_2), ite(M, 0, M_2))$

If *index*(M_2)< *index*(M) ≤*index*($M1_1$):

$ite(M, M_1, M_2) = ite(M_2, ite(M, M_1, 1), ite(M, M_1, 0))$(3.18.b)

If $index(M_1) \leq index(M_2) < index(M)$:

$$ite(M, M_1, M_2) = ite(M_1, ite(M_2, 1, M), ite(M_2, ite(M, 0, 1), 0))$$

If $index(M_2) < index(M) < index(M_1)$:

$$ite(M, M_1, M_2) = ite(M_2, ite(M_1, 1, ite(M, 0, 1)), ite(M_1, M, 0))$$

ANNEX VI CASE STUDIES ON THE CONVERSION FROM LT TO BDD

Some examples are introduced in order to clarify the procedure for converting a LT into a BDD. Different topologies of LT are considered in these case studies.

a) LT without MOEs

A LT with three levels and without MOEs is presented in Figure XIV

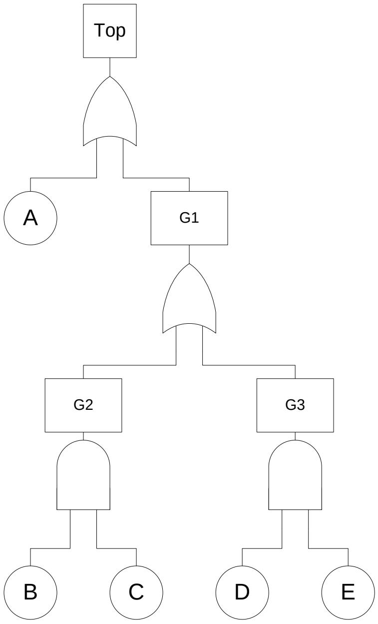


Figure XIV .Simple LT without MOEs (LT01)

Firstly, a variable ranking must be chosen. Due to this LT is very simple, it can be seen the event A has to be placed in first place. The rest of events can be placed in random order because of the symmetry of the LT.

The next ranking is obtained using the Top-Down Left-Right method: $A<B<C<D<E$

$$G_2 = ite(B,1{,}0)\cdot ite(C,1{,}0) = ite(B, ite(C,1{,}0),0)$$

$$G_3 = ite(D,1{,}0)\cdot ite(E,1{,}0) = ite(D, ite(E,1{,}0),0)$$

$$\begin{aligned} G_1 &= ite(B, ite(C,1{,}0),0) + ite(D, ite(E,1{,}0),0) \\ &= ite\big(B, ite(C,1{,}0) + ite(D, ite(E,1{,}0),0), ite(D, ite(E,1{,}0),0)\big) \\ &= ite(B, ite\big(C,1, ite(D, ite(E,1{,}0),0)\big), ite(D, ite(E,1{,}0),0)) \end{aligned}$$

$$\begin{aligned} &Top \\ &= ite(A,1{,}0) \\ &+ ite\left(B, ite\big(C,1, ite(D, ite(E,1{,}0),0)\big), ite(D, ite(E,1{,}0),0)\right) \\ &= ite(A,1, ite(B, ite(B, ite\big(C,1, ite(D, ite(E,1{,}0),0)\big), ite(D, ite(E,1{,}0),0)))) \end{aligned}$$

The resulting BDD is showed in Figure XV

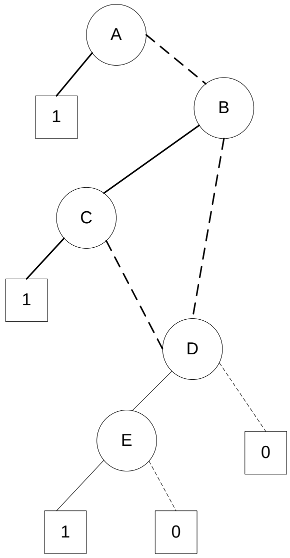


Figure XV *BDD* corresponding to LT01

The probability of the top event obtained by the sum of the probabilities of each CS as:

$$Q_{Sys} = q_A + (1 - q_A)q_Bq_C + (1 - q_A)q_B(1 - q_C)q_Dq_E + (1 - q_A)(1 - q_B)q_Dq_E$$

b) LT withMOEs at the same level

The LT presented in Figure XVI considers that one event has been replaced from the LT given in Figure XIV to generate a MOE.

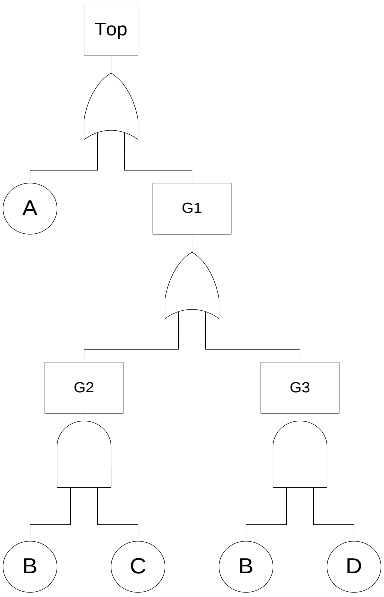


Figure XVI LT with a MOE (LT02)

The MOE B must be more important than the events C and D. Therefore, the variable ranking is: $A < B < C < D$. The BDD in Figure XVII is obtained using this ranking:

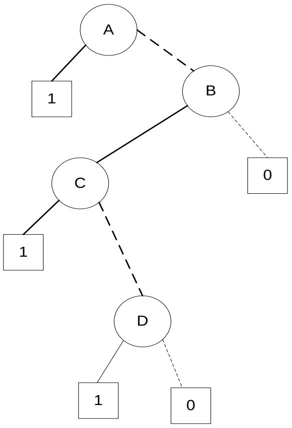


Figure XVII BDD corresponding to LT02

The probability of occurrence of the top event is:

$$Q_{Sys} = q_A + (1 - q_A)q_Bq_C + (1 - q_A)q_B(1 - q_C)q_D$$

c) ***LT with MOEs at different levels***

In this case, see Figure XVIII, the event B is repeated at two different levels:

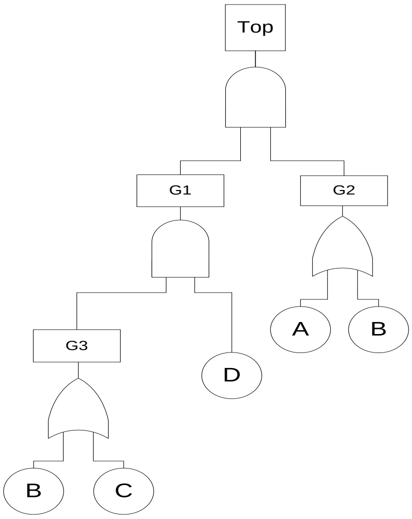


Figure XVIII LT with a MOEs at different levels (LT03)

There is a difference between the different heuristics methods. The appropriate variable ranking is: $B<A<C<D$

Applying the operation rules of BDDs according to the established ranking:

$$G_3 = ite(B,1,ite(C,1,0))$$

$$G_2 = ite(B,1,ite(A,1,0))$$

$$\begin{aligned} G_1 &= ite\big(B,1,ite(C,1,0)\big)\cdot ite(D,1,0) \\ &= ite\big(B,1\cdot ite(D,1,0),ite(C,1,0)\cdot ite(D,1,0)\big) \\ &= ite(B,ite(D,1,0),ite(C,D,0)) \end{aligned}$$

$$\begin{aligned} Top &= ite\big(B,ite(D,1,0),ite(C,D,0)\big)\cdot ite\big(B,1,ite(A,1,0)\big) \\ &= ite\big(B,ite(D,1,0),ite(A,1,0)\cdot ite(C,D,0)\big) \\ &= ite(B,ite(D,1,0),ite(A,ite(C,D,0),0)) \end{aligned}$$

The reduced BDD is shown in the Figure XIX

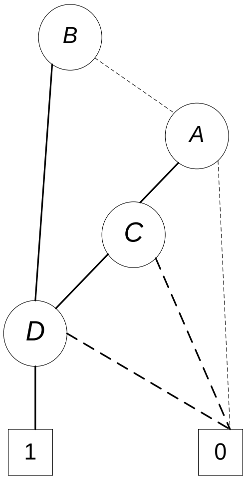


Figure XIX BDD corresponding to LT03

In this case there are only two CSs. The probability of occurrence of the top event is:

$$Q_{Sys} = q_B q_D + (1 - q_B) q_A q_C q_D$$

d) LT alternating AND and OR gates at different levels

In this case, Figure XX shows a LT that alternates AND and OR gates in function of the levels:

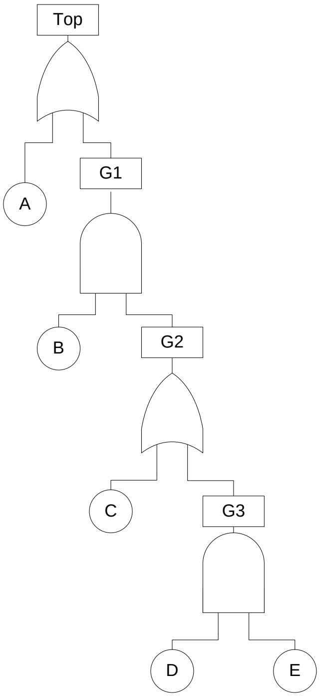


Figure XX LT alternating AND and OR gates (LT04)

The variable ranking will be: A<B<C<D<E

Applying the operation rules of BDDs according to the established ranking:

$$Top = ite(A, 1, ite(B, ite\big(C, 1, ite(D, ite(E, 1, 0), 0)\big), 0))$$

Then the resulting BDD is presented in Figure XXI

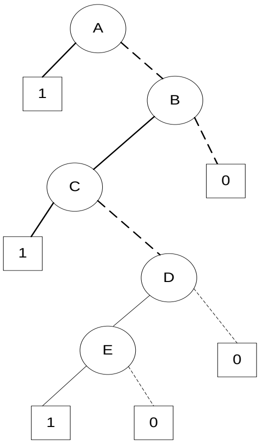


Figure XXI BDD corresponding to LT04

There are three CSs in this case. The probability of occurrence of the top event is:

$$Q_{sys} = q_A + (1 - q_A)q_B q_C + (1 - q_A)q_B(1 - q_C)q_D q_E$$

e) LT with several MOEs at different levels

The Figure XXII presents a LT with several MOEs at different levels.

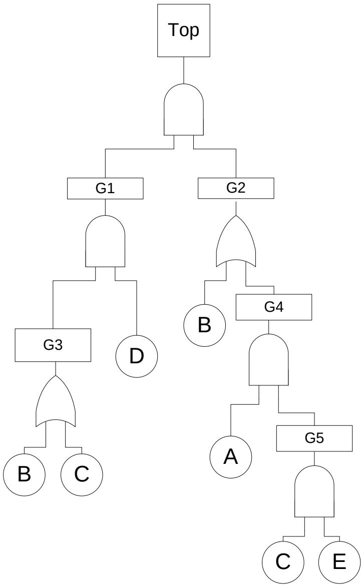


Figure XXII LT with several MOEs at different levels

The variable ranking that must be chosen for this LT is not intuitive. Different ranking methods will be applied in order to establish an appropriate ranking.

Weights method: Weights are assigned to each gate:

$$W(G5) = W(C) + W(E) = 1 + 1 = 2$$

$$W(G4) = W(A) + W(G5) = 1 + 2 = 3$$

$$W(G3) = W(B) + W(C) = 1 + 1 = 2$$

$$W(G2) = W(B) + W(G4) = 1 + 3 = 4$$

$$W(G1) = W(G3) + W(D) = 2 + 1 = 3$$

The logical expressions are rewritten in the output of each gate Gj because the inputs appear in order of increasing weights and the LT is reordered as shown in Figure XXIII:

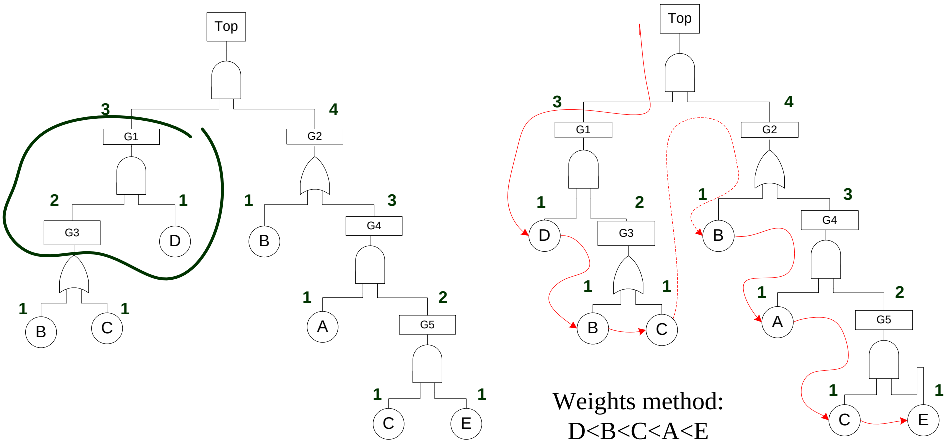
$$G5 = C + E$$

$$G4 = A + G5$$

$$G3 = B + C$$

$$G2 = B + G4$$

$$G1 = D + G3$$



Weights method:
D<B<C<A<E

Figure XXIII Weights method and reordering of the Figure XXII

It is observed that the expression G1 has changed. A DSF exploration is performed according to this new variable ranking.Table AVI shows the rankings obtained for different ranking methods.

Table AVI Comparison of different heuristic methods for LT05

Events	**DFS**	**TDLR**	**Level**	**Weights**	**Structural**	**AND method**	
						nº AND	**(rank)**
A	(4)	(4)	3	(4)	3/64 (4)	2	(3)
B	(1)	(2)	3,2	(2)	30/64 (1)	2,1	(1)
C	(2)	(3)	3,4	(3)	12/64 (3)	2,3	(5)
D	(3)	(1)	2	(1)	27/64 (2)	2	(2)
E	(5)	(5)	4	(5)	3/64 (5)	3	(4)

Different results are obtained for each ranking method. The following order is obtained using the weights method and the TDLR method: D <B <C <A <E. The resulting BDD using this ranking is showed in Figure XXIV.

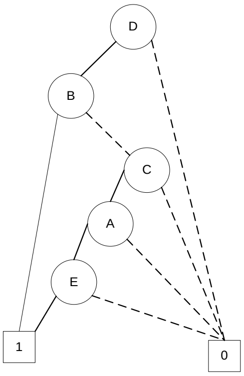
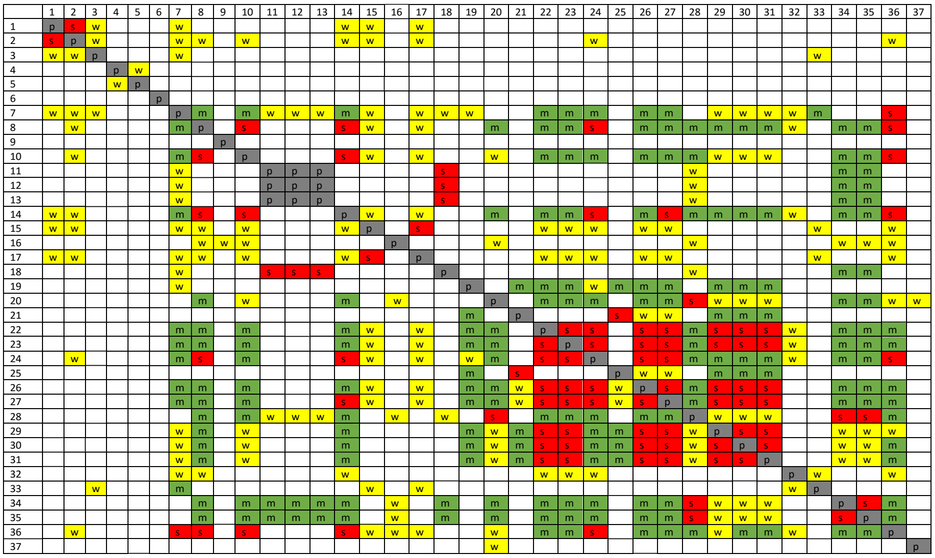


Figure XXIV BDD corresponding to LT05

The probability of occurrence of the Top Event will be:

$$Q_{Sys} = q_D q_B + q_D(1 - q_B) q_C q_A q_E$$

ANNEX VII PEARSON CORRELATION METHOD RESULTS



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
1	p	s	w				w							w	w		w																				
2	s	p	w				w	w		w				w	w		w							w												w	
3	w	w	p				w																										w				
4				p	w																																
5				w	p																																
6						p																															
7	w	w	w				p	m		m	w	w	w	m	w		w	w	w			m	m	m		m	m		w	w	w	w	m			s	
8		w					m	p		s				s	w		w			m		m	m	s		m	m	m	m	m	m	w		m	m	s	
9									p																												
10		w					m	s		p				s	w		w			w		m	m	m		m	m	m	w	w	w			m	m	s	
11							w				p	p	p					s										w						m	m		
12							w				p	p	p					s										w						m	m		
13							w				p	p	p					s										w						m	m		
14	w	w					m	s		s				p	w		w			m		m	m	s		m	s	m	m	m	m	w		m	m	s	
15	w	w					w	w		w				w	p		s					w	w	w		w	w						w			w	
16								w	w	w						p				w								w						w	w	w	
17	w	w					w	w		w				w	s		p					w	w	w		w	w						w			w	
18							w				s	s	s					p										w						m	m		
19							w												p		m	m	m	w	m	m	m		m	m	m						
20								m		w				m		w				p		m	m	m		m	m	s	w	w	w			m	m	w	w
21																			m		p				s	w	w		m	m	m						
22							m	m		m				m	w		w		m	m		p	s	s		s	s	m	s	s	s	w		m	m	m	
23							m	m		m				m	w		w		m	m		s	p	s		s	s	m	s	s	s	w		m	m	m	
24		w					m	s		m				s	w		w		w	m		s	s	p		s	s	m	m	m	m	w		m	m	s	
25																			m		s				p	w	w		m	m	m						
26							m	m		m				m	w		w		m	m	w	s	s	s	w	p	s	m	s	s	s			m	m	m	
27							m	m		m				s	w		w		m	m	w	s	s	s	w	s	p	m	s	s	s			m	m	m	
28								m		m	w	w	w	m		w		w		s		m	m	m		m	m	p	w	w	w			s	s	m	
29							w	m		w				m					m	w	m	s	s	m	m	s	s	w	p	s	s			w	w	w	
30							w	m		w				m					m	w	m	s	s	m	m	s	s	w	s	p	s			w	w	m	
31							w	m		w				m					m	w	m	s	s	m	m	s	s	w	s	s	p			w	w	m	
32							w	w						w								w	w	w								p	w			w	
33			w				m								w		w															w	p				
34								m		m	m	m	m	m		w		m		m		m	m	m		m	m	s	w	w	w			p	s	m	
35								m		m	m	m	m	m		w		m		m		m	m	m		m	m	s	w	w	w			s	p	m	
36		w					s	s		s				s	w	w	w			w		m	m	s		m	m	m	w	m	m	w		m	m	p	
37																				w																	p

ANNEX VIII POINT MACHINE EVENTS , FAULT TREE AND PROBABILITIES

Events Descriptions and Occurrence Probabilities

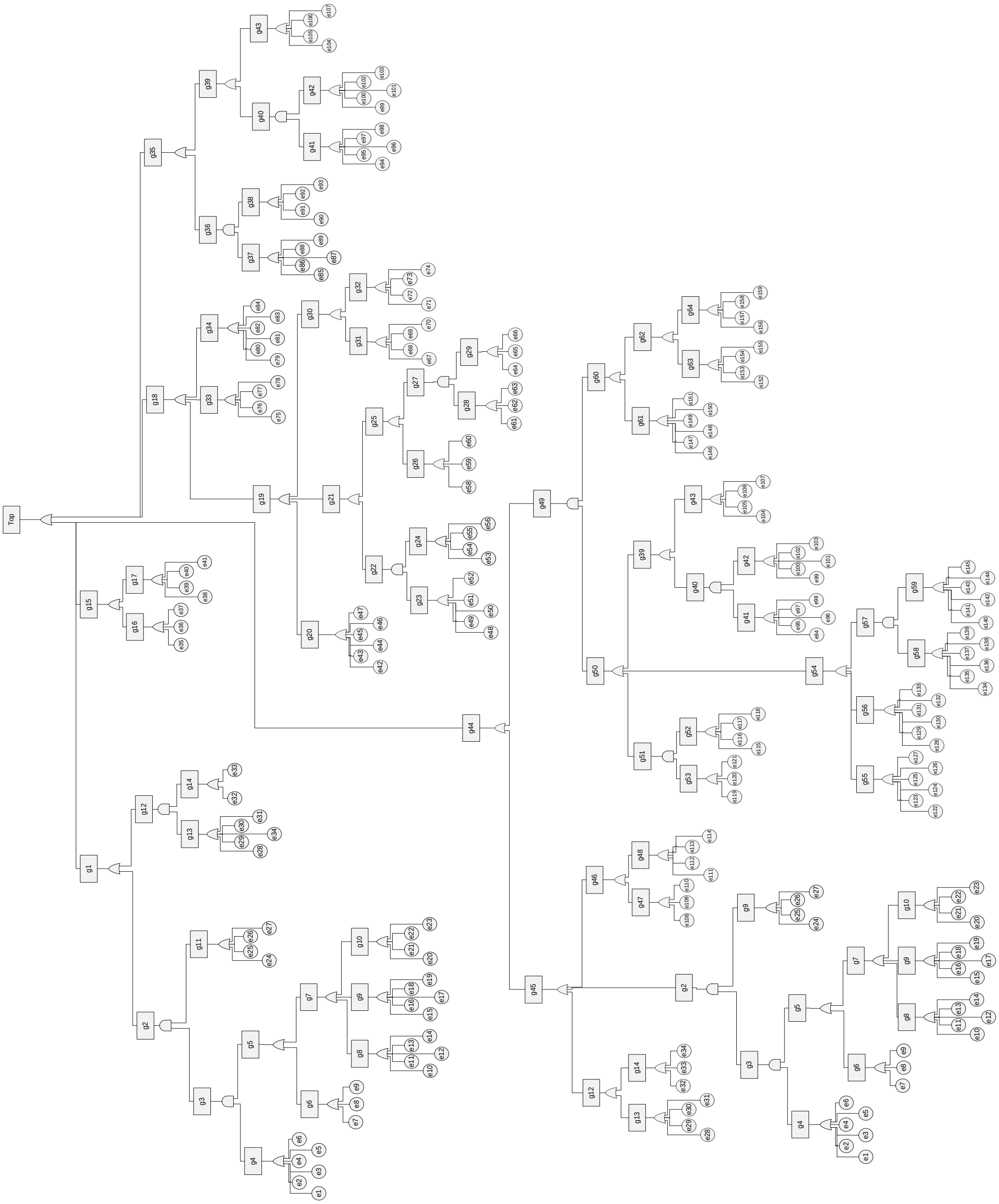
Event	Event Description	Prob %	Event	Event Description	Prob %
Top	**M63 Point Failure**		e102	7/8" Pin Worn	50
g1	Civil & Enviroment		e103	BrokenRollers	25
g2	Civil		g41	Backdrive3	
g3	Vibration		e104	Bush Wear	50
e1	Voiding	50	e105	BrokenDamagedRoller Stocks	25
e2	Damage/Wear	50	e106	BrokenStretcher	10
e3	BrokenChogBolts	50	e107	BrokenWornCranks	10
e4	StudBoltsFouling	10	g42	Rail ClampLocks	
e5	Broken Base Plates	50	g43	Rail Clamp Locks1	
e6	Fitingsbecome a loose	5	g10	Enviroment	
g4	TrackGeometry		g11	Enviroment1	
e7	Cant	5	e28	Broken Clutch Springs.Broken Clutch contact, spring Mechanism	5
e8	TrackComplexity	50	e29	Diode Block FaultyDirectional	5
e9	Radius	10	e30	Clutch Slip Current Set-Up	5
g5	Condition of Track		e31	SeizedClutch	1
g6	Condition of Track1		g12	Enviroment2	
e10	Track Gaude Wide/road Spread	10	e32	Diode Plug not Fully Inserted	1
e11	OversizeHoles/UndersizeBolts	5	e33	WornClutchAssembly	1
e12	Un-Even Switch Rail Contact on a Chairs	25	e34	Clutch Restraint Screw Coming Ou Should be a bolt	1
e13	Twist Faults	1	g2	Civil	
e14	SwitchCrippled	10	g3	Vibration	
g7	Condition of Track2		e1	Voiding	50
e15	Stock Rail Moving	10	e2	Damage/Wear	50
e16	Hogging	25	e3	BrokenChogBolts	50
e17	Dry/RustyChairs	50	e4	StudBoltsFouling	10
e18	MissingComponents	5	e5	Broken Base Plates	50
e19	YellowSlideChairScoring	10	e6	Fitingsbecome a loose	5
g8	Condition of Track3		g4	TrackGeometry	
e20	SolePlateWear	4	e7	Cant	5
e21	MovementLooseFishPlates	5	e8	TrackComplexity	50
e22	WrongComponentsFitted	1	e9	Radius	10
e23	LooseSlideChairs	10	g5	Condition of Track	
g9	P-WayworkingAlone		g6	Condition of Track1	
e24	Lift&Pasking	10	e10	Track Gaude Wide/road Spread	10
e25	Pulling in road	50	e11	OversizeHoles/UndersizeBolts	5
e26	TamperDamage	25	e12	Un-Even Switch Rail Contact on al Chairs	25
e27	ChairBoltsFoulingSwitches	10	e13	Twist Faults	1
g10	Enviroment		e14	SwitchCrippled	10
g11	Enviroment1		g7	Condition of Track2	

e28	Broken Clutch Springs.Broken Clutch contact, spring Mechanisn	5	e15	Stock Rail Moving	10
e29	Diode Block FaultyDirectional	5	e16	Hogging	25
e30	Clutch Slip Current Set-Up	5	e17	Dry/RustyChairs	50
e31	SeizedClutch	1	e18	MissingComponents	5
g12	Enviroment2		e19	YellowSlideChairScoring	10
e32	Diode Plug not Fully Inserted	1	g8	Condition of Track3	
e33	WornClutchAssembly	1	e20	SolePlateWear	4
e34	Clutch Restraint Screw Coming Ou Should be a bolt	1	e21	MovementLooseFishPlates	5
g13	UN-EvenTraficcPatterns		e22	WrongComponentsFitted	1
g14	UN-EvenTraficc Patterns1		e23	LooseSlideChairs	10
e35	Lateral Movement of Track	5	g9	P-WayworkingAlone	
e36	Infrequentoperationmechanism	10	e24	Lift&Pasking	10
e37	Flat TyreDamage	1	e25	Pulling in road	50
g15	UN-EvenTraficc Patterns2		e26	TamperDamage	25
e38	StocK Rail Burns	25	e27	ChairBoltsFoulingSwitches	10
e39	GroovedSlides	5	g44	UN-EvenTraficcPatterns	
e40	SwitchCreep	50	g45	UN-EvenTraficc Patterns1	
e41	Dry Chairs UN-Even Lubrication	25	e108	StocK Rail Burns	25
g16	Machines		e109	GroovedSlides	5
g17	Machines1		e110	SwitchCreep	50
g18	M63 Drive		e111	Dry Chairs UN-Even Lubrication	25
g19	Drive Esternal to Machine	50	g46	UN-EvenTraficc Patterns2	
e42	Worn Drive LugBolt	1	e112	Lateral Movement of Track	5
e43	Loose Drive Shoe	10	e113	Infrequentoperationmechanism	10
e44	Machine Loose	5	e114	groovedSlides	1
e45	Loose Drive Basket	10	g47	Rail Clamp Locks2	
e46	No clearance between Stretcher Rail Kicking	1	g48	ClampLock	
e47	Nuts Run Back on Drive	10	g49	PumpUnit	
g20	Motor		g50	Pump Unit1	
g21	Motor1		e115	DefectivePump	1
g22	Motor 11		e116	VibrationLoose 2bA Nuts	1
e48	Motor Defects	1	e117	LeakyTank	1
e49	WornBrushes	10	e118	StickingValve	10
e50	Motor Wiring Plug High Resistanc	1	g51	Pump Unit2	
e51	Dirty Motor com	5	e119	WornBrushes "HR"	25
e52	LightlyLoadedBrushes	1	e120	FrozenPumpUnit	1
g23	Snubbing	15	e121	Earthy Motors	1
e53	Diode open Circuit	25	g52	Mechanics	
e54	Snubbing Resistor BurntOut	50	g53	Mechanics1	
e55	BurntSnubbingContacts	25	e122	LooseFixedCam	1
e56	Motor Brushes High Resistance	25	e123	Burnt/Worn Damaged Hoses/Pin Holes in Hoses	25
g24	Motor2		e124	WrongLockArmFitted	1
g25	Motor21		e125	BrokenCrimps/Terminals	50
e57	Clutch	7,5	e126	Tie Bar Adjustment/Too Long/Drilling Wrong	10
e58	StickyBrushes 2	1	e127	Displaced Ferrule in Adapter Bloc	1
e59	Broken OP Spring Braid	5	g54	Mechanics2	
e60	Earthy Motor	1	e128	LeakyRams	10
g26	Motor22		e129	Dirt, Grease Build Up on Mechanisms	1
g27	Springs	15	e130	Centre ThrustPackingWrong	25
e61	Burnt Springs	25	e131	Loose Allen Screws	10

e62	Spring Adjustment	50	e132	SwitchAssemblyBushesWear	5
e63	High Resistance "H" Piece	25	e133	FaultyMicroswitches	25
g28	CutoutContact	15	g55	Mechanics3	
e64	SeizedCrankHandleContacts	5	e134	TuningForkWear	25
e65	Operators Not Resetting Cut Out	50	e135	Point Heater Strip Fouls Body	1
e66	High Resistance Cut Out Contact	50	e136	Loose/No spiralFitted	1
g29	Drive internal to Machine		e137	No AdjustmentforWear	5
g30	Drive internal to Machine1		e138	LockStickingMKlOnly	5
e67	Failure to Drive Insufficient Thro	5	e139	Missing "O" Rings From Rams	5
e68	Broken Drive Rod Spring Trailling Points	1	g56	Mechanics4	
e69	Broken/Worn:Worm Drive	1	e140	Worn or Missing Nibs MK1 Only	1
e70	CrankHandleOperationSticking	1	e141	LooseBody	25
g31	Drive internal to Machine2		e142	Open SwitchTappetAdjustment	10
e71	Change Dribe Belt/Lose Ball Bearings	1	e143	Poor CoverSeals	1
e72	SnubbingProblemsBoltBreaks	5	e144	No Lock Wires in Hoses 5.2.19	1
e73	Poor Desing of Throw Bar Couplin Lug Serrations	5	e145	LooseCabling No Glands	1
e74	SnubbingAdjustment	5	g38	Backdrive	
g32	M63 Detection		g39	Backdrive1	
e75	CircuitController Springs	5	e94	Lack of 50mm Flangeway	25
e76	4 FootLock Detector Arrangemen	1	e95	Nuts Run Back	50
e77	Seized Detection Effects Esterna Fittings	1	e96	ObstructionsWeeds/Growth	1
e78	No lift on Push Rod & Push Rod Wear	1	e97	ObstructionBallast	25
g33	M63 Lock		e98	LooseFittings	50
e79	EffectsFromTemperature	25	g40	Backdrive2	
e80	Restrictive Adjustment of Lock Manufactor Design	1	e99	No Escapement on Supplementar Drive	50
e81	Lack of Fine Adjustment Floating Lock	1	e100	ShoesBroken/Loose	25
e82	WornLockDog	1	e101	UN-SupportedChannelRod	25
e83	SeizedLockRod/Nuts	5	e102	7/8" Pin Worn	50
e84	Lock Blade Binding on Machine Case	1	e103	BrokenRollers	25
g34	Machines2		g41	Backdrive3	
g35	SuplementaryDetection		e104	Bush Wear	50
g36	Suplementary Decision1		e105	BrokenDamagedRoller Stocks	25
e85	Chaffed Cable/Wire Missing/Incorrectly FittedCable Glands	1	e106	BrokenStretcher	10
e86	LooseMountingPlate	25	e107	BrokenWornCranks	10
e87	Centre StudRunningLoose	1	g57	S&T Standars	
e88	FaultMocroswitch	25	g58	S&T Standars1	
e89	Blocked Drain Holes/Wear Seal I Lid	1	e146	4 Port Pump	10
g37	Suplementary Detection2		e147	Float&FixedLockConvention	10
e90	BR detector WrongBlades	1	e148	No differential N&R Also F&B Port	10
e91	Lose/MissingPackers	5	e149	SMS'sneed to improve	10
e92	Lack of Stroke Inadequate Maintenance	25	e150	ComponentDesign	10
e93	Tamped	1	e151	4 Port Pump Unable to do correspondence	10
g38	Backdrive		g59	Trainig	
g39	Backdrive1		g60	Training1	

e94	Lack of 50mm Flangeway	25	e152	Poor Maintenance	10
e95	Nuts Run Back	50	e153	Difficulty Fault Finding on Supplementary Detectors	10
e96	ObstructionsWeeds/Growth	1	e154	EquipmentincorrectlyFitted	10
e97	ObstructionBallast	25	e155	M63 Machines Special Tools Needed for Set -Up	10
e98	LooseFittings	50	g61	Trainig2	
g40	Backdrive2		e156	EquipmentincorrectlyMaintained	10
e99	No Escapement on Supplementar Drive	50	e157	"O" Ring Incorrectly Fitted in Supplementary Detector on Replacement of Microswitch	10
e100	ShoesBroken/Loose	25	e158	Difficulty in Identifying "Run Through"	10
e101	UN-SupportedChannelRod	25	e159	RLC LockSlide Inaccesible	10

Point Machine Fault Tree



Parameters of the Events' Probability Functions

Event	K	λ	m	θ	Event	K	λ	m	θ
e1	0,02	0	0	0	e81	0,07	0	0	0
e2	0	0	0,03	0	e82	0	0,023	0	0
e3	0	0,02	0	0	e83	0	0	0,01	0
e4	0	0	0,08	0	e84	0,05	0	0	0
e5	0	0,5	0	0,3	e85	0	0	0,02	0
e6	0	0	0,01	0	e86	0	0	0,02	0
e7	0,01	0	0	0	e87	0	0	0,03	0
e8	0,05	0	0	0	e88	0,02	0	0	0
e9	0,1	0	0	0	e89	0,02	0	0	0
e10	0,1	0	0	0	e90	0,02	0	0	0
e11	0,05	0	0	0	e91	0,03	0	0	0
e12	0,01	0	0	0	e92	0,08	0	0	0
e13	0	0	0,02	0	e93	0	0	0,04	0
e14	0	0,03	0	0	e94	0,05	0	0	0
e15	0,01	0	0	0	e95	0	0	0,05	0
e16	0	0	0,05	0	e96	0,052	0	0	0
e17	0	0,6	0	0,4	e97	0,02	0	0	0
e18	0	0	0,04	0	e98	0	0,2	0	0,15
e19	0,01	0	0	0	e99	0	0	0,05	0
e20	0	0	0,021	0	e100	0	0	0,06	0
e21	0	0	0,02	0	e101	0,04	0	0	0
e22	0,005	0	0	0	e102	0,06	0	0	0
e23	0	0	0,03	0	e103	0	0,23	0	0,22
e24	0,04	0	0	0	e104	0,04	0	0	0
e25	0,02	0	0	0	e105	0	0,4	0	0,15
e26	0	0	0,041	0	e106	0	0,3	0	0,26
e27	0	0	0,01	0	e107	0	0,5	0	0,4
e28	0	0,01	0	0	e108	0,04	0	0	0
e29	0	0	0,01	0	e109	0,04	0	0	0
e30	0,05	0	0	0	e110	0	0,04	0	0
e31	0	0	0,01	0	e111	0,04	0	0	0
e32	0	0	0,02	0	e112	0,05	0	0	0
e33	0	0,01	0	0	e113	0,021	0	0	0
e34	0,05	0	0	0	e114	0,025	0	0	0
e35	0,05	0	0	0	e115	0	0	0,05	0
e36	0,06	0	0	0	e116	0,2	0	0	0
e37	0	0	0,03	0	e117	0	0	0,05	0
e38	0,04	0	0	0	e118	0	0	0,063	0
e39	0	0	0,04	0	e119	0	0,6	0	0,23
e40	0	0	0,01	0	e120	0,3	0	0	0
e41	0,025	0	0	0	e121	0,08	0	0	0
e42	0	0	0,01	0	e122	0,09	0	0	0
e43	0,01	0	0	0	e123	0	0	0,04	0
e44	0	0,015	0	0	e124	0,047	0	0	0
e45	0	0	0,012	0	e125	0	0,01	0	0
e46	0,05	0	0	0	e126	0	0,03	0	0
e47	0,05	0	0	0	e127	0	0,05	0	0
e48	0	0,4	0	0,3	e128	0	0	0,02	0
e49	0	0	0,08	0	e129	0,041	0	0	0
e50	0,04	0	0	0	e130	0,04	0	0	0
e51	0	0,4	0	0,5	e131	0,05	0	0	0

e52	0,03	0	0	0	e132	0	0	0,02	0
e53	0,01	0	0	0	e133	0	0	0,02	0
e54	0,054	0	0	0	e134	0,06	0	0	0
e55	0,045	0	0	0	e135	0,06	0	0	0
e56	0,01	0	0	0	e136	0,2	0	0	0
e57	0,04	0	0	0	e137	0,5	0	0	0
e58	0,02	0	0	0	e138	0,5	0	0	0
e59	0	0	0,07	0	e139	0,04	0	0	0
e60	0	0,35	0	0,15	e140	0	0,55	0	0,2
e61	0	0	0,01	0	e141	0	0	0,01	0
e62	0	0	0,04	0	e142	0,04	0	0	0
e63	0,02	0	0	0	e143	0,2	0	0	0
e64	0,03	0	0	0	e144	0,3	0	0	0
e65	0,014	0	0	0	e145	0	0,06	0	0
e66	0,02	0	0	0	e146	0,01	0	0	0
e67	0,01	0	0	0	e147	0,01	0	0	0
e68	0	0	0,03	0	e148	0,01	0	0	0
e69	0	0	0,014	0	e149	0,01	0	0	0
e70	0,045	0	0	0	e150	0,01	0	0	0
e71	0,052	0	0	0	e151	0,01	0	0	0
e72	0	0	0,02	0	e152	0,01	0	0	0
e73	0,01	0	0	0	e153	0,01	0	0	0
e74	0,01	0	0	0	e154	0,01	0	0	0
e75	0	0	0,01	0	e155	0,01	0	0	0
e76	0,02	0	0	0	e156	0,01	0	0	0
e77	0,03	0	0	0	e157	0,01	0	0	0
e78	0,03	0	0	0	e158	0,01	0	0	0
e79	0	0	0,032	0	e159	0,01	0	0	0
e80	0,03	0	0	0					

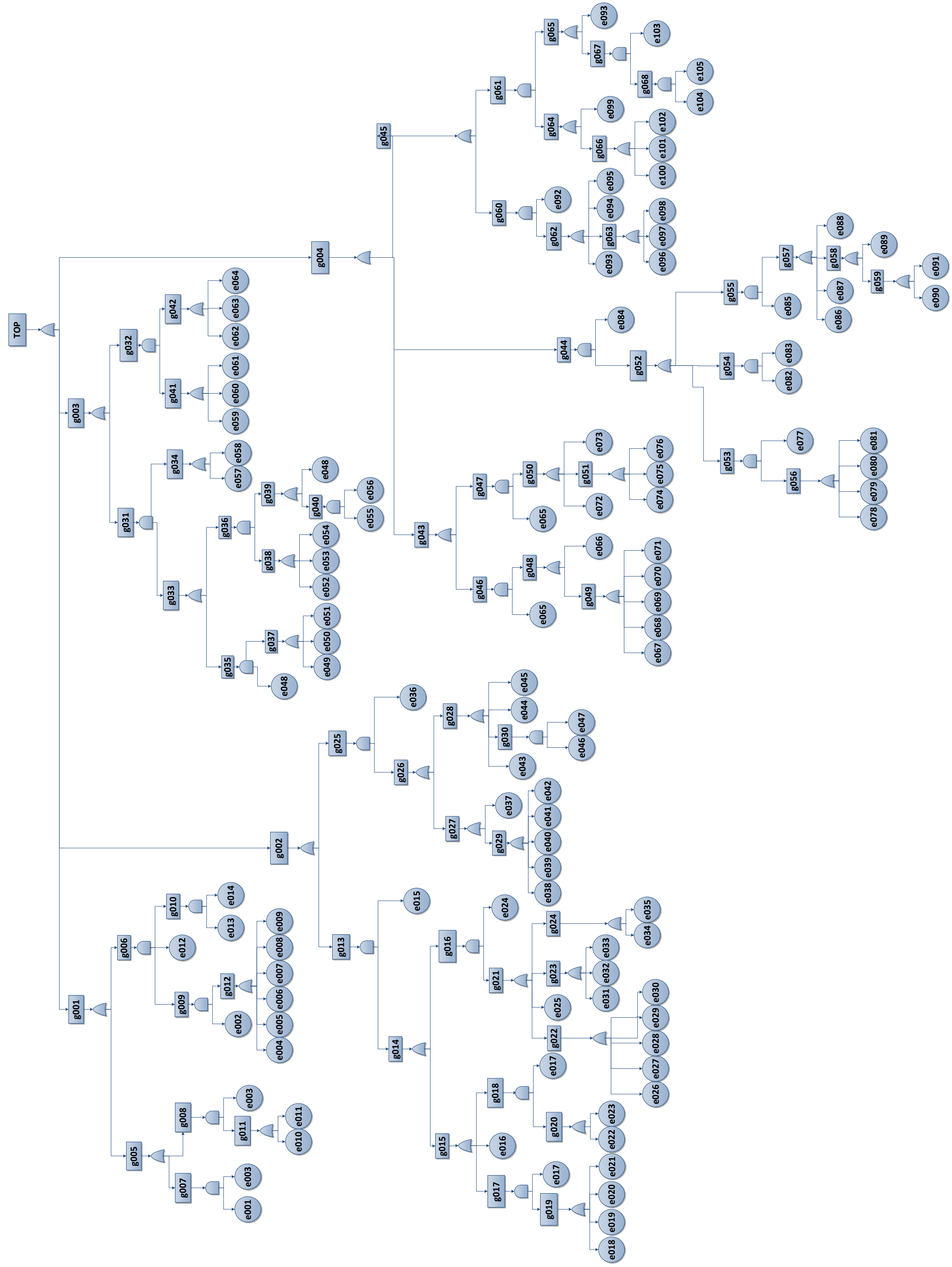
ANNEX IX WIND TURBINE FAULT TREE

Fault Tree 1 Foundation and Tower Failure				Probabilistic Model Assignment
Intermediate Event	**Code**	**Final Event**	**Code**	
Yaw System Failure	g005	Yaw motor fault	e001	Constant
Critical Structural Failure	g006	Abnormal Vibration I	e002	Linear Increasing
yaw motor failure	g007	Abnormal Vibration H	e003	Linear Increasing
Wrong Yaw Angle	g008	Cracks in concrete base	e004	Constant
Structural Failure (Foundation and tower)	g009	Welding damage	e005	Constant
No electric power for yaw motor	g010	Corrosion	e006	Linear Increasing
Metereologhical Unit Failure	g011	Loosen studs in joining foundation and first section	e007	Linear Increasing
Structural Fault (Foundation and tower)	g012	Loosen bolts in joining different sections	e008	Linear Increasing
		Gaps in the foundation section	e009	Exponential Increasing
		Vane damage	e010	Exponential Increasing
		Anemometer damage	e011	Exponential Increasing
		High wind speed	e012	Periodic
		No power supply from generator	e013	Constant
		No power supply from grid	e014	Constant

Fault Tree 2 Critical Rotor Failure				Probabilistic Model Assignment
Intermediate Event	**Code**	**Final Event**	**Code**	
Critical blade failure	g013	High wind speed	e015	Periodic
Blade Failure	g014	Blade Angle asymmetry	e016	Exponential Increasing
Pitch System Failure	g015	Abnormal Vibration A	e017	Exponential Increasing
Critical structural Failure (Blades)	g016	Motor failure	e018	Exponential Increasing
Hydraulic system Failure	g017	Leakages	e019	Constant
Wrong Blade Angle	g018	Over pressure	e020	Constant
Hydraulic system Fault	g019	Corrosion	e021	Exponential Increasing
Metereologhical Unit Failure	g020	Vane damage	e022	Constant
Structural Failure (Blades)	g021	Anemometer damage	e023	Constant
Leading and traililling edges	g022	Abnormal Vibration B	e024	Constant
Shell	g023	Root Cracks	e025	Constant
Tip	g024	Cracks	e026	Constant
Rotor System Failure	g025	Erosion	e027	Exponential Increasing
Rotor System Fault	g026	Delamination in leading edges of blades	e028	Exponential Increasing
Bearings (Rotor)	g027	Delamination in trailing edges of blades	e029	Exponential Increasing
Rotor Hub	g028	Debonding in edges of blades	e030	Exponential Increasing
Wear	g029	Delamination in shell	e031	Exponential Increasing
Imbalance	g030	Crack with structural damage	e032	Constant
		Crack on the beam-shell joint	e033	Constant
		Open tip	e034	Constant
		Lightning strike	e035	Periodic
		Abnormal Vibration C	e036	Constant
		Cracks	e037	Constant
		Corrosion of Pins	e038	Exponential Increasing
		Abrasive Wear	e039	Exponential Increasing
		Pitting	e040	Linear Increasing
		Deformation of face & rolling element	e041	Linear Increasing
		Lubrication Fault	e042	Linear Increasing

		Clearance loosening at root	e043	Exponential Increasing
		Cracks	e044	Constant
		Surface Roughness	e045	Constant
		Mass Imbalance	e046	Exponential Increasing
		Fault in Pitch adjustment	e047	Exponential Increasing
Fault Tree 3 Electrical Components Failure				**Probabilistic Model Assignment**
Intermediate Event	**Code**	**Final Event**	**Code**	
Critical Generator Failure	g031	Abnormal Vibration G	e048	Exponential Increasing
Power Electronics and Electric Controls Failure	g032	Cracks	e049	Constant
Mechanical Failure (Generator)	g033	Imbalance	e050	Exponential Increasing
Electrical Failure (Generator)	g034	Asymmetry	e051	Exponential Increasing
Bearing Generator Failure	g035	Air-Gap eccentricities	e052	Linear Increasing
Rotor and Stator Failure	g036	Broken bars	e053	Linear Increasing
Bearing Generator Fault	g037	Dynamic eccentricity	e054	Linear Increasing
Rotor and Stator Fault	g038	Sensor T error	e055	constant
Abnormal Signals A	g039	T above limit	e056	Periodic
Overwarming generator	g040	Short Circuit (Gen)	e057	Constant
Electrical Fault (PE)	g041	Open Circuit (Gen)	e058	Constant
Mechanical Fault (PE)	g042	Short Circuit	e059	Constant
		Open Circuit	e060	Constant
		Gate drive circuit	e061	linear increasing
		Corrosion	e062	Periodic
		Dirt	e063	Periodic
		Terminals damage	e064	linear increasing
Fault Tree 4 Power Train FAILURE				**Probabilistic Model Assignment**
Intermediate Event	**Code**	**Final Event**	**Code**	
Low speed train Failure	g043	Abnormal Vibration D	e065	Constant
Critical Gearbox Failure	g044	Cracks in main bearing	e066	Constant
High speed train Failure	g045	Spalling	e067	Linear Increasing
Main Bearing failure	g046	Corrosion of Pins	e068	Linear Increasing
Low speed shaft failure	g047	Abrasive Wear	e069	Constant
Main Bearing fault	g048	Deformation of face & rolling element	e070	Linear Increasing
Wear main bearing	g049	Pitting	e071	exponential increasing
Low speed shaft fault	g050	Imbalance	e072	Constant
Wear low shaft	g051	Cracks in l.s. shaft	e073	Linear Increasing
Gearbox Fault	g052	Spalling	e074	Constant
Bearings failure(Gearbox)	g053	Abrasive Wear	e075	Constant
Lubrication fault	g054	Pitting	e076	Constant
Gear Failure	g055	Abnormal Vibration F	e077	Linear Increasing
Wear bearing gearbox	g056	Corrosion of Pins	e078	Exponential Increasing
Gear Fault	g057	Abrasive Wear	e079	Linear Increasing
Tooth Wear	g058	Pitting	e080	Constant
Offset	g059	Deformation of face & rolling element	e081	Linear Increasing
High speed shaft Failure	g060	Oil Filtration	e082	Constant
Critical Brake Failure	g061	Particle Contamination	e083	Exponential Increasing
High speed structural damage	g062	Overwarming gearbox	e084	Linear Increasing
Wear high shaft	g063	Abnormal Vibration E	e085	Periodic
Brake Fault	g064	Eccentricity	e086	Constant
Abnormal Signals B	g065	Pitting	e087	Linear Increasing
Hydraulic brake system Fault	g066	Cracks in gears	e088	Exponential Increasing
Abnormal Signals C	g067	Gear tooth deterioration	e089	Exponential Increasing
Overwarming brake	g068	Poor design	e090	Periodic
		Tooth surface defects	e091	Constant

	Abnormal Vibration J	e092	Constant
	Cracks in h.s. shaft	e093	Linear Increasing
	Imbalance	e094	Periodic
	Overwarming	e095	Exponential Increasing
	Spalling	e096	Constant
	Abrasive Wear	e097	Linear Increasing
	Pitting	e098	Constant
	Cracks in brake disk	e099	Exponential Increasing
	Motor brake fault	e100	Constant
	Oil Leakage	e101	Linear Increasing
	Over pressure	e102	Constant
	Abnormal speed	e103	Linear Increasing
	T sensor error	e104	Periodic
	T above limit	e105	Periodic


TOP
g003
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ANNEX X CASE STUDY FOR DM

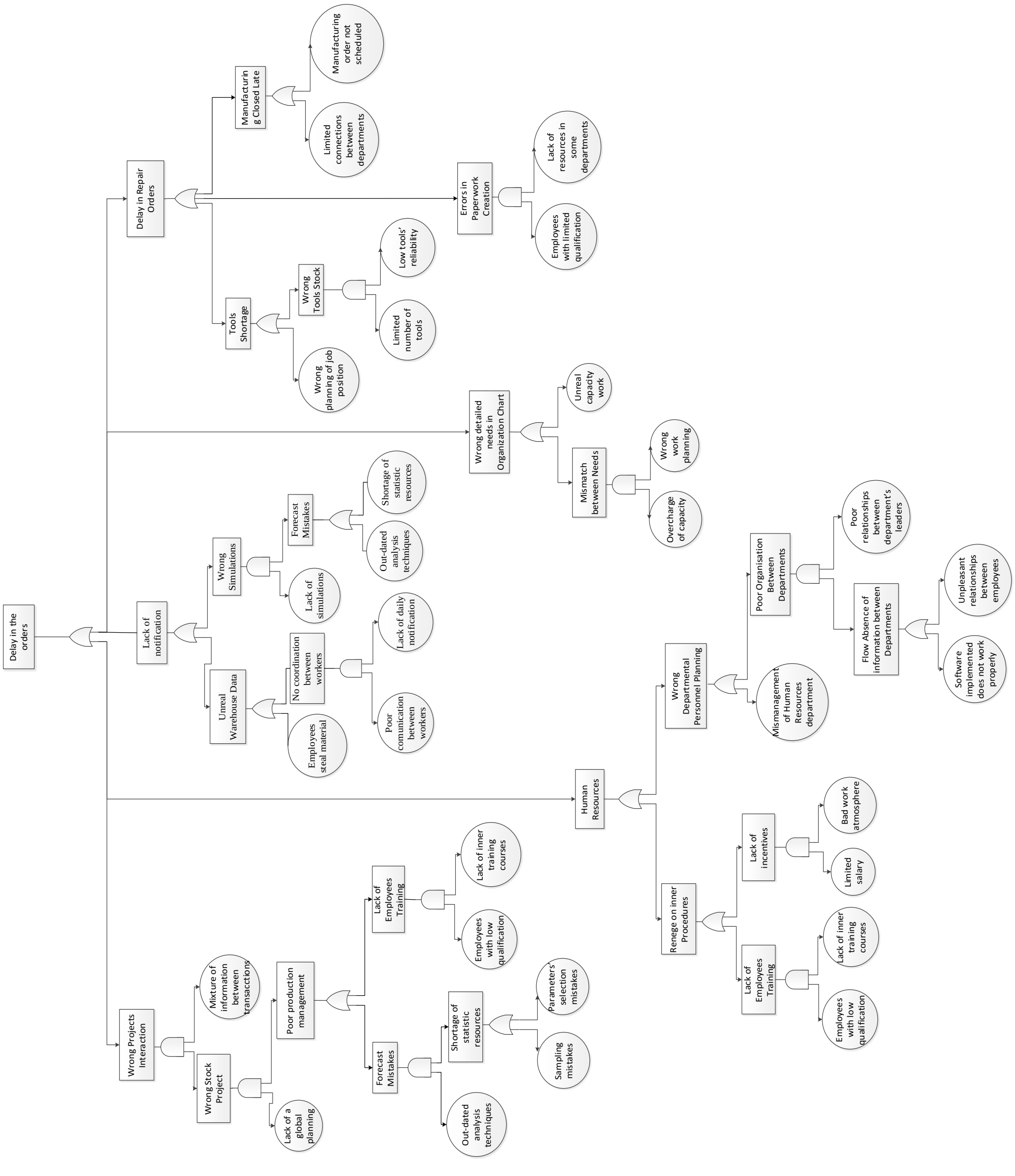

Figure XXV. LDT for DM case study

ANNEX XI LOGICAL TREE FOR TOPOLOGY STUDY