

# **Position Control of a Flexible Link Robot Moving under Gravity with a Novel Gravity Compensation System**



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I dedicate this thesis to my beloved parents, Carlos and Ilse, and my dear brother, Alfredo.



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# Nomenclature

## Variables

$\mathbf{A}_{\text{alg}}(t), \mathbf{A}_{\text{alg}}[k]$  Time varying matrix involved in the algebraic identification procedure expressed in continuous and discrete time, respectively.

$\mathbf{A}_{\text{lms}}[k]$  Time varying matrix involved in the payload identification based on least mean squares.

$a_{ij}(t)$  Time varying coefficients of the matrix  $\mathbf{A}$  of the algebraic identifier ( $i = 1, 2; j = 1, 2$ ).

$\mathbf{B}_{\text{alg}}(t), \mathbf{B}_{\text{alg}}[k]$  Time varying vector involved in the algebraic identification procedure (continuous and discrete form).

$\mathbf{B}_{\text{lms}}[k]$  Time varying vector involved in the payload identification based on least mean squares.

$b_i(t)$  Time varying coefficients of the vector  $\mathbf{B}$  of the algebraic identifier ( $i = 1, 2$ ).

$D(s)$  Equivalent current disturbance at the input of the motor represented in the Laplace domain.

$D_{\text{eff}}(t)$  Distance to which the spring needs to be stretched in order to produce the effective force  $F_{\text{eff}}(t)$ .

$e_i(t)$  Tracking error for the state  $z_i$  ( $i=1,2,3,4$ ).

$F(x, t)$  Force measured at the point  $x$  along the flexible link.

$F_{\text{eff}}(t)$  Effective force that the spring exerts on the flexible link.

$\hat{F}_{\text{eff}}(t)$  Estimation of the effective force that exerts the spring on the flexible link.

$F_{\text{ext}}(t)$  External force acting at the tip position of the flexible link.

- $\vec{F}_{spring}(t)$  Total force that the spring exerts on the flexible link.
- $\mathbf{f}(\mathbf{x}, t)$  Vector field describing dynamics of the flexible link robot system.
- $f_d(t)$  External distributed force acting on the flexible link at time.
- $\mathbf{g}(\mathbf{x}, t)$  Vector field describing dynamics of the flexible link robot system.
- $i_c(t)$  Fictitious motor control signal that takes into account the compensation of the static/Coulomb friction of the motor.
- $i_m(t)$  Motor armature current.
- $k$  Discrete time instance.
- $\hat{m}_{tip}[k]$  Payload estimation for the discrete sample  $k$ .
- $q(t)$  Output of a time-varying linear filter considered in the algebraic identifier.
- $\check{q}(t), \check{q}[k]$  Output of time-varying linear filters considered in the algebraic identifier (continuous and discrete form).
- $R_1(s), R_2(s)$  Transfer functions that form part of the two degrees-of-freedom motor position controller.
- $s$  Laplace complex variable.
- $t$  Time variable.
- $u_{i,j}(t)$  Polynomial coefficients describing the deflection of the flexible link.
- $V_{gauge}(t)$  Output voltage of the strain gauge amplifier.
- $\hat{x}_i(T)$  Estimated state variable  $x$  ( $i=1,2,3,4$ ).
- $\mathbf{x}(t)$  Space-state vector.
- $x(t)$  State variable ( $i=1,2,3,4$ ).
- $\mathbf{z}(t)$  Transformed vector of states.
- $z_i(t)$  Transformed state ( $i = 1,2,3,4$ ).
- $z_i(t)$  Space state variable used to describe the algebraic identifier.
- $z_i^*(t)$  Reference for the transformed state  $z_i$  ( $i = 1,2,3,4$ ).

- $\beta(t), \beta[k]$  Output of time-varying linear filters considered in the algebraic identifier (continuous and discrete form).
- $\Gamma_{st-coul}(t)$  Static/Coulomb friction torque in the motor.
- $\theta_{int}(t)$  Angular position of the intermediate lumped mass in the two lumped mass dynamic model.
- $\Gamma(x, t)$  Torque measured at the point  $x$  along the flexible link.
- $\Gamma_{coup}(t)$  Coupling torque between the flexible link and the motor.
- $\Gamma_{ext}(t)$  External torque acting at the tip position of the flexible link.
- $\Gamma_m(t)$  Motor torque measured at the output of the gearbox.
- $\Gamma_{motor-axis}(t)$  Total torque about the motor axis.
- $\Gamma_{pt-attach}(t)$  Torque at the point of attachment of the spring.
- $\varepsilon(t)$  Cost function of the algebraic identifier.
- $\eta(t), \eta[k]$  Output of time-varying linear filters considered in the algebraic identifier (continuous and discrete form).
- $\hat{\theta}_t(t)$  Estimation of the angular tip position.
- $\theta_a(t)$  Angular position of the attachment point of the spring.
- $\theta_i(t)$  Angular position of the lumped mass  $m_i$  of the general lumped mass dynamic model.
- $\theta_m(t)$  Angular position of the motor measured at the output of the gearbox.
- $\theta_t(t)$  Angular position of the tip of the flexible link.
- $\hat{\mu}_c[k]$  Estimate of the damping coefficient of the flexible link arm described by a single lumped mass for the discrete time sample  $k$ .
- $\xi(t), \xi[k]$  Output of time-varying linear filters considered in the algebraic identifier (continuous and discrete form).
- $\phi_{lms}[k]$  Time varying vector involved in the payload estimation based on least mean squares.
- $\chi(s)$  Variable that represent the Laplace transform of the time function  $\cos(\theta_t(t))$

$\psi_i(\mathbf{z})$  Functions used to express the linearizing control law of the input-state controllers ( $i=1,2$ ).

### Constants

$a_m$  Parameter of the motor model in its Laplace description.

$b_m$  Parameter of the motor model in its Laplace description.

$E$  Young's modulus of the flexible link material.

$m_{int}$  Intermediate lumped mass considered in the two lumped mass flexible link model.

$g$  Gravitational constant.

$I_z$  Area moment of inertia of the cross section of the flexible link.

$J$  Equivalent rotational inertia of the motor and the gearbox measured at the output of the gearbox.

$k_i$  Coefficients of the desired characteristic equation.

$K_p, K_v, K_i$  Gain of PID controllers for the motor position control.

$k_{gauge-attach}$  Proportionality constant relating the strain gauge measurements at the point at which the springs are attached to the flexible link with the torque at the same point.

$k_{gauge-base}$  Proportionality constant relating the strain gauge measurements at the base of the spring to the coupling torque between the link and the motor.

$k_m$  Electromechanical constant of the motor.

$k_{spring}$  Spring constant.

$L_i$  Distance between the gearbox shaft and the far end of the  $i$ -th massless flexible element.

$l_i$  Length of the  $i$ -th massless flexible element considered in the lumped mass dynamic model.

$l_s$  Distance between the motor rotation axis and the attachment point of the spring along the axis  $Y_0$ .

---

|                |                                                                                                                                              |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| $M$            | Length of the window considered in the computation of the moving standard deviation and moving average for the algebraic identifier.         |
| $m_a$          | Overall equivalent contribution made by the link mass and the payload mass to the inertial term of the flexible link dynamics.               |
| $m_g$          | Overall equivalent contribution made by the link mass and the payload mass to the gravitational term of the flexible link dynamics.          |
| $m_i$          | Value of the $i$ – $th$ mass of the lumped mass dynamic model.                                                                               |
| $m_{link}$     | Link mass.                                                                                                                                   |
| $m_{link-ai}$  | Fictitious masses used to model the link mass contributions to the acceleration terms in the dynamics of the flexible link arm ( $i=1,2$ ).  |
| $m_{link-gi}$  | Fictitious masses used to model the link mass contributions to the gravitational terms in the dynamics of the flexible link arm ( $i=1,2$ ). |
| $m_{tip}$      | Payload mass supported by the flexible link at its tip.                                                                                      |
| $m_{tip-cji}$  | Payload mass number ( $i = 1, 2, 3$ ) of the set number ( $j = 1, 2$ ), considered in the experimental setup with the composite link.        |
| $m_{tip-di}$   | Payload mass number $i$ , considered in the experimental setup with the duraluminum link.                                                    |
| $n$            | Number of massless flexible elements considered in the lumped mass dynamics model.                                                           |
| $n_r$          | Gear ratio.                                                                                                                                  |
| $p_i$          | Position of the poles of the inner closed loop.                                                                                              |
| $p_o$          | Position of the poles of the outer closed loop.                                                                                              |
| $\mathbf{R}^n$ | Real coordinate space of $n$ dimensions.                                                                                                     |
| $t_s$          | Sampling time.                                                                                                                               |
| $X$            | Horizontal axis of the rotating coordinate frame.                                                                                            |
| $X_0$          | Horizontal axis of the fixed coordinate frame.                                                                                               |

---

|                              |                                                                                                                                                  |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| $Y$                          | Vertical axis of the rotating coordinate frame.                                                                                                  |
| $Y_0$                        | Vertical axis of the fixed coordinate frame.                                                                                                     |
| $\mathbf{0}_{[ixj]}$         | Null matrix of dimensions $ixj$ .                                                                                                                |
| $\alpha$                     | Parameter describing the closed loop dynamics of the motor.                                                                                      |
| $\gamma_i$                   | Parameters describing the dynamics of the single link flexible arm with gravity compensation ( $i=1,2$ ).                                        |
| $\Gamma_c$                   | Static/Coulomb friction constant.                                                                                                                |
| $\Delta$                     | Tolerance parameter considered in the switching-off considerations of the algebraic identifier.                                                  |
| $\zeta_i$                    | Parameters describing the two-degrees-of-freedom motor controller ( $i=1,2$ ).                                                                   |
| $\iota$                      | Parameters describing the two-degrees-of-freedom motor controller ( $i=1,2$ ).                                                                   |
| $\kappa_i$                   | Parameters describing the two-degrees-of-freedom motor controller ( $i=1,2$ ).                                                                   |
| $\lambda$                    | Proportionality constant relating the total length of the link to the distance between the motor axis and the point of attachment of the spring. |
| $\mu_{ci}$                   | Viscous friction coefficients in the flexible link dynamics ( $i=1,2$ ).                                                                         |
| $\nu$                        | Viscous friction coefficient in the motor dynamics.                                                                                              |
| $\rho_L$                     | Mass linear density of the flexible link.                                                                                                        |
| $\rho_i$                     | Parameters describing the two-degrees-of-freedom motor controller ( $i=1,2$ ).                                                                   |
| $\Omega$                     | Closed connected region in $\mathbf{R}^n$ .                                                                                                      |
| $\omega_{1st}, \omega_{2nd}$ | First and second vibrational frequency of the flexible link.                                                                                     |

### Operators

|                                |                                                                                                                    |
|--------------------------------|--------------------------------------------------------------------------------------------------------------------|
| $adj_{\mathbf{f}}\mathbf{g}$   | Lie brackets of the vector fields $\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{g}(\mathbf{x}, t)$ .                    |
| $adj_{\mathbf{f}}^i\mathbf{g}$ | Repeated Lie brackets of order $i$ the vector fields $\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{g}(\mathbf{x}, t)$ . |
| $E [ ]$                        | Moving average.                                                                                                    |
| $F_{lp} [ ]$                   | Low pass Butterworth filter.                                                                                       |

---

|                        |                                                                 |
|------------------------|-----------------------------------------------------------------|
| $L_{\mathbf{f}} z_1$   | Lie derivative of $z_1$ with respect to $\mathbf{f}$ .          |
| $L_{\mathbf{f}}^i z_1$ | Repeated Lie derivative of $z_1$ with respect to $\mathbf{f}$ . |
| $\mathcal{L}$          | Laplace transform operator.                                     |
| $\sigma [ ]$           | Moving standard deviation.                                      |
| $\nabla$               | Gradient operator.                                              |

### Acronyms and abbreviations

|         |                                                                                                                                                                       |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| FFT     | <b>F</b> ast <b>F</b> ourier <b>T</b> ransform.                                                                                                                       |
| FPI     | Research Staff Training Scholarship ( <i>Beca de <b>F</b>ormación de <b>P</b>ersonal <b>I</b>nvestigador</i> ).                                                       |
| FRF     | <b>F</b> requency <b>R</b> esponse <b>F</b> unction.                                                                                                                  |
| INEI    | Institute of Energy Research and Industrial Applications ( <i>Instituto de <b>I</b>nvestigaciones <b>E</b>nergéticas y <b>A</b>plicaciones <b>I</b>ndustriales</i> ). |
| Labview | <b>L</b> aboratory <b>V</b> irtual <b>I</b> nstrument <b>E</b> ngineering <b>W</b> orkbench Software.                                                                 |
| Matlab  | <b>M</b> atrix <b>L</b> aboratory Software.                                                                                                                           |
| NASA    | The <b>N</b> ational <b>A</b> eronautics and <b>S</b> pace <b>A</b> dministration.                                                                                    |
| PID     | <b>P</b> roportional- <b>I</b> ntegral- <b>D</b> erivative controller.                                                                                                |



# Chapter 1

## Introduction

In the case of most robot manipulators, it is a common practice to consider heavy materials and bulky designs in order to prevent the appearance of vibrations and static deflections in their structure and, therefore, achieve a good accuracy in the positioning of the end-effector. Although these design guidelines have played a key role in the performance of robot manipulators, they are also a source of inefficiency as regards energy consumption, higher building costs, lower operational speeds and higher risk for human operators. Building flexible link robot manipulators, whose main features are lighter materials and slender designs, is an excellent alternative to the traditional manipulators that provides significant benefits in all the aforementioned areas. However, these advantages are attained at the cost of having to deal with the static deflections and vibration induced in their structure owing to the low-stiffness or flexibility of their links. During the attempts to deal with these disadvantages, great efforts have been made to carry out sensor development, flexible dynamic modeling and control strategies.

This thesis is concerned with the dynamic modeling and control of manipulators that have some flexibility distributed along their links. It places particular emphasis on the study of manipulators whose movements are constrained to a vertical plane and, therefore, affected by gravity. It is worth mentioning that no attention is paid to manipulators with flexible joints.

### 1.1 Framework of the thesis

This thesis falls within the scope of two research projects funded by the Spanish Ministry of Economy and Competitiveness undertaken by the Robotics and Automation Group at the University of Castilla-La Mancha, headed by Professor Vicente Feliu Batlle.

The first project is entitled “Design and Control of a very Lightweight Flexible Robot Made of Composite Material. Application to a Robotic based inspection system” and has the reference code DPI2009-09956. The aim of the project is to build and control a multiple degrees-of-freedom flexible link robot made of composite materials and mounted on a mobile platform designed for inspection tasks.

The second project complements and extends the aforementioned project. It is entitled "Development and Comparison of New Control Techniques for Very Large Lightweight Flexible Arms Mounted on Mobile Platforms Under Environmental Disturbances" and has the reference code DPI2012-37062-CO2-01. The aim of this project is to develop controllers that actively cancel the vibrations induced in the aforementioned flexible arm-mobile platforms as they move on an irregular terrain, interact with the wind and bump into objects.

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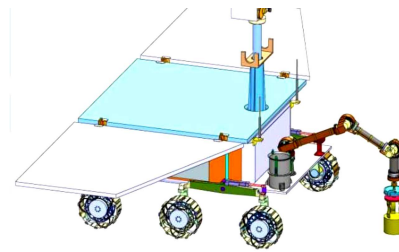
## 1.2 Motivation

The first studies on flexible link robotics were funded by a NASA Research Program in the late seventies. This Research Program was interested in reducing the transporting cost of robot manipulators to outer space. The strategy followed was to reduce the weight of the manipulators by considering slender arms and lighter materials than those that had been used in conventional manipulators. Unfortunately, the same properties that permitted weight reduction were also responsible for significant reductions in accuracy owing to the appearance of undesired vibrations. Since those initial studies the great efforts made in sensor development, dynamics modeling and controller design have led to important improvements in the performance of these robots. The best-known examples of the use of flexible link robots for space applications are currently the Remote Manipulator Systems Canadarm and its successor, Canadarm2 (depicted in Fig 1.1a), installed in the International Space Station (ISS). These manipulators have been used for a wide range of maintenance operations that include moving supplies, equipment and astronauts, and even performing the so-called "cosmic catches", during which unmanned spacecrafts are captured and docked. These spacecrafts normally carry scientific material and other essential items for the crew living aboard the ISS [3]. At the time of writing this thesis, the ESA is carrying out the

DELIAN project (DExterous LIghtweight ANthropomorphicarm) within the ESA's Mars Robotic Exploration Preparation Programme [4]. The objective of the DELIAN project is to develop the next generation of lightweight manipulators to be mounted on spatial rovers and landers, and which would be in charge of the sample acquisition, sample retrieval and instrument deployments in future expeditions to Mars. Given the limitations of the mass, power, volume and the dimensions of these manipulators, very flexible arms are being considered. Fig 1.1b shows a sketch of one of the lightweight manipulators under development, within the framework of the DELIAN project, considered for the task of acquiring of a cached sample.



(a) Photograph of the Remote Manipulator System, Canadarm2 [3].



(b) Sketch of a DELIAN manipulator [4].

Fig. 1.1 Examples of applications of flexible link robots in space.

The flexible link manipulators also have very relevant applications on earth. In this respect, understanding flexible link manipulators not only paves the way toward a new generation of manipulators with enhanced features, but also represents a source of improvement for the existing manipulators. While considering very rigid links is currently a clear requirement in the design of robotics manipulators for industry, the performance of these robots in those tasks that require high precision and reduced cycle times is also severely limited by the appearance of residual vibrations, which may prevail a period of time after a maneuver is completed [5]. This is a clear opportunity to transfer some of the advances in dynamic modeling and control algorithm applied to flexible structures to the already established technologies. Another aspect that could possibly benefit from flexible link robotics is industrial safety. In spite of the progress in robotics cognition, some operations with industrial manipulators currently still involve human supervision and collaboration, especially for those tasks that are not sufficiently structured. Human-robot cooperation in the same workspace without perimeter guarding is unusual for safety reasons, and is severely limited, in part because of the large masses of the links. The EN ISO 10218 "Robots and robotic devices - Safety requirements for industrial robots" regulation establishes highly restricted speed and power limits for robotics manipulators collaborating with humans in the same workspace. Finding

new ways to relax these limits without endangering the health of human operators could be a game changer for the industry that would the human general knowledge, creativity and problem solving skills to be brought to the task execution. There is a great expectancy that flexible link robots could meet these safety requirements in the future given their properties of low inertia and implicit mechanical compliance.

It is also worth mentioning applications other than those that are industrial. In the early 1990s, Russell et al. in [6] and Kaneko et al. in [7] proposed using flexible link structures, referred to as whiskers or antennas, as tactile sensors to explore the environment. This application was biologically inspired, principally by rodents and pinnipeds which rely on whiskers for a wide range of activities that include navigation and object exploration. Artificial whisker sensors are especially useful in dusty and cloudy environments and, unlike the most classical alternatives, do not depend on the emission and capture of light or sound [1]. Fig 1.2 depicts a 2-degrees-of-freedom whisker platform developed in the Robotics and Automation Group at the University of Castilla-La Mancha in Spain. Another potential application is the use of very large and powerful flexible link manipulators to cooperate in maintenance tasks at nuclear power plants and decontamination. The authors of [8, 9] proposed, for example, the use of very large flexible link manipulators to repair leakages in underground storage tanks and reinforced concrete above ground silos containing radioactive waste by-products.

The research area regarding rescue and inspection tasks is especially attractive for flexible link manipulators as they reduce the energy consumption and the overall weight of the platform, two desirable characteristics in a mobile application. A good example of this idea is Gryphon [2], a mobile robot developed to assist in the demining process (see Fig 1.3). It consists of a 3 degrees-of-freedom very long arm manipulator with a 2 degrees-of-freedom wrist mechanism mounted on a four wheeled all terrain vehicle. Given the link flexibility

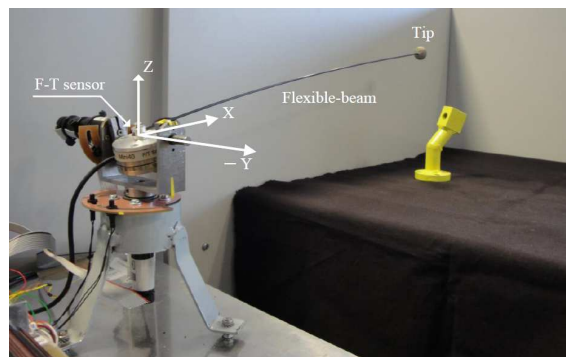


Fig. 1.2 Example of terrestrial applications of flexible link robots: photograph of a 2-DOF whisker tactile sensor [1].

and the motors and vehicle suspension, significant vibrations are induced in the arm structure, which negatively affect the positioning of the detector attached at the end of the arm. A simple solution to this problem is to consider trajectories with very smooth acceleration profiles, but this necessarily increases the time required to complete the detection task. Finding vibration control techniques is an open research problem in these applications which could be particularly challenging if a system's dynamics modeling is considered.

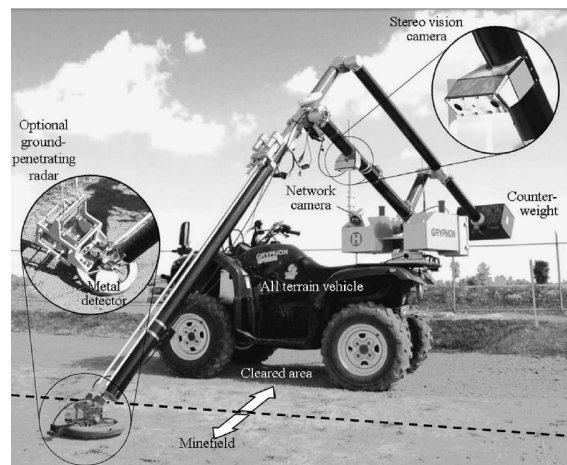


Fig. 1.3 Example of terrestrial applications of flexible link robots: photograph of the Gryphon demining robot [2].

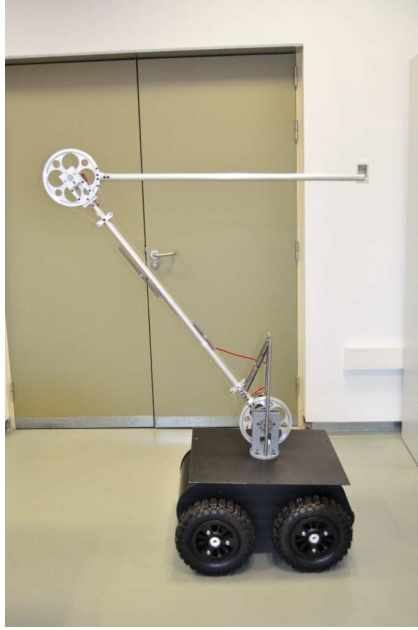
The first step toward considering flexible link manipulators for a wide range of earth applications is to consider these manipulators with movements in a vertical plane and under the action of gravity. However, most of the research carried out to date has been focused on the control of flexible link manipulators moving in a horizontal plane, i.e., under zero gravity conditions. While the dynamic behavior of a single link arm under the assumption of small deflections can be modeled using a linear model, the model of the same platform when subjected to the force of gravity is necessarily non-linear, and is, therefore, a more difficult problem. This problem becomes even more challenging when considering multiple link arms or when considering certain parameter uncertainty. With regard to parameter uncertainty, the payload mass is the most compromised parameter in robots that interact with the environment performing pick and place tasks. In the case of flexible link robots, the uncertainty in this parameter reduces the vibration suppression capabilities, increases the steady-state error and ultimately affects the stability of the closed loop. Since the pick and place task is one of the actions most frequently demanded from a robot manipulator, it is very important to develop control algorithms that are capable of guaranteeing the performance of the robot under these conditions.

One important factor as regard fully exploiting the potential of flexible link robotics for earth applications is that of considering gravity compensation mechanisms. The use of gravity compensation mechanisms leads to benefits in the weight, energy consumption, torque demands and manufacturing costs, and is, therefore, particularly relevant for any flexible robot manipulator, especially those involved in mobile applications. While a considerable amount of literature has been published on the study of gravity compensation mechanisms for rigid manipulators, there has been little research on the use of these mechanism in flexible link robots and only a few applications are reported. The demining Gryphon robot, mentioned previously, is a good example of a gravity compensated flexible link manipulator. In this robot, the gravity affecting the links is compensated by considering a pantograph configuration, whereas the weight of the payload mass is compensated by considering a counter-weight that corresponds to the batteries on the platform. The compensation through counter-weights is achieved by means of an exchange of potential energy between the manipulator and the counter-weights. However, although it is true that the static motor torque demands could be partially or fully canceled, this is achieved at the expense of increasing the dynamic torque requirements in the case of accelerated trajectories, signifying that it is useful only for slow trajectories. This is owing to fact that attaching counter-weights to the link inevitably increases the equivalent rotational moments of inertia that have to be accelerated by the motors. One particular issue of this approach referred in the case of manipulators mounted on mobile robots is that considering a counterweight normally implies raising the location of the center of gravity of the platform, which increases the risk of overturning.

An alternative to the counterweight approach is to consider a gravity compensation mechanism based on springs. In this approach, the gravity compensation is achieved by transforming the gravitational potential energy of the arm into elastic potential energy stored in the springs. The main advantage of using springs is that they do not make an important contribution to the inertia of the manipulator and, consequently, do not significantly alter the position of the center of mass of the system. The greatest disadvantage of this approach is that it normally requires very creative and complex mechanical designs, given the non-linear and pose dependent nature of the gravity affecting the manipulator. No research on the study of spring based gravity compensation mechanisms for flexible link manipulators has been reported to date.

As mentioned in the previous section, this thesis is integrated into the framework of two research projects aligned with the use of flexible link manipulators mounted on mobile robots for inspection task. The research in these projects began with the building of two platforms: (a) a two-link-flexible link robot with two rotational joints and, (b) a single-link prismatic flexible link arm, both mounted on a four-wheeled mobile skid steering car. The

prototypes of these platforms are shown in Fig 1.4. The present thesis is focused on the study of the first degree of freedom of the first of these platforms (manipulator a). The resulting flexible link manipulator considers only one flexible link, and its movements are constrained to a vertical plane and are, therefore, subjected to the force of gravity .



(a) Manipulator A: two-link flexible manipulator with two rotary joints.



(b) Manipulator B: single-link flexible manipulator with a rotary and a prismatic joint.

Fig. 1.4 Photograph of the first prototypes of the flexible link manipulators mounted on a four-wheeled mobile car.

## 1.3 State of the art

This section presents a survey of the main contributions in the field of flexible link robotics. It is organized in two subsections. The first section outlines the main dynamics modeling techniques, while the second provides a summary of the main position control problems and approaches. For a general review of the state-of-the-art the reader is directed to the surveys on flexible link robotics [10, 11].

### 1.3.1 Dynamics modeling

The exact dynamics of a flexible robot is quite complex and is not normally used in the system analysis and the control algorithm design. It is governed by partial differential equa-

tions and, therefore, has an infinite dimension. Scientists working in the field of flexible link robotics have invested a huge amount of effort in the creation of mathematical models that trade accuracy for ease of use. In this section, we describe the most relevant approaches. For a more complete review of the techniques available, please refer to the survey on Dynamic Analysis of Flexible Manipulators by Dwivedy and Eberhard [5].

**Lumped mass approach** In this approach, the flexible link is modeled as a group of massless spring elements interconnected by concentrated masses. In this respect, it is worth mentioning the work of Feliu et al. in [12], in which a dynamic model technique using this approach was proposed to model a single-link flexible arm with an arbitrary number of lumped masses. The work of Yoshikawa in [13] is also relevant, since it considers a three-lumped mass to describe the dynamics of a three degree-of-freedom manipulator with two flexible links and a counterweight. In [14], Konno and Uchiyama followed an approach inspired by the so-called Holzer's method, originally used to study torsional vibration, to develop a general dynamic model for a flexible manipulator with multiple links and multiple degrees-of-freedom.

**Assumed mode approach** Here, the elastic deformation is represented as the sum of modes. Each mode is represented by the product of two functions: a shape function that depends on the distance along the length of the manipulator, and a time varying function. The modes in the sum are, in principle, infinite but are in the practice truncated to a finite number. While the basis of this methodology is well established, there are several ways in which to choose the link boundary conditions and the shape functions. In Belleza et al.[15], a linear model for a single flexible link moving in the horizontal plane is obtained from the pseudo-pinned and pseudo clamped formulations, and subsequently truncated to the most significant modes for its experimental validations. In Pota and Alberts [16], an exact finite order dynamic model for a slewing piezoelectric laminate beam is defined in terms of irrational transfer functions in the Laplace domain variable  $s$ . This finite order and irrational transfer function is commonly converted to a rational transfer function by using Maclaurin series expansion. The result yields on infinite order transfer function that normally needs to be truncated to be considered in the control design. De Luca and Siciliano in [17] presented a closed-form dynamics model for a planar multiple flexible link arm deduced using the assumed mode approach and the Lagrangian approach.

**Finite element approach** The underlying philosophy in this approach is that of discovering an approximate solution to the partial differential equation that describes the flexible link dynamics by dividing the whole domain problem into smaller ones, denominated

as finite elements. The original problem is, therefore, reduced in order to solve a set of second order differential equations defined in the coordinates of the finite elements. In [18], Bayo proposed a finite-element frequency domain formulation for the solution of the equations of motion of a single-link flexible arm moving in a horizontal plane. In [19], Lee and Wang modeled a single link flexible arm that moves in a three-dimensional space. In [20], Tokhi et al. presented theoretical and experimental validation regarding the finite-elements dynamic modeling of single link flexible arm constrained to the horizontal plane.

In all the approaches mentioned above, there is a trade-off between the accuracy and the complexity of the model that is defined in terms of the number of fundamental elements considered (lumped masses, assumed modes or finite elements). The larger the number of fundamental elements, the better the accuracy, but also the greater the complexity. From the control point of view, it is important to choose a reasonable number of fundamental elements that will allow the model to capture the physics of the system with a certain amount of fidelity and without diminishing the stability of the control system. The control algorithm may excite the unmodeled vibrational frequencies and, as a consequence, undermine its own performance and ultimately its stability. This phenomenon is known as *spill-over*, and was first mentioned by Balas in [21].

### 1.3.2 Position control approaches

In this section, we describe the main control techniques used for the end-effector position control of flexible link robots. More citations for each category can be found in [22] and [23]. The techniques are divided into two categories:

**Open loop control** All these schemes have in common that no sensor measurements are used for the computation of the control action. Depending on how the control action is computed, these can be broken down into model-based techniques and those techniques that require only a very limited amount of information from the flexible robot, i.e., the vibrational frequencies, but not its mathematical description. Within the model-based-techniques, there are well known strategies that consider system dynamics inversion for the computation of the input that makes the system follow a desired trajectory. These techniques are widely used in rigid robotics but cannot be directly applied to flexible link robots owing to their non-minimum phase nature. A flexible link robot that considers the actuator torque or position as input and the end-effector pose as output is non-minimum phase and is, therefore, non-invertible in a

stable manner. De Luca et al. in [24] proposed a solution to this problem by defining an alternative output along the length of the flexible arm respect of which the system behaves as a minimum phase system and makes stable inversion of the dynamics possible. In [25], Feliu defined an approximate dynamic inversion based on the Wiener filter. The techniques that require only a limited amount of information about the model are usually simpler than those first mentioned but are, nevertheless effective. The Input Shaping technique, for instance, marked a milestone as regards the control of flexible link manipulators when achieved a reduction factor of 25 in the vibration of the end-effector of the Space Remote Manipulator System (SRMS) simulator. The SRMS arms are a family of robotics flexible manipulators used by NASA to deploy and retrieve payloads in space. The Input Shaping technique uses only the knowledge concerning the natural vibrational frequencies and the damping ratios of the system to shape the actuator commands in order to significantly reduce the residual vibration at the end point [26]. Several known variants of this algorithm currently exist, as those shown in [27], whose objective is to improve the robustness and time performance.

**Closed loop control** In contrast with the previous category, in the closed loop control approaches the control action computation relies on feedback sensors that allow not only positioning of the end-effector but also the suppression of disturbances and the compensation of modeling errors. These techniques are, in general, more complex than those mentioned previously, and require a mathematical model of the system. Most of the control algorithms are found in the model based techniques. In [24], De Luca adapted the well known Computed Torque technique from rigid robotics, based fundamentally on Feedback Linearization, to flexible link robots through the same output redefinition used for stable dynamic inversion mentioned in the previous paragraph. Wang and Vidyasagar, in [28], also considered an alternative output which they denominated as the 'reflected tip position', but in this case with the intention of controlling the resulting system by passivity. In [12], Feliu et al. also proposed a composed control law in which the dynamics of the system was divided into two subsystems: the actuator dynamics and the flexible link dynamics. This separation defines a control architecture which is different to that considered in the previous approach in which a single loop was taken into account. This simplifies the control law design, improves the robustness of the overall system and allows the compensation of nonlinear disturbances acting on the actuators e.g., Coulomb friction.

Siciliano and Book in [29] proposed an approach based on the Singular Perturbation strategy. In this approach, the flexible link dynamics is decomposed into slow and fast dynamics: the slow dynamics corresponds to an equivalent rigid link manipulator

and the fast dynamics includes the elastic variables and their time derivatives. The strategy consists of designing a control law for the slow subsystem on the basis of the well established control schemes available for rigid robots, and then designing a control law for the fast dynamics considering the state variables of the slow subsystem as parameters. In [30], O'Connor formulated the Wave-Based Control strategy, in which the vibration suppression is studied as a problem of the traveling and reflection of waves. In this strategy, for a specific rest-to-rest motion, the controller first injects energy into the system to produce the link displacement and then extracts it all once the end-effector has reached its goal. This energy travels as a wave through the link, is then reflected as it reach the end of the link and if finally absorbed at the actuator by minimizing the dynamic impedance at this point. The wave based control assumes that the superposition theorem can be applied and, therefore, linearity, but in [31] it was shown that it is also robust in the case of non-linear elasticity owing to large deflections. In the control of flexible link robots there are several other techniques that originate from almost every control strategy: Robust Control [32], Adaptive Control [33, 34], Intelligent Control [35], and many others.

The open loop and closed loop approaches are not mutually exclusive and have been combined several times as, for example, occurs in [36].

## 1.4 Objectives of the thesis

This thesis attempts to advance the prospect of flexible link manipulators for terrestrial applications by contributing mainly in two principal respects. First, it proposes a new solution to the problem of end-effector trajectory tracking of a single-link arm moving in the presence of gravity and under conditions of payload mass uncertainty. Second, it proposes a spring-based gravity compensation mechanism for a single-link flexible manipulator as an alternative to the gravity compensation mechanism based on counterweights and a control algorithm for the gravity compensated manipulator.

The general objective of the thesis is to control one-degree-of-freedom flexible link robots moving in a vertical plane and, therefore, affected by gravity. The core contributions of the thesis are:

1. The development of dynamic models that describe the behavior of the flexible link arm moving under gravity.
2. The formulation of a non-linear control law with which to perform the tip position trajectory tracking of the flexible link arm in the presence of joint friction.

3. The design of a fast identification algorithm with which to estimate the payload mass carried by the flexible link arm.
4. The design of an adaptive control law, which extends that formulated in Objective 2 to one that is capable of perform in conditions of payload mass uncertainty.
5. The design and construction of a spring based gravity compensation mechanism for the single link flexible arm moving under gravity.
6. The formulation of a dynamic model for the single link flexible arm with the gravity compensation mechanism.
7. The design of a non-linear control law with which to perform the tip position trajectory tracking of the flexible link arm with the gravity compensation mechanism.

## 1.5 Research stay overview

As mentioned in Section 1.1, the author of this thesis took part in a research stay at the Institute of Control Theory and Systems Engineering at the Technical University of Dortmund between April and September 2014. During this period, the author acquired important experience in the field of dynamic modeling and systems identification, which allowed him to significantly improve the accuracy of the dynamics models shown in the thesis. This experience impacted very positively on all the results obtained in this thesis, but more directly on Objectives 1 and 6. The most explicit result of this period was the modification of the lumped mass dynamic model for the single link flexible arm moving under gravity. The experimental observations made during this period indicated that the effect that the distributed nature of the link mass has on its dynamic behavior cannot be accurately modeled by a lumped-based dynamic model that considers only one lumped mass, particularly when the link mass is very significant with regard to the payload mass. The reason for this is that a single lumped mass cannot correctly model both the inertial and gravitational effect at the same time. As a result of this observation, in this thesis we have considered a modified lumped mass modeling model into which the effect of the link mass was introduced by means of two additional constants added to the corresponding inertial and gravitational terms of the original dynamics (more details in Chapter 3). In the case of the single lumped mass model, this strategy substantially reduced the modeling errors, and enabled us to preserve the minimum phase property of the model. The two-lumped mass dynamic model was also greatly improved by applying the same modeling principle, this time by separating the intermediate lumped mass constant into two parts, each of which represents the inertial and gravitation

effects. The preservation of the minimum phase property of the single lumped mass model played a fundamental role in the complexity of the algorithms developed in Chapters 5-7 (Objectives 2,4,7). In addition to the aforementioned results, during this period the author learned about the use of Matlab Toolboxes for Optimization and Identification, both of which were used throughout this thesis.

## 1.6 Organization of the manuscript

The manuscript will be divided into eight chapters:

**Chapter 2** The preparation of this thesis involved the installation of a one-degree-of-freedom flexible link robot with movements in the vertical plane. The chapter introduces a complete description of the platform used.

**Chapter 3** This chapter establishes the dynamics model and identification procedure of the one-degree-of-freedom flexible robot. It covers only the case of the flexible link robot without gravity compensation and contemplates the use of two links of different dimensions and materials.

**Chapter 4 and 5** These chapters cover the end-effector trajectory tracking problem of the one degree-of-freedom flexible link robot moving in the vertical plane and in the presence of joint friction. No gravity compensation mechanism is considered in these chapters yet. As a general approach, a two-nested loop cascade control was used to manage with the joint friction: the inner loop considered an two-degrees of freedom controller for the joint positioning and the outer loop employed an input-state feedback linearization control law to track the trajectory and to cancel the vibrations. Chapter 4 explains the design a joint positioning control algorithm that is robust to joint friction disturbances. Chapter 5 uses the results obtained in Chapter 4 and formulates a non-linear control law based on feedback linearization techniques to perform the end-effector trajectory tracking. Both chapters conclude with the experimental validation of the algorithms.

**Chapter 6** This chapter extends the controller developed in the previous chapter in order to compensate payload mass variations through the use of an adaptive control strategy. The adaptive control law combines the previous controller with a payload mass identifier based on the algebraic identification framework. First, the algebraic identification algorithm is developed, compared with other identification procedures and experimentally validated. The payload algebraic identifier is subsequently combined

with the previously obtained end-position trajectory tracking algorithm and is experimentally tested.

**Chapter 7** This chapter introduces the design and construction of a spring gravity compensation mechanism for the one-degree-of-freedom flexible link robot considered in the previous chapters. In addition, it presents the development of a dynamic model based on the lumped mass approach along with an identification methodology for the overall system (flexible link with the gravity compensation mechanism). Afterward, a control algorithm based on feedback linearization is developed to control the tip position of the flexible arm. Both the dynamic model and the control algorithm are experimentally validated.

**Chapter 8** This chapter discusses the results attained, details the core contributions and suggests future works.

## Chapter 2

### Experimental Setup

A platform designed and built by the Robotics and Automation Group at the University of Castilla-La Mancha was used for the experimental validation of the theoretical developments. The platform corresponds to a single-link flexible robot whose movements are constrained to the vertical plane. This chapter describes the main components of this platform, which are grouped according to the following categories: mechanical structure, actuators and sensors, and hardware and software. Figure 2.1 shows the platform configured with the two different links considered throughout this thesis.

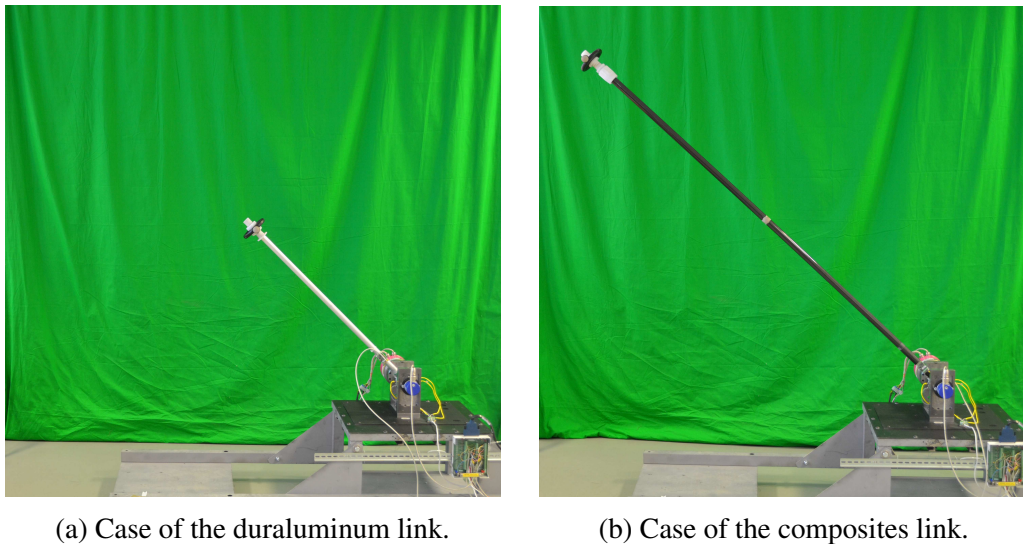


Fig. 2.1 Experimental setup: photographs of the single-link flexible arm prototype configured with two different links.

Before proceeding with the description of the platform, it is important to state that a first prototype of the mechanical part of the platform was built before the start of this thesis (Fig 1.4a). The author, in collaboration with D.José Abel Chocoteco Campos, was mainly

in charge of the subsequent mechanical modifications that have resulted in the current mechanical prototype. It is relevant to emphasize that the author was primarily responsible for the electrical wiring, the installation and configuration of sensors and actuators, and for the programming of the system.

## 2.1 Mechanical structure

The main elements of the structure of the flexible link arm are indicated in Fig 2.2. These elements are: the mechanical support, the flexible link, the payload, and the actuator and sensors. This section is devoted to describing the first three elements mentioned. The actuator and sensors will be described separately in the following section.

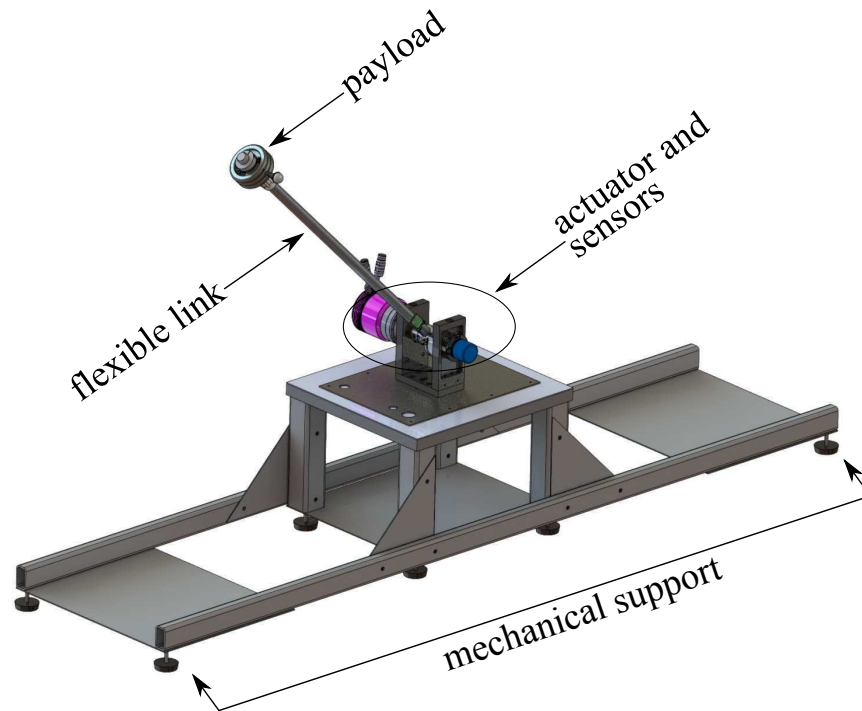
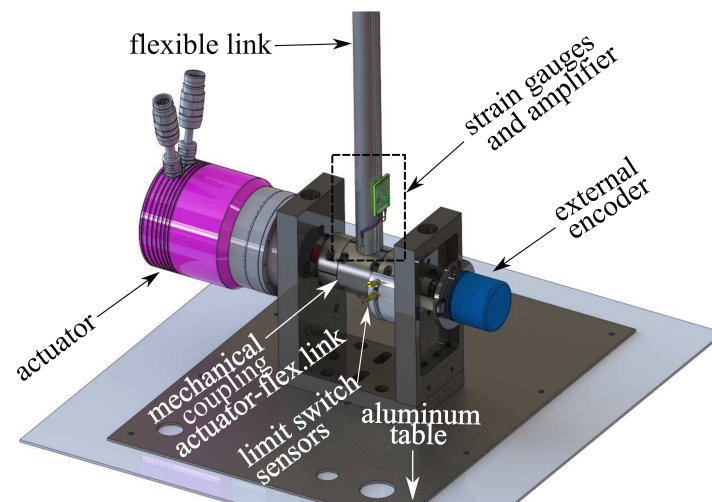
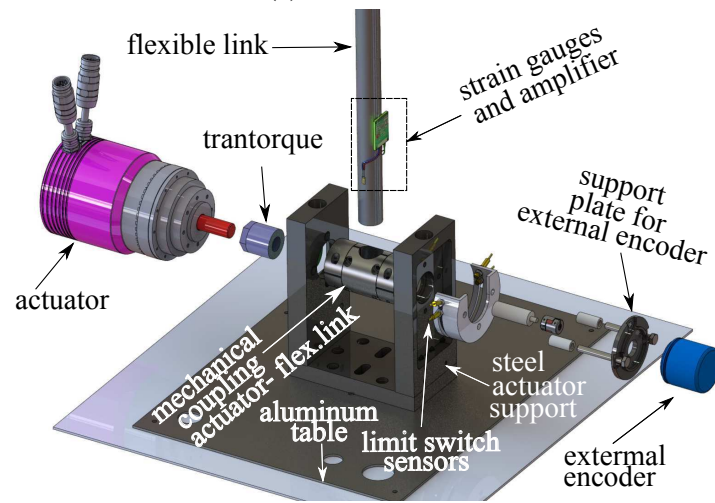


Fig. 2.2 Experimental setup: CAD representation of the single-link flexible arm.

The mechanical support of the platform is a reinforced aluminum table with height adjustable feet that compensate for the uneven floor. A u-shaped mechanical structure, made of steel, supports the body of the actuator and enables the connection between the actuator shaft and the flexible link by means of a trantorque. The table and the actuator support have been designed to be very rigid in order to avoid the transmission of the vibrations produced by the movement of the flexible link. Fig 2.3 shows a more detailed view of these parts and some others, which will be described in the following sections.



(a) Standard view



(b) Exploded view

Fig. 2.3 Experimental setup: CAD representation of the coupling between the flexible link and the actuator.

Two different flexible links with tubular cross sections were considered in the experiments. The first link is 1 m long, has an external cross-sectional diameter of 25.4 mm, a wall thickness of 3.7 mm, and is made of an aluminum alloy called duraluminum. The second link is 2 m long, has an external cross-sectional diameter of 30 mm, a wall thickness of 1.5 mm, and is built of a carbon composite material. Aluminum alloys constitute currently the most frequently used building materials in flexible link robotics given that they are durable, lightweight, ductile and relatively low cost. The use of composite materials in flexible link robotics is, on the other hand, more recent and has been fundamentally motivated by their superior strength to weight ratio, stiffness to weight ratio and lower thermal conductivity coefficient in comparison with more traditional metallic materials, such as aluminum or steel. In addition to this, composite materials also have almost perfect elastic deflections, i.e., they have relatively no plastic deformations. Tables A.1 and A.2 in Appendix A provide a summary the main characteristics of these links. For more detailed discussion on the use of composite materials in flexible link robots refer to [37].

The payload is essentially a set of weight plates stacked on top of each other and attached to the end of the flexible link by a plastic structure. This plastic structure is firmly fixed at one side to the end of the flexible link by means of a clamping collar, whereas on the other side has a solid threaded rod with a hexagonal nut that secures the weight plates. The most relevant feature of this plastic structure is that the flexible link does not need to be drilled in order to attach the payload mass. This is especially important in the case of the composites link, in which a drilling operations may cause the delamination of the material and, as a consequence, some alterations to the mechanical properties. Fig. 2.4 shows a CAD representation of the payload fixing structure.

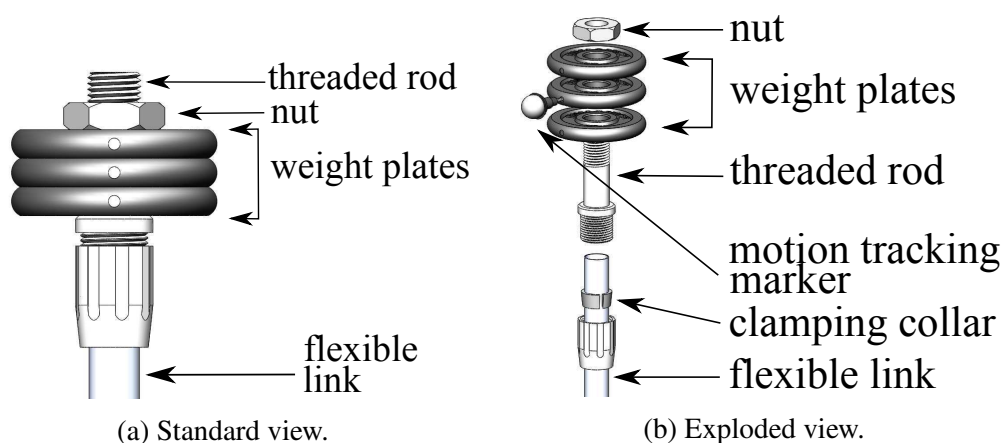


Fig. 2.4 Experimental setup: CAD representation of the payload fixing structure.

## 2.2 Actuator and sensors

### 2.2.1 Actuator

The actuator considered is a DC Brushless Motor with a zero backlash gearbox and mechanical break fabricated by Harmonic Drive under the CHA-25A-100-D-SP model. The motor is commanded by a DPCANTR-040B080 four quadrant operation digital servo drive produced by Advanced Motion Controls and is fundamentally used for current control. Tables A.4 and A.5 in Appendix A show the main features of the actuator and the digital servo drive. The actuator and the servo drive are depicted in Fig 2.5.



(a) CAD Representation of Actuator Harmonic Drive CHA-25A-100-D-SP.



(b) Photograph of the Servo Drive Advanced Motion Controller DPCANTR-040B080.

Fig. 2.5 Experimental setup: photographs of the actuator and the servo drive.

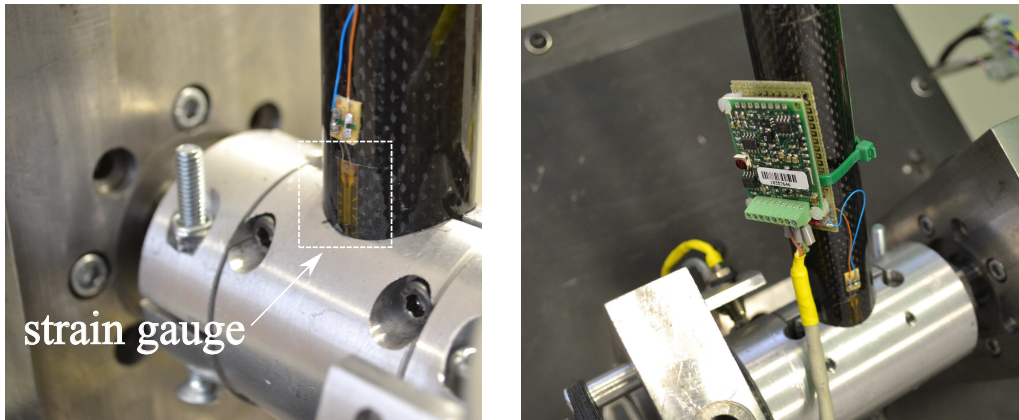
### 2.2.2 Sensors

In closed loop control approaches it is important to know how certain variables evolve over time so as to be able to move the flexible link arm in a rapid, precise and efficient manner. The control algorithms developed in this thesis are fundamentally closed loop and require continuous measurements of the actuator position and the deformation of the link.

In order to measure the actuator position, a resolver at the motor shaft and an auxiliary encoder located at the output of the gearbox were installed (see Fig 2.3). The auxiliary

encoder was used to validate that there is no backlash or sliding between the gearbox shaft and the coupling piece that interfaces with the flexible link. The accuracy of the resolver and auxiliary encoder are  $\pm 10'$  spread and 10000 pulses/turn, respectively. More relevant technical data concerning the resolver and the auxiliary encoder are contained in Tables A.6 and A.7, respectively.

The deformation of the link was measured at two points along the arm: at the base of the link and at the attachment point of the gravity compensation mechanism (See Chapter 7). For this purpose, in each point of measurement an arrangements of strain gauges were pasted. Each arrangement is composed by two strain gauges placed at opposing positions and in the plane of movement of the arm. Fig 2.6 shows a strain gauge used to measured the deformation at the base of the link. Kyowa KFRP-5-120-C1-1 strain gauges were used for the duraluminum flexible link, whereas Kyowa KFG-5-120-C1-23 strain gauges were considered in the case of the composite flexible link. Vishay BA 660 voltage amplifiers provided the signal conditioning for the strain gauges. Refer to Tables A.8-A.10 in Appendix A for more detailed information on the strain gauges and the strain gauges amplifier. Fig 2.6 shows a photograph of the strain gauges installed at the base of the composites link and a photograph of the strain gauges amplifier.



(a) Photograph of a strain gauge installed at the base of the flexible link. (b) Photograph of the strain gauges amplifier connected to the strain gauges.

Fig. 2.6 Experimental setup: photographs of the installation of the strain gauges and the voltage amplifier.

In addition to the above mentioned sensors, a motion capture system was used to verify the real movements performed by the flexible link robot. The system was manufactured by Qualysis and consists of three cameras model ProReflex MCU 240 capable of tracking the position of moving markers. The operating principle behind this system is the emission and the subsequent detection of infrared light. The infrared light is initially emitted through

an array of LEDs located in each camera. The light is then reflected by spherical markers specifically designed for this purpose, and is eventually captured by the camera. Each camera records the movement of the markers in 2-D and then, by triangulation, the system computes the spatial position in 3-D. In our experiments, the motion capture system was used to measure the movements of the flexible link robot at its tip position. The precision of these measurements were about 0.5 mm for the setup with the duraluminum link, and 1 mm for the case of composite link. It is important to clarify that these measurements were used only for validation, and not for control purposes. For a more detailed description of the motion capture system, refer to Table.A.12 in Appendix A. Fig 2.7 displays a photograph of the whole experimental setup, including the Qualisys ProReflex MCU 240 motion capture system.

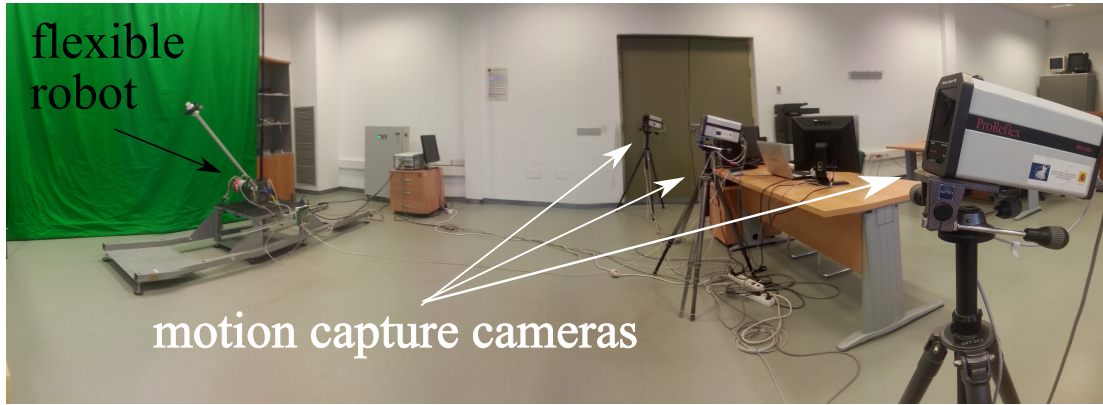


Fig. 2.7 Experimental setup: photograph of the complete view of the experimental setup.

For safety reasons the work space of the robot is defined for motor positions between 0 and 180 degrees, as shown in Fig 2.8. The limitation of the robot movement to this working space is achieved by means of a safety stopping mechanism. This mechanism consists of a magnet and four magnetic reed switches, working as limit switches, which are used to detect four different orientations at the base of the link. The main characteristics of magnetic reed sensors are presented in Tables A.11, while the arrangement of these sensors on the platform is shown in Fig 2.9.

The safety mechanism is activated at two levels, depending on which angular positions have been reached. The first limit is when the motor position is less than  $-5$  degrees or greater than 185 degrees. The second safety limit is defined by the angular positions of  $-15$  and 195 degrees. When the motor position surpasses the first limit, the servo drive, which initially works in current control mode, is set to the velocity control mode and a zero velocity is commanded. This action slows down the arm. However, it is not always sufficient to completely stop it before reaching the second limit. In the case that the second

limit is crossed, the servo drive activates the mechanical break in order to completely stop the robot. The mechanical break in the actuator is intended for a static break, and the use of the two-level safety stop mechanism prevents the unnecessary activation or the misuse of the mechanical break. There is also the possibility of manually activating of the stopping mechanism by means of an emergency stop button. In this case, the two stopping levels mentioned previously are activated with a certain time delay between them.

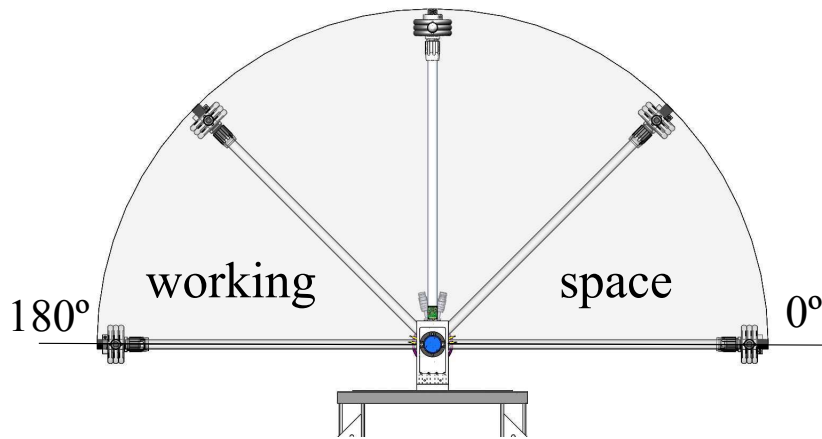


Fig. 2.8 Experimental setup: working space of the flexible link manipulator.

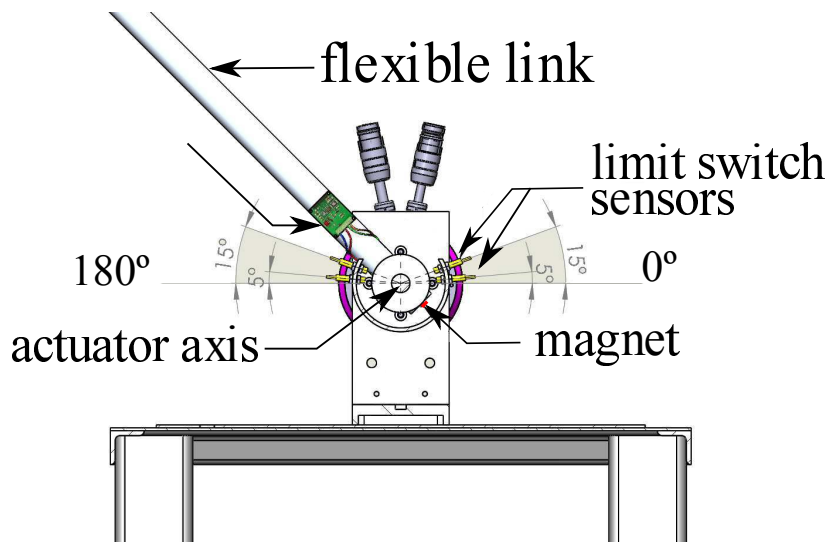


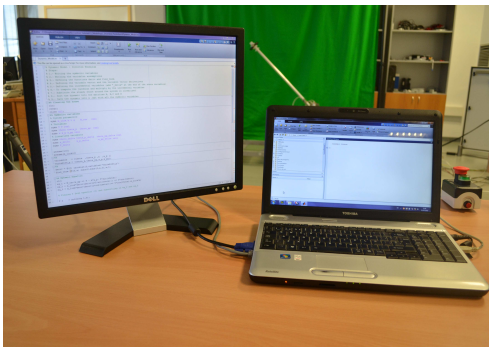
Fig. 2.9 Experimental setup: diagram representing the automatic safety stopping mechanism.

## 2.3 Computer hardware and software

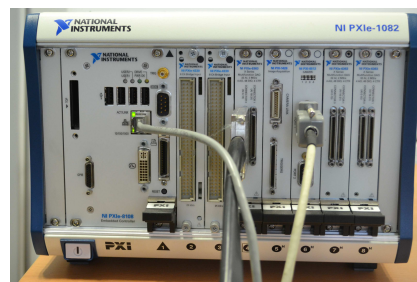
This section describes the hardware and software involved in the simulation and experimental validation. A commercial computer Toshiba Satellite L500-20z with a Intel Dual Core processor with 2.26 GHz and 4 GB of RAM was used for both tasks. Table A.14 provides an overview of the main characteristics of this computer and Fig 2.10a shows a photograph of it.

The simulation of the dynamic models and the control algorithms developed were carried out using MATLAB 2013.b with the following toolboxes: Simulink toolbox, Control System toolbox, Identification toolbox and Symbolic toolbox. The MSC Adams 2014 with the Adams/Flex and Adams/Viewflex modules was also used in combination with MATLAB/Simulink as an additional tool to validate the dynamics models and the control algorithm developed.

For the experimental validation, the algorithms were implemented in the computer using Labview 2010 and subsequently executed by a National Instrument PXI real time system working at a sampling time of 2 ms (see Fig 2.10b). During the operation of the platform, the computer played the role of the master and the PXI played the role of the slave. The computer was in charge of the high level operation of the platform (execution of start/stop commands, switching modes, monitoring of the signals), which was achieved by means of the ongoing communication via Ethernet with the PXI, whereas the PXI interpreted these high level commands and translated them into low level control actions. The chassis, the embedded control and data acquisition cards installed in the aforementioned PXI correspond to the NI-PXIE-1082, NI-PXIE-8108 and NI-PXIE-6363 models. For additional information about these components, refer to the Appendix A.



(a) Photograph of the computer Toshiba Satellite L500-20z.



(b) Photographs of the Real-Time NI PXI.

Fig. 2.10 Experimental setup: photographs of the computers used for the simulation and the experimental validation of the algorithms.

To conclude this chapter, Fig 2.11 shows a schematic representation of most important parts of the experimental platform and the existing connections between them.

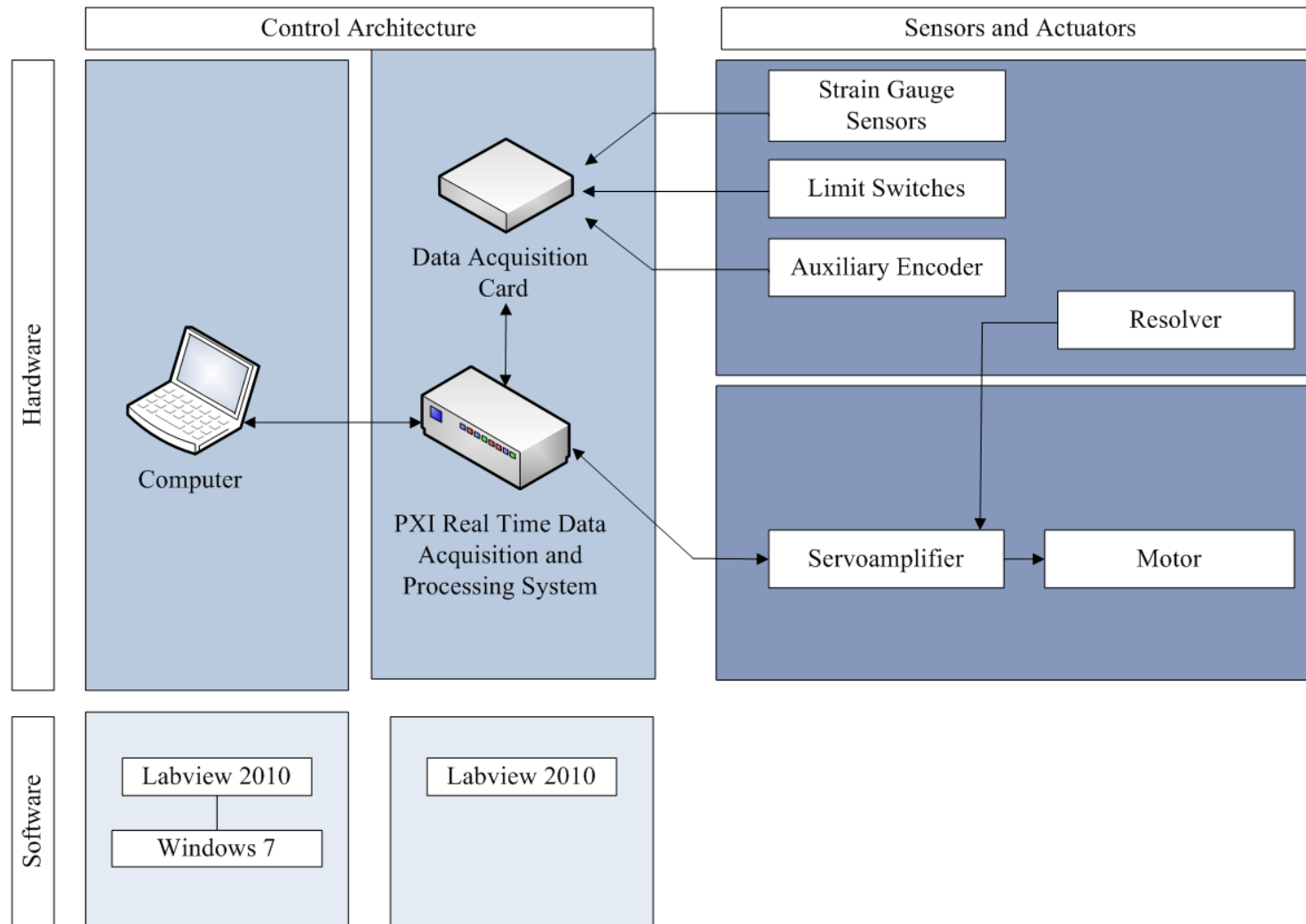


Fig. 2.11 Experimental setup: diagram representing the interconnection of the parts composing the robotic platform.



# Chapter 3

## Dynamic modeling

The dynamics of flexible link arms is governed by nonlinear coupled ordinary partial differential equations, whose exact solution is not feasible in practice. Although the original dynamics of these systems possesses infinite dimensions, it is most commonly discretized and truncated to some finite dimensional models by using the assumed modes, finite elements or lumped mass methods. The choice of the discretization method and the dimension of the model depends on the accuracy required and the complexity that the researcher is willing to assume. In this chapter, two dynamics models are derived for a single link flexible robot constrained to the vertical plane using the lumped mass approach and considering one and two lumped masses. Some modifications to the classical formulation of the lumped mass approach have been included in order to improve the accuracy of the model in the case of movements in a vertical plane and under the effects of the force of gravity.

The results presented in this chapter have been partly published in

- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Feedback linearizing controller for a flexible single-link arm under gravity and joint friction,” in *ROBOT2013: First Iberian Robotics Conference*, (Madrid, Spain), pp. 169–184, Springer, 2014
- J. C. Cambera and V. Feliu-Batlle, “Input-state feedback linearization control of a single-link flexible robot arm moving under gravity and joint friction,” *Robotics and Autonomous Systems*, vol. 88, pp. 24–36, Elsevier, 2017

### 3.1 Dynamic model of a single link flexible robot

The dynamic model of the robot can be divided into two submodels, the first of which represents the motor dynamics and the second of which describes the behavior of the flexible link. These submodels are linked by the torque exerted by the motor on the base of the

flexible link, hereafter referred to as the coupling torque. The dynamic models of the actuator and the flexible link arm are presented in the following sections.

### 3.1.1 Actuator model

In a DC motor, the differential equation that relates the armature current ( $i_m$ ) in the motor position ( $\theta_m$ ), expressed at the output of the gear, can be written as

$$\Gamma_m = k_m n_r i_m = J\ddot{\theta}_m + v\dot{\theta}_m + \Gamma_{st-coul} + \Gamma_{coup}, \quad (3.1)$$

where  $\Gamma_m$  is the output torque of the motor,  $k_m$  represents the torque constant,  $n_r$  is the gear ratio,  $J$  is the rotor inertia,  $v$  is viscous friction coefficient and  $\Gamma_{st-coul}$  represents the static/Coulomb friction and  $\Gamma_{coup}$  is the coupling torque of the link on the motor. In this formula, it is assumed that the servo drive closes a current control loop, and that the resulting closed loop dynamics is so fast that it is negligible with respect to the dynamics of the mechanical part of the actuator. In other words, it is assumed that the armature current can be set instantaneously.

The friction model considered throughout this thesis is defined in terms of the static/Coulomb friction constant ( $\Gamma_c$ ) as

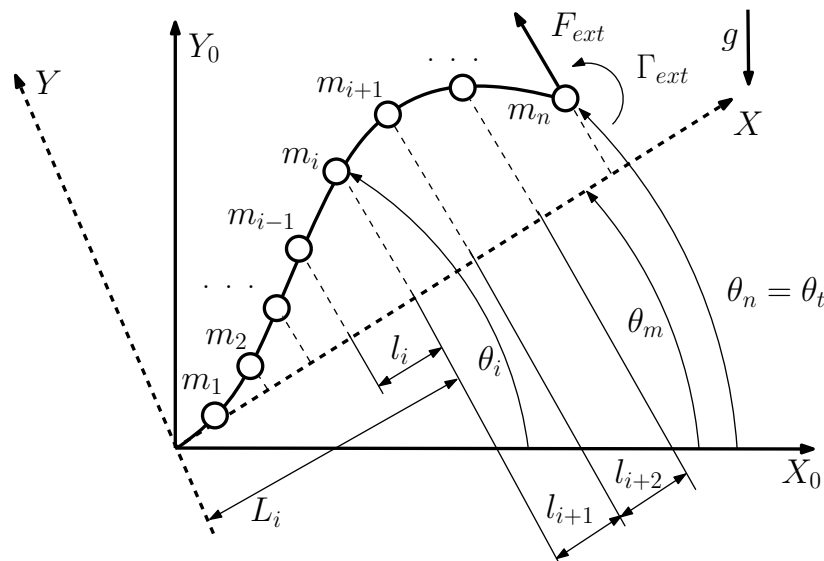
$$\Gamma_{coul} = \begin{cases} \text{if } \dot{\theta}_m \neq 0 \\ \Gamma_c \operatorname{sgn}[\dot{\theta}_m], \\ \text{if } \dot{\theta}_m = 0 \text{ and } \|k_m n_r i_m - \Gamma_{coup}\| < \Gamma_c \\ k_m n_r i_m - \Gamma_{coup}, \\ \text{if } \dot{\theta}_m = 0 \text{ and } \|k_m n_r i_m - \Gamma_{coup}\| > \Gamma_c \\ \Gamma_c \operatorname{sgn}[k_m n_r i_m - \Gamma_{coup}]. \end{cases} \quad (3.2)$$

This friction model and others can be found in [40].

### 3.1.2 General formulation of the lumped mass dynamic model for the flexible link arm

The flexible link dynamic model is obtained by extending the lumped mass procedure proposed by Feliu in [12], which was originally developed for a single link flexible arm with movements in a horizontal plane. The development here presented here considers a single-link flexible arm with movements constrained to a vertical plane and, therefore, affected by the force of gravity. In the following section, we formulate the general model for an arbitrary

The lumped mass model assumes that the dynamics can be approximated by a set concentrated masses distributed along the link and connected by massless flexible elements. Although the real flexible link arm has infinite vibrational modes, only a few of them are significant and, therefore, relevant from the point of view of a control algorithm. The lumped mass model captures a finite number of vibrational modes, which is equal to the number of lumped masses considered. Fig. 3.1 shows a general diagram of the lumped-mass approach when applied to a single link flexible arm.



The lumped-mass model presented here is formulated under the following assumptions:

**Assumption 2** The link’s deflection is small in comparison to its length.

**Assumption 4** Torsion and compression effects are negligible in comparison to the deflection effects.

Let us consider the flexible link depicted in Fig. 3.1. It represents a massless flexible link with  $n$  point masses distributed along its length  $l$ . Let  $m_i$ ,  $1 \leq i \leq n$  denote the  $i$ -th

mass, with the last mass  $m_n$  located at the tip position. Let  $l_i$  be the distance between two consecutive masses  $i - 1$  and  $i$ , and  $l_1$  be the distance between the base of the link (gearbox shaft) and the first mass. Let  $L_i$  be the distance between the mass  $m_i$  and the base of the link (gearbox shaft). Two coordinate frames, whose origins coincide with the actuator axis, are established: the coordinate frame  $\mathbf{X}_0 - \mathbf{Y}_0$  is fixed, while the coordinate frame  $\mathbf{X} - \mathbf{Y}$  rotates with the motor shaft at an angle of  $\theta_m$ . The deflection  $y(x, t)$  is, therefore, defined as the distance along the  $\mathbf{Y}$  axis between any point  $x$  measured along the link and the  $\mathbf{X}$  axis. It is assumed that the deflections are very small (Assumption 2), signifying that the distance between two consecutive masses along the length of the link is equal to its projection on the  $\mathbf{X}$  axis.

We denote as  $F(x)$  and  $\Gamma(x)$  the force and the torque at the point  $x$ . We define the gravity as a force acting on the  $\mathbf{Y}_0$  direction and determined by the gravitational constant  $g$ . We also consider an external force  $F_{ext}$  and torque  $\Gamma_{ext}$  applied at the tip of the beam.

The Euler-Bernoulli equation governs the dynamic behavior of a flexible link arm under the assumption stated above. It corresponds to the following fourth order partial differential equation

$$E I_z \frac{\partial^4 y(x, t)}{\partial x^4} + \rho_L \frac{\partial^2 y(x, t)}{\partial t^2} = f_d(x, t), \quad (3.3)$$

where  $E$  denotes Young's modulus,  $I_z$  is the moment of inertia of the link cross section,  $\rho_L$  is the linear density, and  $f_d(x, t)$  is an external distributed force. Let us assume that the link is massless ( $\rho_L = 0$ ), and that the force gravity is not treated as a distributed force but rather as an external force acting on the lumped masses ( $f_d(x, t) = 0$ ). The resulting equation is

$$E I_z \frac{\partial^4 y(x, t)}{\partial x^4} = 0. \quad (3.4)$$

In order to simplify the notation, in the equations that follow the time variable  $t$  will be omitted and used only when it is strictly necessary. Assuming that  $E$  and  $I_z$  are constant throughout the length of the link, the deflection in the interval  $[i-1, i]$  is given by

$$y_i(x) = u_{i,0} + u_{i,1}(x - L_{i-1}) + u_{i,2}(x - L_{i-1})^2 + u_{i,3}(x - L_{i-1})^3, \quad (3.5)$$

where  $u_{i,j}$  are polynomial coefficients that vary for each interval and that are not necessarily constant in time. The polynomial form of the deflection and the way its coefficients vary in time define the dynamical behavior of the flexible link. In this equation is assumed that  $L_0 = 0$ . For the sake of simplicity, the time dependency is omitted in the nomenclature of the deflection ( $y(x)$ ) and of the polynomial coefficients  $u_{i,j}$ . The deflection at specific points  $i$  in terms of these polynomials is expressed as

$$u_{i,0} + u_{i,1} l_i + u_{i,2} l_i^2 + u_{i,3} l_i^3 = y(L_i) \quad 0 \leq i \leq n. \quad (3.6)$$

The value of the coefficients is obtained from the boundary conditions of the problem and from the dynamic equations resulting from the application of Newton's laws. In order to attain a complete description of the robot, apart from the coefficients, it is necessary to define the coupling torque, as long as it is the variable that relates the flexible link model to the actuator model. All these points are covered in the following sections.

**Boundary conditions** It is necessary to define the boundary conditions between the motor and the link, the continuity conditions between two consecutive intervals, and the boundary conditions at the free end. The boundary condition between the motor shaft and the link is

$$y_i(0) = \frac{dy(0)}{dx} = 0 \implies u_{1,0} = u_{1,1} = 0. \quad (3.7)$$

The deflection, and its first and second derivatives are assumed to be continuous between two consecutive intervals, which yields

$$\begin{aligned} y_i(l_i) &= y_{i+1}(0) \implies u_{i,0} + u_{i,1} l_i + u_{i,2} l_i^2 + u_{i,3} l_i^3 = u_{i+1,0}, \\ \frac{dy_i(l_i)}{dx} &= \frac{dy_{i+1}(0)}{dx} \implies u_{i,1} + 2 u_{i,2} l_i + 3 u_{i,3} l_i^2 = u_{i+1,1}, \\ \frac{d^2 y_i(l_i)}{dx^2} &= \frac{d^2 y_{i+1}(0)}{dx^2} \implies u_{i,2} + 3 u_{i,3} l_i = u_{i+1,2}. \end{aligned} \quad (3.8)$$

The boundary conditions at the free-end are defined in terms of the external torque ( $\Gamma_{ext}$ ) as

$$\Gamma_n = \Gamma_{ext} = E I_z \frac{d^2 y_n}{dx^2} \implies u_{n,2} + 3 u_{n,3} l_n = \frac{\Gamma_{ext}}{2 E I_z}. \quad (3.9)$$

**Dynamic equations and coupling torque definition** Upon applying Newton's laws to the  $n$  lumped masses, we obtain

$$\begin{aligned}
-\Gamma(L_i) = & \sum_{j=i+1}^n (m_j (L_j - L_i) \frac{d^2 \hat{y}(L_j)}{dt^2} + m_j g (L_j - L_i) \cos(\theta_j)) \\
& - \Gamma_{ext} - F_{ext} (L_n - L_i) \quad 0 \leq i \leq n, \quad (3.10)
\end{aligned}$$

$$F(L_i) = F_{ext} - \sum_{j=i+1}^n (m_j \frac{d^2 \hat{y}(L_j)}{dt^2} + m_j g \cos(\theta_j)) \quad 0 \leq i \leq n, \quad (3.11)$$

where  $\hat{y}(L_i) = y(L_i) + \theta_m L_i$  is the position of the mass  $j$  measured along the axis  $\mathbf{Y}_0$  and  $\theta_i = \frac{\hat{y}(L_i)}{L_i} = \frac{y(L_i)}{L_i} + \theta_m$  is the angular position with respect to the axis  $\mathbf{X}_0$ . Note that only the normal component of the gravity force affects the deflection of the link; the tangential component would produce some compression effects, but these are ignored (Assumption 4). Considering that

$$u_{i,2} = \frac{\Gamma(L_{i-1})}{2 E I_z}, \quad (3.12)$$

$$u_{i,3} = -\frac{F(L_{i-1})}{6 E I_z}, \quad (3.13)$$

and combining (3.10)-(3.13), we obtain

$$\frac{d^2 \hat{y}(L_i)}{dt^2} + g \cos(\theta_i) = \frac{6 E I_z}{m_i} (u_{i,3} - u_{i+1,3}), \quad 0 \leq i \leq n-1, \quad (3.14)$$

where  $u_{n+1,3} = -\frac{F_{ext}}{6 E I_z}$ . The coupling torque can be computed from (3.12) as

$$\Gamma_{coup} = -\Gamma(0) = -\Gamma(L_0) = 2 E I_z u_{1,2}. \quad (3.15)$$

The number of lumped masses to consider in a dynamic model depends mainly on the number of vibrational modes that need to be described. The significance of a vibrational mode for a control algorithm is determined by its amplitude and the frequency with which it occurs. In this thesis, we refer to a vibrational mode as being insignificant if it can be only weakly excited by the actuator and, therefore, does not greatly affect the precision of the positioning and, above all, does not compromise the stability of the overall system. The capability of the actuator to excite a vibrational mode depends not only on the physical limits of the motor but can also be adjusted to suit convenience.

On our experimental platform, the number of significant vibrational modes varies depending on the link considered. In the case of the duraluminum link, only one significant vibrational mode appears at most, and it does so at a low frequency, whereas the composite link has two significant vibrational modes at low and intermediate frequencies. Refer to Tables A.1 and A.2 in Appendix A to see the properties of each of these links.

In the following sections we present four dynamics models: The first two models represent the dynamics of the platform with only one significant vibrational mode, while the second two models describe the platform with two significant vibrational modes. In each case, in the models with one and two vibrational modes, first a regular model is developed by following the formulas above, and then a modified version of the first is presented in order to incorporate some of the effects ignored in the first one.

### 3.1.3 Single Lumped Mass Flexible Link Model

The single lumped-mass model considers two additional assumptions to those established in the general formulation presented above: (1) it assumes that only one significant vibrational mode exists and (2) it considers that the link mass is negligible with respect to the payload mass. This last assumption will, hereafter, be denominated as "the link mass assumption". The model is obtained from the general formulation by setting the number of elements to  $n = 1$ . We use the following notation: the link length is  $l$ , the value of the only lumped mass is  $m_{tip}$ , the motor position is  $\theta_m$  and the angular tip position is  $\theta_t$ . Fig. 3.2 shows a representation of the model. Since this thesis is concerned with the control of a flexible link robot moving freely, i.e., without interacting with the environment, the model presented here does not consider any external force or torque acting on the structure.

#### General equations and conditions

- Imposing the free motions conditions

$$\Gamma_{ext} = 0, F_{ext} = 0. \quad (3.16)$$

- From equation (3.5)

$$y(x) = u_{1,0} + u_{1,1}(x) + u_{1,2}(x)^2 + u_{1,3}(x)^3. \quad (3.17)$$

#### Geometric equations

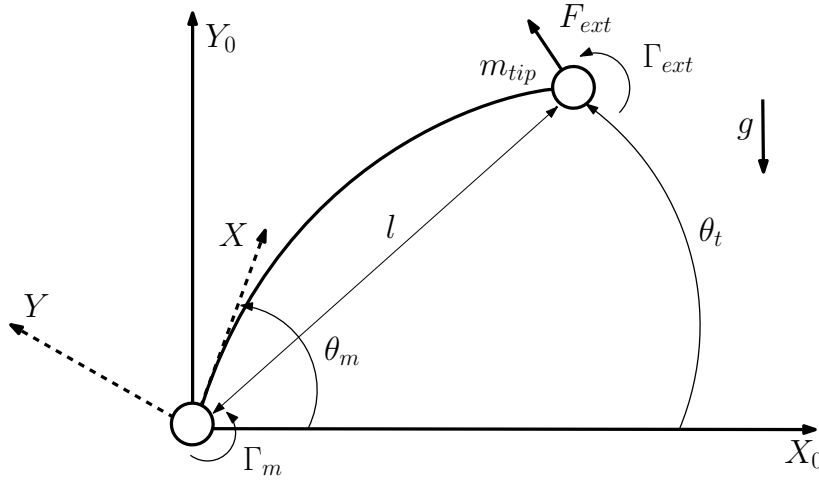


Fig. 3.2 Dynamic modeling: diagram of the single link flexible robot described by 1 lumped-mass model.

- From equation (3.6)

$$y(l) = u_{1,0} + u_{1,1} l + u_{1,2} l^2 + u_{1,3} l^3. \quad (3.18)$$

- From equation (3.7)

$$u_{1,0} = u_{1,1} = 0. \quad (3.19)$$

- There is no equation (3.8).

- From equation (3.9)

$$u_{1,2} + 3 l u_{1,3} = \frac{\Gamma_{ext}}{2 E I_z}. \quad (3.20)$$

### Dynamic equations

- From equation (3.14), and considering  $\hat{y}(L_i) = \frac{\theta_i}{L_i}$ , we obtain

$$l \frac{d^2 \theta_t}{dt^2} + g \cos(\theta_t) = \frac{6 E I_z}{m_{tip}} \left( u_{1,3} + \frac{F_{ext}}{6 E I_z} \right). \quad (3.21)$$

- Upon substituting  $u_{1,3}$  obtained from the geometric equations (3.18-3.20), and considering the expressions  $\hat{y}(L_i) = y(L_i) + \theta_m L_i$ ,  $\hat{y}(L_i) = \frac{\theta_i}{L_i}$ , we obtain

$$l \frac{d^2 \theta_t}{dt^2} + g \cos(\theta_t) = \frac{6 E I_z}{m_{tip}} \left( \frac{\theta_m - \theta_t}{2 l^2} + \frac{\Gamma_{ext}}{4 E I_z l} + \frac{F_{ext}}{6 E I_z} \right). \quad (3.22)$$

**Coupling torque** From equation (3.15), and substituting  $u_{1,2}$  computed from the geometric equations (equations 3.18-3.20), we obtain

$$\Gamma_{coup} = -2 E I_z u_{1,2} = \frac{3 E I_z}{l} (\theta_m - \theta_t) + \frac{\Gamma_{ext}}{2}. \quad (3.23)$$

Upon rearranging equations (3.22) and (3.23) and applying the conditions for a flexible link moving freely ( $\Gamma_{ext} = 0$  and  $F_{ext} = 0$ ), the resulting equations are

$$\Gamma_{coup} = m_{tip} l^2 \ddot{\theta}_t + m_{tip} g l \cos(\theta_t), \quad (3.24)$$

$$\Gamma_{coup} = \frac{3 E I_z}{l} (\theta_m - \theta_t). \quad (3.25)$$

One interesting feature of this model, from the control design point of view, is that the relationship between the motor position and the tip position is minimum phase. This is a unique property of the single lumped-mass model. A minimum phase description of the robot is a desirable simplification of the dynamics of the system that eases the design of the control algorithm, as will be discussed in Chapter 5. However, this property is useful only if the resulting model accurately describes the dynamic behavior of our platform.

### 3.1.4 Modified Single Lumped Mass Flexible Link Model

The single lumped-mass model reproduces only one vibrational mode and completely ignores the mass of the link, therefore, it is useful only when two conditions are met: (1) the higher vibrational modes are not significant, which typically implies that they occur at very high frequencies, are very damped, and cannot be easily excited by the actuator; (2) the link mass is negligible with respect to the payload mass, which is the same as saying that the payload mass to link mass ratio is high. A model based on a single lumped mass approach was developed by Castillo in [41] to successfully model a flexible antenna sensor with two degrees of freedom, with only one significant vibrational model and whose link mass was negligible with regards the payload mass. In our platform, the first condition is met for the duraluminum link, which has only one significant vibrational mode, whereas it is not totally accurate in the case of the composite link on which two significant vibrational modes appear. This model is still useful in the second case depending on the bandwidth requirements of the system, as will be shown in Chapter 6. With regard to the second condition, it is not fulfilled by our platform. On our platform, the smallest payload mass to link mass ratios considered are only  $\frac{1.13 \text{ Kg}}{0.61 \text{ Kg}} = 1.85$  and  $\frac{0.73 \text{ Kg}}{0.27 \text{ Kg}} = 2.70$  for the duraluminum link and the composites link respectively, which are not sufficiently high to be negligible.

Having explained the underlying assumptions of the single lumped-mass model are exposed, there are two options as regards determining a model that properly reproduces the dynamic behavior of the platform: (1) Consider a higher order model with more lumped masses, or (2) modifying the current model to include the effects of the link mass. By considering a higher order model, the resulting dynamics might be capable of including the link mass and reproducing higher vibrational modes, but at the expense of dealing with a more complex model with a non-minimum phase dynamics. By considering the second option, we would still be capable of reproducing the most significant vibrational behavior of the platform with a minimum phase model, but under the assumption that such a reliable modified model can be found. In this thesis, we have chosen the second option for the sake of simplicity. A modified single lumped-mass model was obtained and is presented below.

We propose to make a modification to the single lumped-mass model in which acknowledge separately the contribution of the link mass  $m_{link}$  to the inertia and the gravity terms of the dynamics. Two constants  $m_{link-a}$  and  $m_{link-g}$  are, therefore, added to equation (3.24), which represent the contributions of the link mass to the inertia and the gravity terms, respectively. The resulting dynamics for the modified model can be written as

$$\begin{aligned}\Gamma_{coup} &= (m_{tip} + m_{link-a}) l^2 \ddot{\theta}_t + (m_{tip} + m_{link-g}) g l \cos(\theta_t) \\ &= (m_a) l^2 \ddot{\theta}_t + (m_g) g l \cos(\theta_t),\end{aligned}\tag{3.26}$$

$$\Gamma_{coup} = \frac{3 E I_z}{l} (\theta_m - \theta_t).\tag{3.27}$$

Although this modification to the single lumped-mass model allows us to include the contribution of the link mass to the dynamics, this modification is only valid for a certain range of payload-mass to link-mass ratios that depends on the properties of the links.

In summary, the complete dynamic model of the experimental platform is formed of equations (3.1), (3.26) and (3.27). Before concluding this section, it is important to define the relationship that exists between the sensors and the variables in the model. The motor position ( $\theta_m$ ) is measured directly from the resolver and the incremental encoder. With regards to the coupling torque ( $\Gamma_{coup}$ ), the output voltage of the strain gauge amplifier ( $V_{gauge}$ ) is related to the coupling torque by means of a proportionality constant ( $k_{gauge-base}$ )

$$\Gamma_{coup} = k_{gauge-base} V_{gauge}.\tag{3.28}$$

Here, the constant  $k_{gauge-base}$  depends on the characteristics of the link, the strain gauges and the strain gauges amplifier. The tip position cannot be measured directly but it can be

estimated from the motor position and the coupling torque measurements. The estimation is achieved by employing (3.27), and can be written as

$$\hat{\theta}_t = \theta_m - \Gamma_{coup} \frac{l}{3 E I_z}. \quad (3.29)$$

Note that the tip position estimation is independent of the gravity and the payload mass. This characteristic will be particularly useful in Chapter 6, in which an adaptive controller is designed for the tip position control of the robot in a situation of payload mass uncertainty.

### 3.1.5 Two lumped masses flexible link model

The main difference between the dynamic model that considers two lumped masses and the previous, which considers only one, is that the former is capable of reproducing the two vibrational modes instead of one. The derivation of this model is achieved using the general formulation presented in Section 3.1, by considering the number of elements as  $n = 2$ . Fig. 3.3 shows a diagram representing the single-link flexible robot described by two lumped masses. The following notation is considered:  $l_1$  represents the distance from the base of the link to the position of the first lumped mass;  $l_2$  denotes the distance between the two lumped masses;  $l$  is the sum of the distances  $l_1$  and  $l_2$ ;  $m_{int}$  and  $m_{tip}$  are the values of the lumped masses located at an intermediate point and at the tip of the flexible link, respectively, and  $\theta_{int}$  and  $\theta_t$  are the angular position of these lumped masses, respectively. In this model, the robot is considered to be moving freely, i.e., without interacting with the environment and, therefore, with no external force or torque acting on the robot apart from the force of gravity.

#### General equations and conditions

- Imposing the free motion conditions

$$\Gamma_{ext} = 0, F_{ext} = 0, \quad (3.30)$$

- Equation (3.5)

$$y_1(x) = u_{1,0} + u_{1,1}(x) + u_{1,2}(x)^2 + u_{1,3}(x)^3 \quad 0 \leq x \leq l_1, \quad (3.31)$$

$$y_2(x) = u_{2,0} + u_{2,1}(x - l_1) + u_{2,2}(x - l_1)^2 + u_{2,3}(x - l_1)^3 \quad l_1 \leq x \leq l_1 + l_2. \quad (3.32)$$

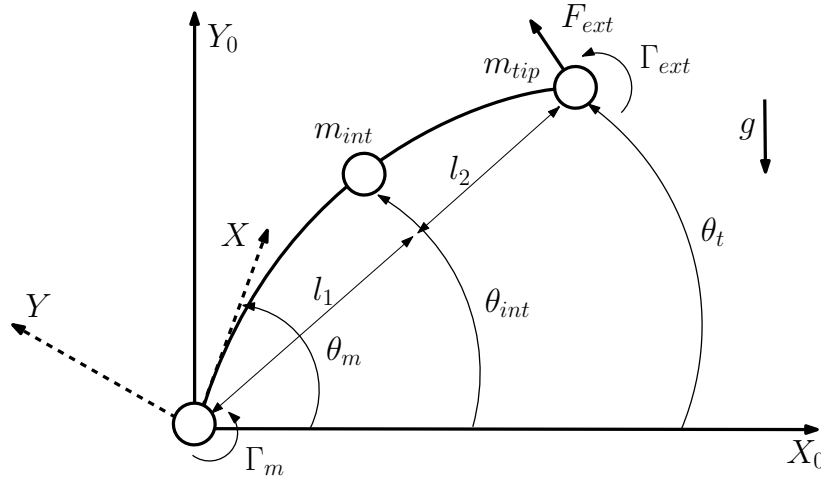


Fig. 3.3 Dynamic modeling: diagram of the single link flexible robot described by two lumped-mass model.

### Geometric equations

- Equation (3.6)

$$y(l_1) = u_{1,0} + u_{1,1} l_1 + u_{1,2} l_1^2 + u_{1,3} l_1^3, \quad (3.33)$$

$$y(l_1 + l_2) = u_{2,0} + u_{2,1} l_2 + u_{2,2} l_2^2 + u_{2,3} l_2^3. \quad (3.34)$$

- Equation (3.7):  $u_{1,0} = u_{1,1} = 0$ .

- Equation (3.8)

$$u_{1,0} + u_{1,1} l_1 + u_{1,2} l_1^2 + u_{1,3} l_1^3 = u_{2,0}, \quad (3.35)$$

$$u_{1,1} + 2 u_{1,2} l_1 + 3 u_{1,3} l_1^2 = u_{2,1}, \quad (3.36)$$

$$2 u_{1,2} + 6 u_{1,3} l_1 = 2 u_{2,2}. \quad (3.37)$$

- Equation (3.9)

$$u_{2,2} + 3 l_2 u_{2,3} = \frac{\Gamma_{ext}}{2 E I_z}. \quad (3.38)$$

**Dynamic equations** Upon considering  $\hat{y}(L_i) = \frac{\theta_i}{L_i}$  and the equation (3.14),

$$l_1 \ddot{\theta}_{int} + g \cos(\theta_{int}) = \frac{6 E I_z (u_{1,3} - u_{2,3})}{m_{int}}, \quad (3.39)$$

$$(l_1 + l_2) \ddot{\theta}_t + g \cos(\theta_t) = \frac{6 E I_z (u_{2,3} + \frac{F_{ext}}{6 E I_z})}{m_{tip}}, \quad (3.40)$$

Upon substituting the coefficients  $u_{1,3}$  and  $u_{2,3}$  obtained from the geometric equations in the previous equation and considering  $l_1 = l_2 = \frac{l}{2}$  we obtain

$$\frac{l}{2} \ddot{\theta}_{int} + g \cos(\theta_{int}) = \frac{24 E I_z}{7 l^2 m_{int}} (6 \theta_m - 16 \theta_{int} + 10 \theta_t - \frac{\Gamma_{ext} l}{E I_z}), \quad (3.41)$$

$$l \ddot{\theta}_t + g \cos(\theta_t) = \frac{E I_z}{7 l^2 m_{tip}} (-24 \theta_m + 120 \theta_{int} - 96 \theta_t + \frac{18 \Gamma_{ext} l}{E I_z} + \frac{7 F_{ext} l^2}{E I_z}). \quad (3.42)$$

**Coupling torque** From equation (3.15), and upon substituting  $u_{1,2}$  obtained from the geometric equations (equations 3.33-3.38), we obtain

$$\Gamma_{coup} = -2 E I_z u_{1,2} = \frac{24 E I_z}{7 l} (2 \theta_m - 3 \theta_{int} + \theta_t - \frac{\Gamma_{ext} l}{24 E I_z}). \quad (3.43)$$

Upon rearranging equations (3.39-3.43), and imposing the free motion conditions (equation 3.30), we obtain

$$m_{int} \frac{l^2}{4} \ddot{\theta}_{int} + m_{int} g \frac{l}{2} \cos(\theta_{int}) = \frac{24 E I_z}{7 l} (-8 \theta_{int} + 5 \theta_t + 3 \theta_m), \quad (3.44)$$

$$m_{tip} l^2 \ddot{\theta}_t + m_{tip} g l \cos(\theta_t) = \frac{24 E I_z}{7 l} (5 \theta_{int} - 4 \theta_t - \theta_m), \quad (3.45)$$

$$\Gamma_{coup} = \frac{24 E I_z}{7 l} (-3 \theta_{int} + \theta_t + 2 \theta_m). \quad (3.46)$$

It is demonstrable that the relationship between the motor position ( $\theta_m$ ) and the tip position ( $\theta_t$ ) for this model is non-minimum phase. This property adds significant difficulties to the control algorithm design that could be avoided by considering the single lumped-mass dynamic model presented in the previous section.

### 3.1.6 Modified Two Lumped Masses Flexible Link Model

The two lumped mass dynamic model lacks two characteristics required to successfully reproduce the behavior of the experimental setup with the composite link. The first charac-

teristic is the capacity to describe the distributed nature of the flexible link arm by means of the lumped masses. As discussed previously, the effects of gravity and inertial forces acting on the link cannot be correctly modeled by a single lumped mass, even if it is located in an intermediate location. However, it is possible to approximate the distributed nature of the link by splitting the lumped mass  $m_{int}$  into two terms,  $m_{link-a}$  and  $m_{link-g}$ , which represent the inertial effects and the gravitational effects of the link. The second characteristic corresponds to the capacity to model the damping of the vibrational modes. A flexible link arm has infinite vibrational modes, but not all are significant from the point of view of the control algorithm. Most of the vibrational modes are in the highest part of the frequency spectrum and are very damped. The control algorithms normally take into account only those vibrational modes that could potentially be excited by the motor. In the case of the setup with the composite link, the first two vibrational modes are significant, and the second is notably damped. We have opted to include a linear damping model which is added to the equations (3.44) and (3.45) by means of the terms  $(\mu_{c1} \dot{\theta}_{int})$  and  $(\mu_{c2} \dot{\theta}_t)$ . The resulting model is given by

$$m_{link-a} \frac{l^2}{4} \ddot{\theta}_{int} + m_{link-g} g \frac{l}{2} \cos(\theta_{int}) = \frac{24 E I_z}{7 l} (-8\theta_{int} + 5\theta_t + 3\theta_m) + \mu_{c1} \dot{\theta}_{int}, \quad (3.47)$$

$$m_{tip} l^2 \ddot{\theta}_t + m_{tip} g l \cos(\theta_t) = \frac{24 E I_z}{7 l} (5\theta_{int} - 4\theta_t - \theta_m) + \mu_{c2} \dot{\theta}_t, \quad (3.48)$$

$$\Gamma_{coup} = \frac{24 E I_z}{7 l} (-3\theta_{int} + \theta_t + 2\theta_m), \quad (3.49)$$

Another way of writing the expression for the coupling torque is obtained by replacing equations (3.47) and (3.48) to (3.49)

$$\begin{aligned} \Gamma_{coup} = & \frac{m_{link-a} l^2 \ddot{\theta}_{int}}{4} + \frac{m_{link-g} g l \cos(\theta_{int})}{2} + \mu_{c1} \dot{\theta}_{int} \\ & + m_{tip} l^2 \ddot{\theta}_t + m_{tip} g l \cos(\theta_t) + \mu_{c2} \dot{\theta}_t. \end{aligned} \quad (3.50)$$

## 3.2 Dynamic model identification

In this section, we summarize the identification process of the actuator (Section 3.1.1) and the modified version of the lumped-mass flexible link dynamic models (Sections 3.1.4 and 3.1.6). This section begins by identifying of the actuator. It then goes on to the Modified Single Lumped Mass Model and, finally, deals with the identification of the Modified Two Lumped Mass Dynamic Model.

### 3.2.1 Identification of the actuator

The motor torque constant  $k_m$  was obtained from the catalog, while its viscous friction parameter ( $v$ ) and its rotor inertia ( $J$ ) were estimated from the velocity time response of the motor to step current commands when no flexible link was attached to the motor shaft. The static/Coulomb friction parameter ( $\Gamma_c$ ) was computed by measuring the break-away current that makes the motor move from an initial state of rest. The parameters identified for the Harmonic Drive actuator are presented in Table A.4 in Appendix A. The static/Coulomb friction represents about 11.6 % of the maximum motor torque and 16.8 % of the continuous stall torque.

### 3.2.2 Identification of the Modified Single Lumped Mass Flexible Link Model

The flexible links which are made of duraluminum and composite materials, were both identified by considering the Modified Single Lumped Mass Model. Given the limitations of the model, only the first significant vibrational mode was captured in each case. The identification procedure was divided into two stages that are explained below. Three payload masses for each link setup were considered. These payload masses are denoted throughout the thesis as  $m_{tip-d1}$ ,  $m_{tip-d2}$  and  $m_{tip-d3}$ , in the case of the duraluminum link, and  $m_{tip-c11}$ ,  $m_{tip-c12}$  and  $m_{tip-c13}$  in the case of the composite link. These constants particularize the original constant  $m_{tip}$  used in the dynamic equations. Their values are indicated in Tables A.1 and A.2 in Appendix A. In all the stages, it is assumed that the motor is controlled in position (see 4 for motor position controller alternatives).

**First stage:** in this stage, the constant  $k_{gauge-base}$  and the link mass contribution to the gravitational term ( $m_{link-g}$ ) were identified. The input data considered during the identification procedure considered as input data the strain measurements from the strain gauges located on the base of the link, and the tip position measured by the motion capture system. This data was collected from static experiments in which the robot held the different payload masses at 20 evenly spaced static poses where the motor position was between 90 and 180 degrees. The parameters were obtained by linear regression of the following expression

$$k_{gauge-base} V_{gauge} = \Gamma_{coup} = (m_{tip} + m_{link-g}) g l \cos(\theta_t), \quad (3.51)$$

resulting from combining equations (3.26) and (3.28) and considering conditions of zero acceleration. The results of this identification stage are presented in Tables 3.1 and 3.2.

**Second stage:** here, the flexural rigidity ( $E I_z$ ) and the link mass contribution to the inertial term ( $m_{link-a}$ ) were identified from the frequency response around an equilibrium point, where  $\theta_{m0} = 135$  degrees. For the identification of the model under consideration, only the first vibrational frequencies of the links around the equilibrium point were required. However, we intended to characterize the frequency spectrum to the point at which the second vibrational mode appears in order to assess its relevance in terms of the location in the spectrum and its amplitude. In the following paragraph, we present two methods with which to obtain the frequency description of the system.

The first method corresponds to the computation of the Frequency Response Function (FRF) of the system between the motor position ( $\theta_m$ ) and the coupling torque ( $\Gamma_{coup}$ ). In order to compute this, we considered a chirp signal with an amplitude of 1 degree added to a sinusoidal signal with an amplitude of 5 degrees and a constant frequency of 0.1 Hz. The chirp signal was used to excite the dynamics of the flexible link, whereas the sinusoidal signal was used to overcome the static friction in the motor so as to prevent its non-linear nature from interfering with the excitation of the flexible arm. Owing to the limitations of the motor control signal, the maximum frequency of the chirp signal that was successfully tracked by the motor in the case of the duraluminum link was about 80 Hz, and about 60 Hz in the case of the composite link. In this range, the results of these experiments show only one vibrational frequency in the case of the duraluminum link and two vibrational frequencies in the case of the composite link. The nominal parameters of the duraluminum link make it possible to know that the second vibrational frequency is located at a higher frequency than 80 Hz and, therefore, cannot be properly detected by the above method. This observation is valid to argue that the second vibrational frequency cannot be significantly excited by the actuator and cannot, therefore, in principle, notably impact on the performance of a controller based on a dynamic model that considers only the first vibrational mode. In contrast, in the case of the composite link, the second vibrational frequency is significant and may negatively affect a controller based on a dynamic model that considers only the first vibrational mode. These negative effects can, to certain extent, be avoided by reducing the bandwidth of the overall closed loop system (see, for example, Section 6.5 in Chapter 6).

In order to overcome the limitations of the first method and to excite a broader range of the frequency spectrum, we considered a second method for the excitation of the system. With this method, the robot was initially at rest at  $\theta_{m0} = 135$  degrees. Different parts of the link were then gently hammered in an attempt to excite at least the first two vibrational modes, which could be identified subsequently from the FFT.

The results of this approach were also unable to allow us to trace either the location of the second vibrational frequency of the duraluminum link, or the positions of higher

order vibrational frequencies in the case of the composite link. These results indicate that, regardless of the position of these vibrational frequencies in the spectrum, their amplitudes do not appear to be very significant.

The frequency response of the system for the duraluminum and the composite link are presented in Fig 3.4 for all the payload masses considered in the previous stage. In the case of the duraluminum link, the frequency response of the system presented is obtained from the second method as long as it allows us to represent a broader range of frequencies. In this method, since the excitation force applied with the hammer was not exactly the same for each experiment, the amplitude of the FFT response was normalized for the sake of clarity. In the case of the composite link, the FRF between the motor position and the coupling torque is presented. The corresponding first vibrational frequencies for the duraluminum link were 7.32 Hz, 5.43 Hz and 4.53 Hz for the payload masses  $m_{tip-d1}, m_{tip-d2}, m_{tip-d3}$ , respectively. In the case of the composite link, the corresponding first vibrational frequencies were 2.97 Hz, 2.34 Hz, 2.15 Hz for the payload masses  $m_{tip-c11}, m_{tip-c12}, m_{tip-c13}$ , whereas the second vibrational frequencies were all grouped around 44 Hz. The two methods commented above were repeated for different initial resting poses  $\theta_{m0}$ . The results indicated that the vibrational frequencies are hardly affected by the orientation of the link, at least in the case of the links studied. The largest differences are about 0.1 Hz for the first vibrational frequencies.

Once the value of the first vibrational frequencies is known, the expression in the model to be fitted is obtained by linearizing the dynamic model for a resting position of the tip ( $\theta_{t0}$ ) corresponding to  $\theta_{m0} = 135$  degrees. The frequency expression corresponds to the imaginary part of the poles of the linearized system expressed by

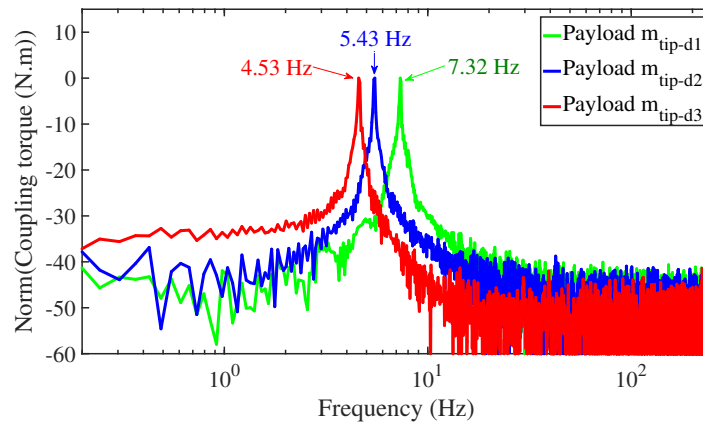
$$\omega_{1st}(\theta_{t0}) = \sqrt{\frac{3 E I_z}{(m_{tip} + m_{link-a}) l^3} - \frac{(m_{tip} + m_{link-g})}{(m_{tip} + m_{link-a})} \frac{g}{l} \sin(\theta_{t0})}, \quad (3.52)$$

which can be approximated by means of the following expression

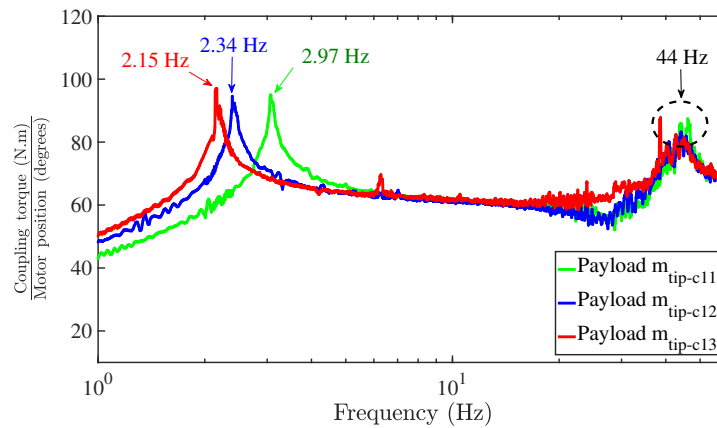
$$\omega_{1st} \approx \sqrt{\frac{3 E I_z}{(m_{tip} + m_{link-a}) l^3}}, \quad (3.53)$$

since there are no important changes in the vibrational frequencies with respect to the orientation of the link. The results of this identification stage are contained in Tables 3.1 and 3.2.

Although the second vibrational frequencies do not even appear in the FFT of the duraluminum link, the Euler Bernoulli beam theory (see B.2 in Section B.2), and the parameters



(a) Identification - duraluminum link: FFT of the strain gauge measurements after hitting the link.



(b) Identification - composites link: Frequency Response Function between the motor position and the coupling torque.

Fig. 3.4 Identification - duraluminum and composite links: Frequency Response Function between the motor position and the coupling torque.

Table 3.1 Identification - duraluminum link: parameters for the Modified Single Lumped Masses Model.

| Description                                           | Value                   |
|-------------------------------------------------------|-------------------------|
| Identified Parameters                                 |                         |
| Strain gauge gain constant ( $k_{gauge-base}$ )       | 39.55 N m/V             |
| Flexural rigidity ( $E I_z$ )                         | 803.67 N m <sup>2</sup> |
| Link mass gravitational contribution ( $m_{link-g}$ ) | 0.305 Kg                |
| Link mass inertial contribution ( $m_{link-a}$ )      | 0.151 Kg                |

Table 3.2 Identification - composite link: parameters for the Modified Single Lumped Masses Model.

| Description                                           | Value                   |
|-------------------------------------------------------|-------------------------|
| Identified Parameters                                 |                         |
| Strain gauge gain constant ( $k_{gauge-base}$ )       | 23.14 N m/V             |
| Flexural rigidity ( $E I_z$ )                         | 746.57 N m <sup>2</sup> |
| Link mass gravitational contribution ( $m_{link-g}$ ) | 0.138 Kg                |
| Link mass inertial contribution ( $m_{link-a}$ )      | 0.035 Kg                |

identified, enable us to know that they are located at approximately 97.61 Hz, 96.27 Hz and 95.80 Hz for the payload masses  $m_{tip-d1}$ ,  $m_{tip-d2}$  and  $m_{tip-d3}$ , respectively.

### 3.2.3 Identification of the Modified Two Lumped Mass Flexible Link Model

In this section, we consider only the experimental setup with the composite link, given that it is the only flexible link that has two significant vibrational modes in the range of frequencies that could be excited by the motor (0-60 Hz). The identification procedure is divided into two stages, which are explained in the following paragraphs. The experiments carried out considered the payload masses  $m_{tip-c11}$ ,  $m_{tip-c12}$ ,  $m_{tip-c13}$  indicated in Appendix A in Table A.2.

**First stage:** In this stage, the strain gauge constant ( $k_{gauge-base}$ ) and the link mass contribution to the gravitational term ( $m_{link-g}$ ) were identified. The identification procedure took into account the strain measurements obtained from the strain gauges placed on the base of the link, and the angular position  $\theta_{int}$  and  $\theta_t$  measured by the motion capture system. The data was collected from experiments where the system was in 20 different static poses with the motor position between 90 and 180 degrees. The parameters were obtained by linear regression of the expression (3.50) for static conditions, which is presented below

$$k_{gauge-base} V_{gauge} = \frac{m_{link-g} g l \cos(\theta_{int})}{2} + m_{tip} g l \cos(\theta_t), \quad (3.54)$$

**Second stage:** In this stage, we identified the flexural rigidity ( $E I_z$ ), the link mass contribution to the term of the acceleration ( $m_{link-a}$ ) and the damping coefficients ( $\mu_{c1}$  and  $\mu_{c2}$ ). The identification procedure consisted of fitting the FRF between the motor position and the coupling torque for the aforementioned payload masses, and around an equilibrium point of

$\theta_{m0} = 135$  degrees. Fig 3.4b presents the FRF for each of the payload masses. This fitting was carried out in two steps. The first step consisted of ignoring the damping coefficients and adjusting the values of the parameters  $E I_z$  and  $m_{link-a}$  that match the frequency observed in the experiments with the vibrational frequency of the linearized model around the equilibrium point. Taking into account that the vibrational frequencies in the experiments did not substantially change with the selection of different equilibrium points, we ignored the terms that depend on gravity in the expression for the vibrational frequencies of the model. The expressions considered were the following

$$\omega_{1st} \approx \sqrt{\frac{48 E I_z}{7 l^3} \left( \frac{m_{link-a} - \sqrt{m_{link-a}^2 + 9 m_{link-a} m_{tip} + 64 m_{tip}^2} + 8 m_{tip}}{m_{link-a} m_{tip}} \right)}, \quad (3.55)$$

$$\omega_{2nd} \approx \sqrt{\frac{48 E I_z}{7 l^3} \left( \frac{m_{link-a} + \sqrt{m_{link-a}^2 + 9 m_{link-a} m_{tip} + 64 m_{tip}^2} + 8 m_{tip}}{m_{link-a} m_{tip}} \right)}. \quad (3.56)$$

The second step consisted of manually adjusting the values of the damping coefficients  $\mu_{c1}$  and  $\mu_{c2}$  to match the amplitude of the peaks on the FRF. During the adjustment, it was necessary to slightly modify the previous parameters in order to maintain the location of the frequencies.

The comparison between the FRF in the experimental data and the dynamic model is presented in Fig. 3.5. The parameters identified are presented in Table 3.3. Observe that the damping coefficient ( $\mu_{c2}$ ) is equal to zero, which confirms the fact that the first vibration mode is very weakly damped. This can be clearly seen in equation (3.50), in which the corresponding term is ( $\mu_{c2} \dot{\theta}_t$ ).

Table 3.3 Identification - composite link: parameters for the Modified Two Lumped Mass Model.

| Description                                           | Value                   |
|-------------------------------------------------------|-------------------------|
| Strain gauge gain constant ( $k_{gauge-base}$ )       | 23.27 N m/V             |
| Flexural rigidity ( $E I_z$ )                         | 720.05 N m <sup>2</sup> |
| Link mass gravitational contribution ( $m_{link-g}$ ) | 0.303 Kg                |
| Link mass inertial contribution ( $m_{link-a}$ )      | 0.035 Kg                |
| Damping coefficient 1 ( $\mu_{c1}$ )                  | 2.57 N m s/rad          |
| Damping coefficient 2 ( $\mu_{c2}$ )                  | 0 N m s/rad             |

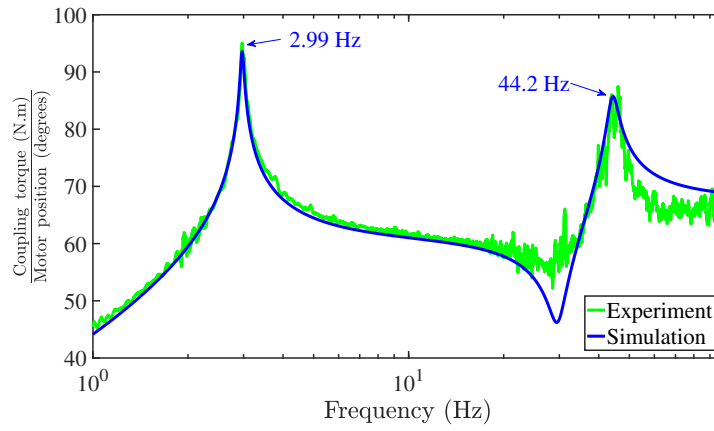
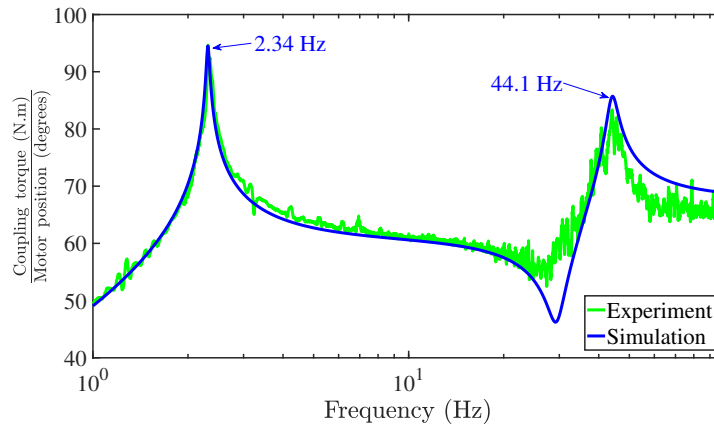
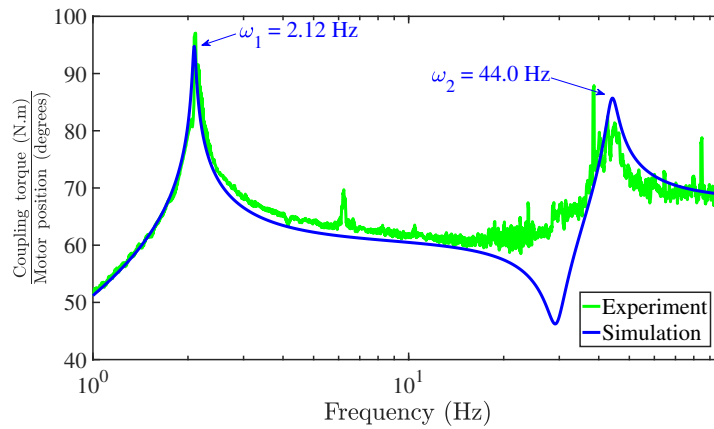
(a) Payload mass  $m_{tip-c11}$ (b) Payload mass  $m_{tip-c12}$ (c) Payload mass  $m_{tip-c13}$ 

Fig. 3.5 Identification - composite link: fitting of the Frequency Response Function between the coupling torque and the motor position.

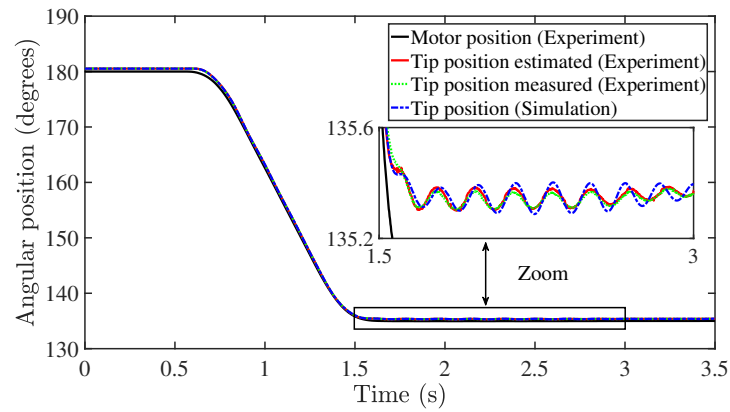
### 3.3 Validation of the dynamic model

In this section, we validate the modified dynamic models with one and two lumped masses presented in Sections 3.1.4 and 3.1.6. In the first part, we compare the time response predicted by the models with the actual experimental response of the platform. In the second part, we study the limits of the validity of the Modified Single Lumped Mass Model in terms of the link mass to payload mass ratio.

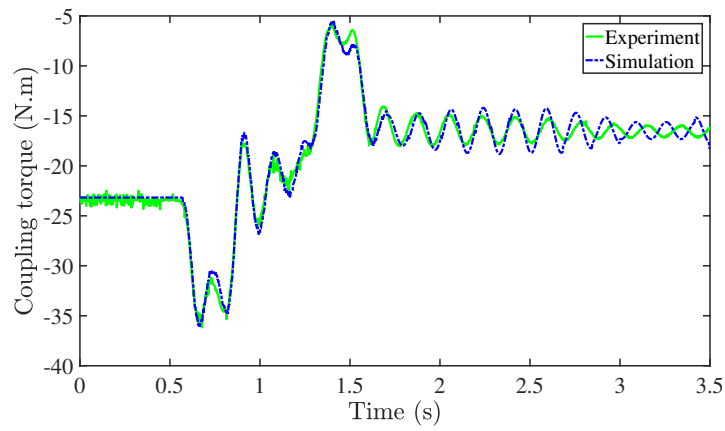
#### 3.3.1 Validation of the dynamic models for the flexible link arm based on experiments

The flexible link dynamic models were validated by means of experiments in which the motor tracked a fourth order trajectory that went from 180 to 135 degrees in 1 and 2 seconds in the case of the duraluminum and the composite links, respectively. Likewise, the payloads were respectively  $m_{tip-d2} = 2.16$  Kg and  $m_{tip-c2} = 1.20$  Kg. The position of the motor was controlled by means of the two degrees-of-freedom motor controller, explained in Chapter 4, whereas no controller was considered for the cancellation of the vibrations or for the positioning of the tip. The simulations of the flexible link arm were carried out by considering the motor position measured from the experiments as input. The validation of the Modified Single Lumped Mass Model is presented for both links (the duraluminum and the composite links, Figs. 3.6 and 3.7), whereas the validation of the Modified Two Lumped Mass Model is presented only for the composite link (Fig.3.8). This is justified by the number of vibrational modes that exist for each setup. In these figures, in the angular position comparison: "Motor position" is obtained from the motor encoder measurements, "Tip position estimated (Experiment)" corresponds to the estimation of the tip position based on the coupling torque and motor position measurements expressed by equation (3.25), "Tip position measured (Experiment)" is obtained from the motion capture system; "Tip position (Simulation)" is the result of the simulation of the model with the parameters identified in the previous sections. In the case of the Modified Two Lumped Mass Model, the signal of the "Tip position estimated (Experiment)" is omitted since no expression could be obtained for the estimation of the tip position from the sensors available at the moment of the validation: an encoder and a pair of strain gauges installed at the base of the link. With regard to the coupling torque comparison, the experimental results correspond to the measurements obtained from the strain gauges installed at the base of the link.

Apart from the visual similarity between the experiments and the simulation of the model, there are certain observations that we wish to highlight regarding the figures. First, it is important to note that the first vibrational mode of the duraluminum seems to be lightly

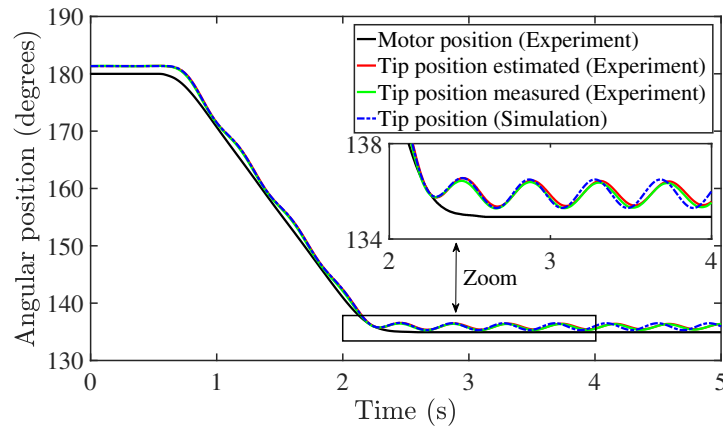


(a) Angular position comparison.

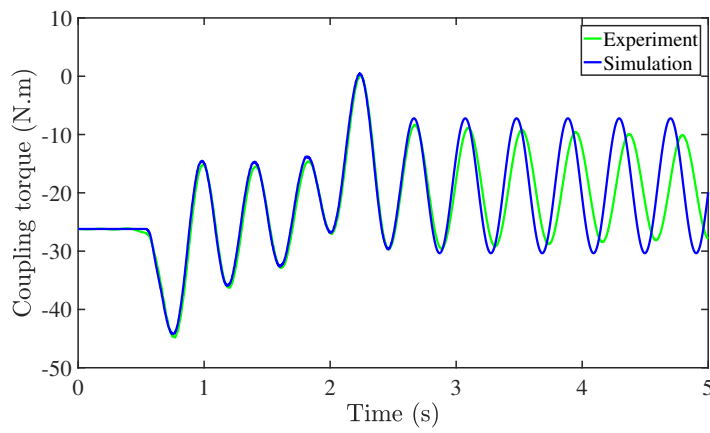


(b) Coupling torque comparison.

Fig. 3.6 Validation - duraluminum link: comparison of the angular tip position and the coupling torque for the Modified Single Lumped Dynamic Model (payload mass  $m_{tip-d2}$ )

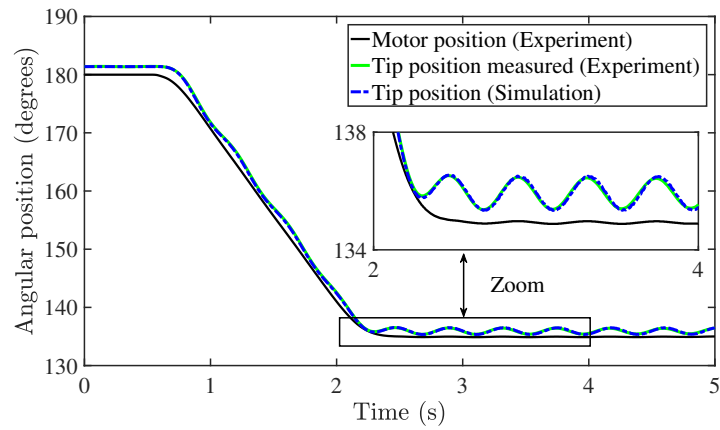


(a) Angular position comparison.

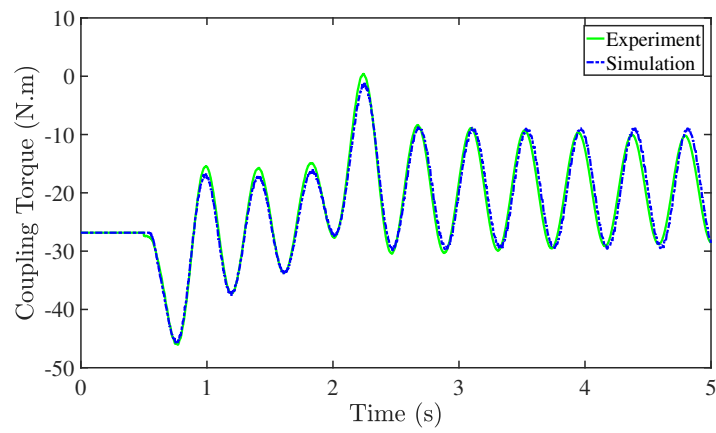


(b) Coupling torque comparison

Fig. 3.7 Validation - composite link: comparison of the angular position and the coupling torque for the Modified Single Lumped Mass Model (payload mass  $m_{tip-c2}$ )



(a) Angular position comparison



(b) Coupling torque comparison

Fig. 3.8 Validation - composite link: comparison of the angular position and the coupling torque for the Modified Two Lumped Masses Model (payload mass  $m_{tip-c2}$ )

damped, whereas the composite link did not appear to have any significant damping. In our dynamic models, a linear damping model was only considered in the case of the Modified Two Lumped Mass Model to approximate the damping of the second vibrational mode, which was quite significant. An accurate damping description of a single flexible link is normally non-linear, but is normally avoided in order to prevent the addition of unnecessary complexity to the model. In particular, it should be taken into consideration that the damping does not normally pose a real danger for the performance of the controller but contributes to the cancellation of the vibrations and the stability of the system. A second observation is that there is a certain minimal mismatch between the vibrational frequencies of the experimental setup and the model, especially in the case of the Modified Single Lumped Mass Models (Figs.3.6 and 3.7). Fortunately, this mismatch is almost negligible, and only noticeable at the end of the plot. Alternatively, the Modified Two Lumped Mass Model matches the experimental first vibrational frequency more precisely, which is the only frequency that can be seen in the experiment.

To conclude this section, Table 3.4 presents the maximum tip position errors between the experiments and the simulation of the model. The maximum tip position errors were computed from the beginning of the trajectory until the moment at which three cycles of residual vibrations had occurred. The metrics was defined in this way in order to attain a real sense of the precision of the model, ignoring the mismatches regarding the the damping and vibrational frequencies commented on previously.

Table 3.4 Validation - duraluminum and composite links: tip position errors in the experimental validation of the dynamics models.

|             |                   | With respect to<br>the estimation based<br>on strain gauges | With respect to<br>the measurements based<br>on motion capture system |
|-------------|-------------------|-------------------------------------------------------------|-----------------------------------------------------------------------|
| Duraluminum | L.(1 lumped mass) | 0.8 mm                                                      | 1 mm                                                                  |
| Composite   | L.(1 lumped mass) | 3.9 mm                                                      | 6 mm                                                                  |
| Composite   | L.(2 lumped mass) | (not estimated)                                             | 3 mm                                                                  |

The results obtained demonstrate the validity and the accuracy of the dynamic model considered very clearly, especially during the trajectory itself. In the case of the Single Lumped Mass Model, some differences in the amplitude of the signals and in the frequency of the signals start to be evident in a certain period of time after the end trajectory. These differences are mainly owing to the approximations made by the model, the precision of the parameters identified and as a result of completely ignoring the damping in this model. These errors are more noticeable in the configuration with the duraluminum link than in the case of the composites link, since the damping is more significant in the first case and given

that, as will be shown in the following section, the model loses accuracy as the link mass to payload mass ratio increases. As was expected, the model based on two lumped masses was more accurate as regards both the vibrational frequency and the amplitude of the signals.

### 3.3.2 Study of the link mass assumption for the duraluminum link

One of the underlying assumptions of the single lumped-mass model was what was denominated as "link mass assumption" (see Section 3.1.3). This assumption states that the single lumped-mass model can only describe with precision those platforms on which the mass of the flexible link is negligible with regard to the payload mass. Although this assumption greatly simplifies the complexity of the model, it really does not describe the reality of some flexible link manipulators on which the mass of the arm is quite significant. As an alternative to the original model, in Section 3.1.4, the Modified Single Lumped Mass Flexible Link Model was proposed in order to approximate the effects of the distributed link mass in the dynamics of the manipulator in those cases in which it cannot be completely ignored. In this section, we examine how considering non negligible link masses with respect to the payload mass affects the precision of the Modified Single Lumped Mass Model. More specifically, the idea is to assess how well our model recreates some relevant indicators of the system when the link mass is between  $\frac{1}{4}$  to 4 times the real mass of the duraluminum link. The indicators taken into account in this study were: the first vibrational frequency, the coupling torque in static conditions when the motor position is  $\theta_m = 0$ , and the static vertical deflection at the tip for the same configuration. In terms of these features, we compared our Modified Single Lumped Mass Model with a more complex finite element model, and also with the values obtained directly from the standard formulas of the Euler Bernoulli theory applied to the study of cantilever beams and presented in Appendix B.2.

This validation is limited to the case of the duraluminum link holding the smaller payload mass considered,  $m_{tip-d1} = 1.13$  Kg. This consideration of the payload mass represents the worst case scenario, i.e., in which the mass is more significant with regard to the payload mass. The cases examined contemplate link mass values of  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 1, 2 and 4 times the actual link mass on our platform, while the remaining parameters of the platform were the same as those shown in Table A.1 in Appendix A. The corresponding link-mass-to-payload-mass ratios ( $\frac{m_{link}}{m_{tip-d1}}$ ) for these link masses are 0.14, 0.27, 0.55, 1.08 and 2.16, respectively.

The finite element model was built using MSC Adams software and considered 100 discrete flexible elements. Given that the finite element model considers more elements than ours, it is supposed that the former is more precise. The finite element model was configured by employing experimentally identified parameters collected in Table 3.1. When defining the Modified Single Lumped Mass Model, no relationship was

established between the values of the link mass ( $m_{link}$ ) and the equivalent contributions of the link mass to the inertia ( $m_{link-a}$ ) and the gravity ( $m_{link-g}$ ) terms of the model. The computation of these terms relied completely on the identification of the platform and not on the nominal value of the link mass. In order to recreate the Modified Single Lumped Mass Flexible Link Model for the different link masses, these contributions were adjusted from the finite element model by following a similar procedure to that followed when identifying the platform, shown in Section 3.2.2.

The parameters adjusted for each link mass are shown in Table 3.5. In this table, the contribution of the link mass to the inertia ( $m_{link-a}$ ) and the gravity ( $m_{link-g}$ ) are approximately  $\frac{1}{4}$  and  $\frac{1}{2}$  times the link mass considered, respectively. Once the parameters had been defined, the model was validated in terms of the aforementioned indicators. In the following tables, the terms "E-B formula" or "E-B f." refer to the results obtained from the corresponding Euler Bernoulli formula.

Table 3.5 Identification - duraluminum link: parameters for a single lumped-mass model and different link mass values.

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-d1}}$ ) | Link<br>Mass           | Link<br>Mass<br>Value<br>(Kg) | $m_{link-a}$<br>Lumped<br>Model<br>(Kg) | $m_{link-g}$<br>Lumped<br>Model<br>(Kg) | $EI$<br>Lumped<br>(N.m <sup>2</sup> ) |
|----------------------------------------------------------------------------|------------------------|-------------------------------|-----------------------------------------|-----------------------------------------|---------------------------------------|
| 0.14                                                                       | $\frac{1}{4} m_{link}$ | 0.153                         | 0.036                                   | 0.077                                   | 815.35                                |
| 0.27                                                                       | $\frac{1}{2} m_{link}$ | 0.305                         | 0.072                                   | 0.154                                   | 815.37                                |
| 0.55                                                                       | $m_{link}$             | 0.610                         | 0.144                                   | 0.306                                   | 815.47                                |
| 1.08                                                                       | $2 m_{link}$           | 1.220                         | 0.289                                   | 0.613                                   | 815.80                                |
| 2.16                                                                       | $4 m_{link}$           | 2.440                         | 0.581                                   | 1.222                                   | 816.75                                |

Table 3.6 compares the first vibrational frequencies of the three proposed models. In this table, the errors of the finite element model and the lumped-mass model with regard to the ideal values predicted by using Euler Bernoulli formulas were under 1%. Because the parameters of the lumped model were fitted from the finite element model by following the identification procedure presented in the previous section, the first vibrational frequencies of the lumped-mass model are exactly the same.

Table 3.7 presents a comparison of the models in terms of the static coupling torque at zero motor position. It will be noted that, for this indicator, the errors in relation to the Euler Bernoulli formula are quite small and are of the same order of magnitude (under 0.26 %). The results show that when using a much more complex model based on finite elements, no significant improvements are made to this indicator.

Table 3.6 Simulation - duraluminum link: comparison of the first vibrational frequency for different link masses.

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-d1}}$ ) | E-B<br>formula<br>Model<br>(Hz) | Finite<br>Elements<br>Model<br>(Hz) | Lumped<br>Mass<br>Model<br>(Hz) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|---------------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.14                                                                       | 7.69                            | 7.75                                | 7.75                            | 0.78                                         | 0.78                                        |
| 0.27                                                                       | 7.57                            | 7.63                                | 7.63                            | 0.79                                         | 0.79                                        |
| 0.55                                                                       | 7.36                            | 7.41                                | 7.41                            | 0.64                                         | 0.64                                        |
| 1.08                                                                       | 6.97                            | 7.02                                | 7.02                            | 0.71                                         | 0.71                                        |
| 2.16                                                                       | 6.35                            | 6.40                                | 6.40                            | 0.79                                         | 0.79                                        |

Table 3.7 Simulation - duraluminum link: comparison of the static coupling torque for different link masses at  $\theta_m = 0$ .

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-d1}}$ ) | E-B<br>formula<br>(Nm) | Finite<br>Elements<br>Model<br>(Nm) | Lumped<br>Mass<br>Model<br>(Nm) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.14                                                                       | 11.35                  | 11.37                               | 11.38                           | 0.18                                         | 0.26                                        |
| 0.27                                                                       | 12.07                  | 12.10                               | 12.09                           | 0.25                                         | 0.17                                        |
| 0.55                                                                       | 13.50                  | 13.52                               | 13.52                           | 0.15                                         | 0.15                                        |
| 1.08                                                                       | 16.37                  | 16.40                               | 16.41                           | 0.18                                         | 0.24                                        |
| 2.16                                                                       | 22.11                  | 22.14                               | 22.14                           | 0.14                                         | 0.14                                        |

Table 3.8 compares the model in terms of the static vertical deflection. This table shows the most significant differences between the models. Whereas the finite element model maintained a uniform performance for the different link masses with an average error of 1.36 % with regard to the E-B formula, the performance of the Modified Single Lumped Mass Model diminished when the link mass increased, reaching an error of 13.47 % ( $\simeq 1 \text{ mm}$ ) in the worst case. In the nominal case, our model had an error of 4.29 % ( $\simeq 0.21 \text{ mm}$ ), which is a reasonable value, although it is a slightly higher than the error of the finite element model 1.43 % ( $\simeq 0.07 \text{ mm}$ ).

The results of this study allow us to conclude that the Modified Single Lumped Mass Model is a reasonable approximation of our plant, and that its performance is similar, in the nominal case, to the more complex model based on finite elements. The Modified Single

Table 3.8 Simulation - duraluminum link: comparison of the static vertical tip deflection for different link masses at  $\theta_m = 0$ .

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-d1}}$ ) | E-B<br>formula<br>(mm) | Finite<br>Elements<br>Model<br>(mm) | Lumped<br>Mass<br>Model<br>(mm) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.14                                                                       | -4.27                  | -4.22                               | -4.29                           | -1.17                                        | 0.47                                        |
| 0.27                                                                       | -4.48                  | -4.42                               | -4.57                           | -1.34                                        | 2.01                                        |
| 0.55                                                                       | -4.89                  | -4.82                               | -5.10                           | -1.43                                        | 4.29                                        |
| 1.08                                                                       | -5.71                  | -5.63                               | -6.18                           | -1.40                                        | 8.23                                        |
| 2.16                                                                       | -7.35                  | -7.24                               | -8.34                           | -1.50                                        | 13.47                                       |

Lumped Mass Model performs better when the payload mass is greater than the link mass, but it is still useful for systems in which the payload mass is in the same order of magnitude and only one significant vibrational mode exists. However, a more complex lumped-mass model could also be used depending on the precision required and on the number of significant vibrational modes.

### 3.3.3 Study of the link mass assumption for the composites link

As occurred in the previous section, here we are interested in studying how considering non negligible link masses with regard to the payload mass affects the precision of the Modified Single Lumped Mass Flexible Link Model in the case of the composites link. In order to put the quality of our results into perspective, our model is compared with a more complex finite element model, with 100 elements, in terms of three indicators: the first vibrational frequency, the static coupling torque at zero motor position ( $\theta_m = 0$ ) and the static vertical deflection at the tip, also at zero motor position. The nominal values of these indicators, also included in the comparison, were computed according the Euler Bernoulli theory as indicated in Appendix B.2.

In this study only the lowest payload mass available for the composite link was considered ( $m_{tip-cl1} = 0.73$  Kg), as it represents the worst case scenario in which the link-mass-to-payload-mass ratios ( $\frac{m_{link}}{m_{tip-cl1}}$ ) are more significant. With regard to the link masses, the cases covered were link masses of  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 1, 2 and 4 times the actual mass of the composites link. The corresponding link-mass-to-payload-mass ratios ( $\frac{m_{link}}{m_{tip-cl1}}$ ) for theses link masses are 0.09, 0.19, 0.37, 0.74 and 1.48, respectively.

The finite element model was simulated using MSC Adams and considered the parameters of the composite link previously identified. The composite link parameters are shown in Table A.2 in Appendix A. As was mentioned in the previous section, no relationship has been established between the values of the link mass ( $m_{link}$ ) and the equivalent contributions of the link mass to the inertia ( $m_{link-a}$ ) and the gravity ( $m_{link-g}$ ) terms of the model. The computation of these contributions relies completely in the identification of the platform. In the case of this study, they are adjusted using simulated data obtained from the MSC Adams finite element model and considering a procedure similar to that followed when identifying the platform (Section 3.2).

The parameters adjusted for each link mass are shown in Table 3.9. As a general remark, the contribution of the link mass to the inertia ( $m_{link-acc}$ ) and the gravity ( $m_{link-g}$ ) are about  $\frac{d1}{4}$  and  $\frac{1}{2}$  times the link mass considered, respectively. In the following tables, the terms "E-B formula" or "E-B f." refer to the results obtained from the corresponding Euler Bernoulli formula. Table 3.10 depicts the first vibrational frequencies of the two models compared and the nominal values obtained from the corresponding Euler-Bernoulli formulas. In summary, these results show that, in all the cases, the finite element model and the Modified Single Lumped Mass Model had an error of less than 0.73% in relation to the nominal frequency. Table 3.11 compares the static coupling torque predicted by the Modified Single Lumped Mass Model and the finite element model with respect to the corresponding nominal value (Euler Bernoulli formula). In both models, the error maintained the same order of magnitude (less than 0.09 %) and did not significantly deteriorate in relation to the link-mass-to-payload-mass-ratio.

Table 3.9 Simulation - composite link: parameters adjusted for a single lumped-mass model and different link mass values

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-cl}}$ ) | Link<br>Mass           | Link<br>Mass<br>Value<br>(Kg) | $m_{link-a}$<br>Lumped<br>Model<br>(Kg) | $m_{link-g}$<br>Lumped<br>Model<br>(Kg) | $EI$<br>Lumped<br>( $N.m^2$ ) |
|----------------------------------------------------------------------------|------------------------|-------------------------------|-----------------------------------------|-----------------------------------------|-------------------------------|
| 0.09                                                                       | $\frac{1}{4} m_{link}$ | 0.068                         | 0.018                                   | 0.034                                   | 759.90                        |
| 0.19                                                                       | $\frac{1}{2} m_{link}$ | 0.135                         | 0.034                                   | 0.068                                   | 759.34                        |
| 0.37                                                                       | $m_{link}$             | 0.270                         | 0.065                                   | 0.135                                   | 758.04                        |
| 0.74                                                                       | $2 m_{link}$           | 0.540                         | 0.124                                   | 0.269                                   | 754.15                        |
| 1.48                                                                       | $4 m_{link}$           | 1.080                         | 0.235                                   | 0.538                                   | 737.54                        |

A comparison of the models in terms of the static vertical deflection is presented in Table 3.12. This indicator reflects the most important differences between the models. While

Table 3.10 Simulation - composite link: comparison of the first vibrational frequency for different link masses.

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-cl}}$ ) | E-B<br>formula<br>Model<br>(Hz) | Finite<br>Elements<br>Model<br>(Hz) | Lumped<br>Mass<br>Model<br>(Hz) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|---------------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.09                                                                       | 3.01                            | 3.04                                | 3.04                            | 0.75                                         | 0.70                                        |
| 0.19                                                                       | 2.98                            | 3.01                                | 3.01                            | 0.75                                         | 0.73                                        |
| 0.37                                                                       | 2.92                            | 2.94                                | 2.94                            | 0.70                                         | 0.70                                        |
| 0.74                                                                       | 2.81                            | 2.84                                | 2.83                            | 0.78                                         | 0.71                                        |
| 1.48                                                                       | 2.63                            | 2.65                                | 2.64                            | 0.81                                         | 0.43                                        |

Table 3.11 Simulation - composites link: comparison of the static coupling torque for different link masses at  $\theta_m = 0$ .

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-cl}}$ ) | E-B<br>formula<br>(Nm) | Finite<br>Elements<br>Model<br>(Nm) | Lumped<br>Mass<br>Model<br>(Nm) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.09                                                                       | 15.28                  | 15.29                               | 15.29                           | 0.06                                         | 0.09                                        |
| 0.19                                                                       | 15.94                  | 15.95                               | 15.96                           | 0.06                                         | 0.09                                        |
| 0.37                                                                       | 17.27                  | 17.28                               | 17.29                           | 0.05                                         | 0.08                                        |
| 0.74                                                                       | 19.94                  | 19.95                               | 19.95                           | 0.05                                         | 0.07                                        |
| 1.48                                                                       | 25.26                  | 25.27                               | 25.28                           | 0.05                                         | 0.06                                        |

the finite element model maintained a small and uniform error for the different links masses (less than 1.60 %), the Modified Single Lumped Mass Model deteriorated as the link mass increased, presenting an error of 13.22 % in the worst case. In the case of our actual platform, this error was of 2.68 % which is a little higher than the error presented of the more complex finite element model. This error is equivalent to 0.81 mm and can be considered an acceptable value when considering the simplicity of the model proposed.

The results of the study of the link mass assumption for the composite link lead to the same conclusions found in the case of the duraluminum link. First, the Modified Single Lumped Mass Model is a reasonable approximation of our physical system, with a performance comparable to a more complex finite element model with 100 elements. Second, the Modified Single Lumped Mass Model performs better when the payload mass is larger than

Table 3.12 Simulation - composite link: comparison of the static vertical tip deflection for different link masses at  $\theta_m = 0$ .

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip-cl}}$ ) | E-B<br>formula<br>(mm) | Finite<br>Elements<br>Model<br>(mm) | Lumped<br>Mass<br>Model<br>(mm) | Finite<br>Elements<br>E-B f.<br>Error<br>(%) | Lumped<br>Mass M.<br>E-B f.<br>Error<br>(%) |
|----------------------------------------------------------------------------|------------------------|-------------------------------------|---------------------------------|----------------------------------------------|---------------------------------------------|
| 0.09                                                                       | -27.65                 | -27.25                              | -27.46                          | -1.44                                        | -0.70                                       |
| 0.18                                                                       | -28.56                 | -28.15                              | -28.71                          | -1.45                                        | 0.53                                        |
| 0.37                                                                       | -30.39                 | -29.94                              | -31.20                          | -1.48                                        | 2.68                                        |
| 0.74                                                                       | -34.04                 | -33.52                              | -36.14                          | -1.53                                        | 6.16                                        |
| 1.48                                                                       | -41.35                 | -40.69                              | -46.82                          | -1.60                                        | 13.22                                       |

the link mass, but it is still a good approximation in those cases in which the payload mass is in the same order of magnitude as the link mass and only one vibrational mode needs to be modeled. Finally, depending on the precision required and the number of vibrational modes that need to be reproduced, a higher order lumped-mass model can be considered.



# Chapter 4

## Motor position control

Static/Coulomb friction may negatively affect the precision of flexible link robots if is not managed properly. In rigid robotics, the static/Coulomb friction phenomenon can be ignored without important implications by considering a controller with very high feedback gain for tip positioning of the arm. In the case of flexible robotics, however, using an equivalent controller with a very high feedback gain is not a viable alternative since the dynamics of the flexible link arm has a non-minimum phase characteristic, which limits the maximum gains for which the system remains stable. Another reason for limiting the feedback gain of the end-position controller is to avoid exciting the high order dynamics, which has not been included in the dynamic model. The excitation of this dynamics, known in literature as *spill-over*, introduces disturbances into the controlled system, thus producing undesired vibrations and, ultimately, causing instability in the system.

In this thesis, we consider a double loop cascade control as a general control architecture with which to control the end position of a flexible link arm. This methodology allows us to divide the general control problem into two simpler ones with specific constraints. In the inner loop, the motor position control is addressed in such a way that the problems associated with static/Coulomb friction are solved with total independence of the the flexible link dynamics. In the outer loop, the flexible link end-effector position control problem is tackled by considering the resulting inner loop dynamics and the flexible link dynamics. This chapter is devoted to the design of a joint position controller.

### 4.1 Design requirements

Certain conditions need to be met by the motor position controller in order to guarantee a good performance of the overall system and to facilitate the design of the end-effector position control in the outer loop:

**Condition 1** Performance entirely independent of the link dynamics.

**Condition 2** Cancellation of the static/Coulomb friction effects.

**Condition 3** Tracking of sharp references with neither steady state error nor overshoot.

**Condition 4** Closed loop dynamics with no zeros.

This latter requirement is a necessary condition if the appearance of overshoot in the response of the system is to be completely avoided. Furthermore, it is necessary to guarantee the condition of an absence of the inner dynamics required in the design of the control algorithm for the flexible link position control described in Chapter 5. In the following paragraph, we discuss (in general terms) how to meet the first two conditions. The remaining conditions are discussed during the design of the motor position control in the following section.

Let us consider the motor dynamics, previously expressed in (3.1),

$$\Gamma_m = k_m n_r i_m = J \ddot{\theta}_m + v \dot{\theta}_m + \Gamma_{st-coul} + \Gamma_{coup} ,$$

where the motor position  $\theta_m$  is the variable to be controlled, the armature current  $i_m$  represents the control signal, and the coupling torque  $\Gamma_{coup}$  acts as a linking variable between the motor and flexible link subsystems.

In order to control the motor position with total independence from the link dynamics (Condition 1), the coupling torque needs to be canceled in the motor dynamics. This first step is especially important in experimental setups such as ours, in which very long link flexible arms and heavy payload masses are considered. Since the coupling torque can be measured using the strain gauges, it can be directly compensated. This is achieved by considering the following fictitious control signal  $i_c$

$$i_c = i_m - \frac{\Gamma_{coup}}{K_m n} , \quad (4.1)$$

whose substitution in the original motor dynamics (3.1) provides a new differential equation describing the motor dynamics that is completely independent from the coupling torque and, therefore, from the link dynamics

$$K_m n i_c = J \ddot{\theta}_m + v \dot{\theta}_m + \Gamma_{st-coul} . \quad (4.2)$$

There are two main approaches with which to deal with the cancellation of the static/Coulomb frictions effects in the motor position control (Condition 2: first, the cancellation based on a model of the friction; and second, the design of controllers that are robust to these effects.

The first of these approaches is difficult to apply since accurate models of static/Coulomb friction are complicated, keeping in mind that this friction might vary in a complex manner depending on factors such as motor position, temperature and operating conditions. A more practical approach might consist of considering simple models that are identified in real time but that normally need to be combined with an adaptive control algorithm. The second approach, which is that considered in this thesis, considers static/Coulomb friction as a disturbance acting on the motor dynamics and develops control algorithms capable of rejecting this type of disturbances. This approach is more straightforward to apply, is less sensitive to variations in the friction and does not require an accurate model of the friction. By considering Coulomb friction as a disturbance, the motor dynamics expressed in (4.2) can be written in the Laplace domain as

$$\theta_m(s) = \frac{\frac{K_m n}{J}}{s^2 + \frac{v}{J}s} (I_c(s) + D(s)) = \frac{a_m}{s^2 + b_m s} (I_c(s) + D(s)) , \quad (4.3)$$

where  $D(s)$  represents a disturbance current owing to static/Coulomb friction,  $a_m = \frac{k_m n_r}{J}$  and  $b_m = \frac{v}{J}$ . The Coulomb friction model presented in 3.2 allow us to assume that it behaves as a step disturbance acting on the input of the motor subsystem. Upon taking these considerations into account, the inner loop design process is reduced to the design of a linear controller that is capable of rejecting this type of disturbances.

## 4.2 Two degree-of-freedom controller design

The two degree-of-freedom controller shown in Fig. 4.1 is used to meet the design requirements established in Section 4.1. The two degree-of-freedom setting is advantageous when dealing with disturbances since it decouples the response of the system to command signals  $\theta_m^*$  from the response to disturbances  $D(s)$  acting on the input. The resulting transfer function is given by

$$\theta_m(s) = \frac{R_1(s) a_m}{s^2 + b_m s + (R_1(s) + R_2(s)) a_m} \theta_m^*(s) + \frac{a_m}{s^2 + b_m s + (R_1(s) + R_2(s)) a_m} D(s) , \quad (4.4)$$

where  $R_1(s)$  and  $R_2(s)$  define the controller. From the final-value theorem, it is known that to completely cancel a step disturbance  $D(s)$  in the steady state, owing to static/Coulomb friction (Condition 2), it is necessary the presence of at least of one multiplicative term 's' in the numerator of the transfer function  $\frac{\theta_m(s)}{D(s)}$ . This condition is met by assuming that  $R_1(s) = \frac{R_{n1}(s)}{s R_d(s)}$  and  $R_2(s) = \frac{R_{n2}(s)}{s R_d(s)}$ . If these expressions are substituted in (4.4), we obtain

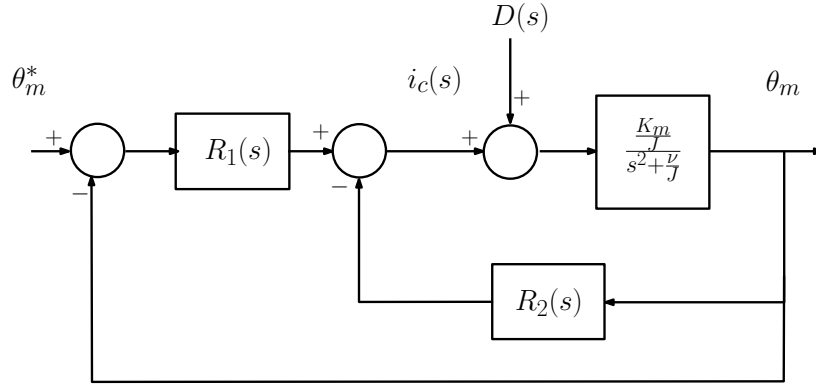


Fig. 4.1 Controller design: two degree-of-freedom motor position controller.

$$\theta_m(s) = \frac{R_{n1}(s) a_m}{s^3 + b_m s^2 + (R_{n1}(s) + R_{n2}(s)) a_m} \theta_m^*(s) + \frac{s a_m}{(s^3 + b_m s^2) + (R_{n1}(s) + R_{n2}(s)) a_m} D(s). \quad (4.5)$$

The order of the polynomial  $R_{n1}(s)$ ,  $R_{n2}(s)$  and  $R_d(s)$  should be imposed by respecting the condition of causality of the transfer function  $R_1(s)$  and  $R_2(s)$  and considering a sufficient number of parameters to set the remaining conditions (Condition 3 and Condition 4). In the following paragraph, we shall show that by considering  $R_{n1}(s) = \kappa_1 s^2 + \zeta_1 s + \rho_1$ ,  $R_{n2}(s) = \kappa_2 s^2 + \zeta_2 s + \rho_2$  and  $R_d(s) = s + \iota$  all the conditions can be fulfilled. The substitution of these transfer functions in (4.5) leads to

$$\theta_m(s) = \frac{(\kappa_1 s^2 + \zeta_1 s + \rho_1) a_m}{s^4 + (\iota + b_m) s^3 + (\iota b_m + \kappa_1 a_m + \kappa_2 a_m) s^2 + (\zeta_1 a_m + \zeta_2 a_m) s + (\rho_1 a_m + \rho_2 a_m)} \theta_m^*(s) + \frac{s(s + \iota) a_m}{s^4 + (\iota + b_m) s^3 + (\iota b_m + \kappa_1 a_m + \kappa_2 a_m) s^2 + (\zeta_1 a_m + \zeta_2 a_m) s + (\rho_1 a_m + \rho_2 a_m)} D(s). \quad (4.6)$$

The controllers have 7 parameters and, therefore, 7 equations are required to tune it:

- The first equation is obtained by establishing a unity steady state gain for step inputs in  $\frac{\theta_m(s)}{\theta_m^*(s)}$ . This condition is necessary to achieve step tracking with no steady state error (Condition 3). The final value theorem makes it easy to demonstrate that this condition is met when the independent terms of the numerator polynomial and the

denominator polynomial are equal. In the transfer function  $\frac{\theta_m(s)}{\theta_m^*(s)}$  in (4.6), this implies that  $\rho_2 = 0$ .

- four other equations are derived by allocating the closed loop poles in the same position  $p_i$  in the real axis. This allows us to set the rate of convergence of the controller at convenience and it is a sufficient condition, together with the absence of zeros in the expression  $\frac{\theta_m(s)}{\theta_m^*(s)}$ , to avoid the overshoot in the response (Condition 3). The following expression must, therefore, be met

$$(s - p_i)^4 = s^4 + (\iota + b_m) s^3 + (\iota b_m + \kappa_1 a_m + \kappa_2 a_m) s^2 + (\zeta_1 a_m + \zeta_2 a_m) s + (\rho_1 a_m + \rho_2 a_m) \quad (4.7)$$

Upon equating the polynomial coefficients on both sides, we obtain

$$\iota + b_m = -4 p_i, \quad (4.8) \quad (\zeta_1 + \zeta_2) a_m = -4 p_i^3, \quad (4.10)$$

$$\iota b_m + \kappa_1 a_m + \kappa_2 a_m = 6 p_i^2, \quad (4.9) \quad (\rho_1 + \rho_2) a_m = p_i^4. \quad (4.11)$$

- The remaining two equations are obtained by setting the two zeros in  $\frac{\theta_m(s)}{\theta_m^*(s)}$  such that they are in the same position of the poles  $p_i$ , so be canceled. This condition can be written as

$$(s - p_i)^2 = s^2 + \frac{\zeta_1}{\kappa_1} s + \frac{\rho_1}{\kappa_1}. \quad (4.12)$$

Upon equating the coefficients on both sides of the equation, we obtain

$$\frac{\zeta_1}{\kappa_1} = -2 p_i, \quad (4.13) \quad \frac{\rho_1}{\kappa_1} = p_i^2. \quad (4.14)$$

The controller parameters can be expressed in term of the closed loop poles as

$$\kappa_1 = \frac{p_i^2}{a_m}, \quad (4.15)$$

$$\kappa_2 = \frac{5 p_i^2}{a_m} + \frac{(4 p_i + b_m) b_m}{a_m}, \quad (4.16)$$

$$\iota = -4 p_i - b_m, \quad (4.17)$$

$$\zeta_1 = -\frac{2 p_i^3}{a_m}, \quad (4.18)$$

$$\zeta_2 = -\frac{2 p_i^3}{a_m}, \quad (4.19)$$

$$\frac{\rho_1}{\kappa_1} = p_i^2, \quad (4.20)$$

$$\rho_2 = 0. \quad (4.21)$$

Similarly, the relationship between the reference, the disturbance and the output can be expressed as

$$\theta_m(s) = \frac{p_i^2}{(s + p_i)^2} \theta_m^*(s) + \frac{s(s - 4 p_i - b_m) a_m}{(s + p_i)^2} D(s), \quad (4.22)$$

and, in a more compact way, as

$$\theta_m(s) = \frac{1}{(\alpha s + 1)^2} \theta_m^*(s) + \frac{s(s - 4 \alpha^{-1} - b_m) a_m}{(\alpha s + 1)^2} D(s), \quad (4.23)$$

where  $\alpha = \frac{1}{p_i}$ .

### 4.3 Comparison with other controllers

In this section, we compare the two degree-of-freedom motor position controller presented above with the following alternatives: (i) a PD controller, (ii) a P controller with velocity feedback, (iii) a PI with velocity feedback. Table 4.1 presents, respectively, the block diagrams and the resulting transfer functions between the reference and the output  $\frac{\theta_m(s)}{\theta_m^*(s)}$  and between the disturbance and the output  $\frac{D(s)}{\theta_m^*(s)}$ . The main properties of each controller are discussed in the following paragraphs.

Table 4.1 Controller design: transfer functions for the motor position controllers.

| Controller                                 | Parameters                                                        | $\frac{\theta_m(s)}{\theta_m^*(s)}$                                                                                                                                   | $\frac{\theta_m(s)}{D(s)}$                                                                                                                  |
|--------------------------------------------|-------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| PD controller                              | $K_p$<br>$K_v$                                                    | $\frac{a_m (K_p + K_v s)}{s^2 + (b_m + a_m K_v) s + a_m K_p}$                                                                                                         | $\frac{a_m}{s^2 + (b_m + a_m K_v) s + a_m K_p}$                                                                                             |
| P controller<br>with velocity<br>feedback  | $K_p$<br>$K_v$                                                    | $\frac{a_m K_p}{s^2 + (b_m + a_m K_v) s + a_m K_p}$                                                                                                                   | $\frac{a_m}{s^2 + (b_m + a_m K_v) s + a_m K_p}$                                                                                             |
| PI controller<br>with velocity<br>feedback | $K_p$<br>$K_v$<br>$K_i$                                           | $\frac{a_m (K_p s + K_i)}{s^3 + (b_m + a_m K_v) s^2 + a_m K_p s + a_m K_i}$                                                                                           | $\frac{a_m s}{s^3 + (b_m + a_m K_v) s^2 + a_m K_p s + a_m K_i}$                                                                             |
| Two degree-<br>of-freedom<br>controller    | $\kappa_1$ $\kappa_2$<br>$\rho_1$ $\rho_2$<br>$\zeta_1$ $\zeta_2$ | $\frac{(\kappa_1 s^2 + \zeta_1 s + \rho_1) a_m}{s^4 + (1 + b_m) s^3 + (1 + b_m + a_m (\kappa_1 + \kappa_2)) s^2 + a_m (\zeta_1 + \zeta_2) s + a_m (\rho_1 + \rho_2)}$ | $\frac{s (s + 1) a_m}{s^4 + (1 + b_m) s^3 + (1 + b_m + a_m (\kappa_1 + \kappa_2)) s^2 + a_m (\zeta_1 + \zeta_2) s + a_m (\rho_1 + \rho_2)}$ |

**The PD controller** , shown in Fig. 4.2, is the simplest controller in the comparison. Its parameters could be set in such a way that the a zero/pole cancellation occurs in the transfer function  $\frac{\theta_m(s)}{\theta_m^*(s)}$  and no overshoot appears in the transient response. However, this controller has two drawbacks. First, the control signal saturates easily owing to the presence of the derivative action in the path of the reference. This phenomenon is called "set-point kick". Second, the controller does not completely cancel step-like disturbances  $D(s)$ , e.g. Coulomb friction, in the steady state, which is equivalent to saying that it produces some steady state error in the response. This is evident if the final value theorem is applied to the relationship  $\frac{\theta_m(s)}{D(s)}$  when a step input is considered. The best result can be obtained by considering a very high  $K_p$  gain, but it might also imply the saturation of the control signal.

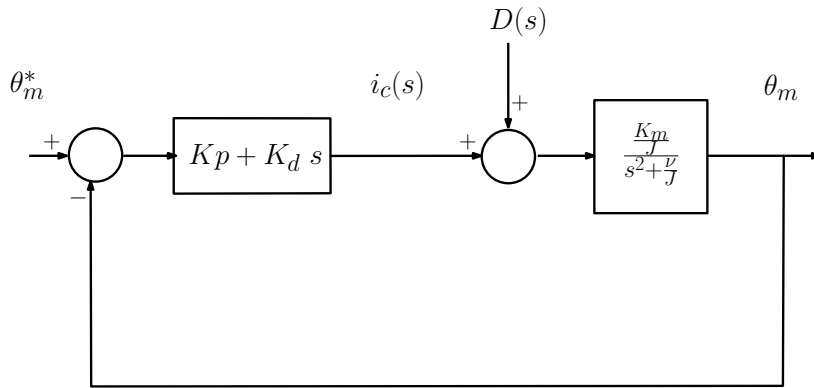


Fig. 4.2 PD motor position controller.

**The P controller with velocity feedback** , depicted in Fig. 4.3, corrects the "set-point kick" problem by considering the derivative action only in the feedback path and thus avoiding the differentiation of the reference signal. However, in this controller the Coulomb friction produces a steady state error in the response for the same reason as the previous controller.

**The PI controller with velocity feedback** , presented in Fig. 4.4, eliminates the steady state error owing to the Coulomb friction disturbances but it also adds some overshoot to the transient response as a result of the presence of a zero in the relationship  $\frac{\theta_m}{\theta_m^*}$ . Unlike the case of the PD controller, in this case a zero-pole cancellation is not possible without making the system unstable. The absence of zeros in the closed loop dynamics is a desirable feature that simplifies the design of the controllers for the end-effector position that will be studied in the subsequent chapters.

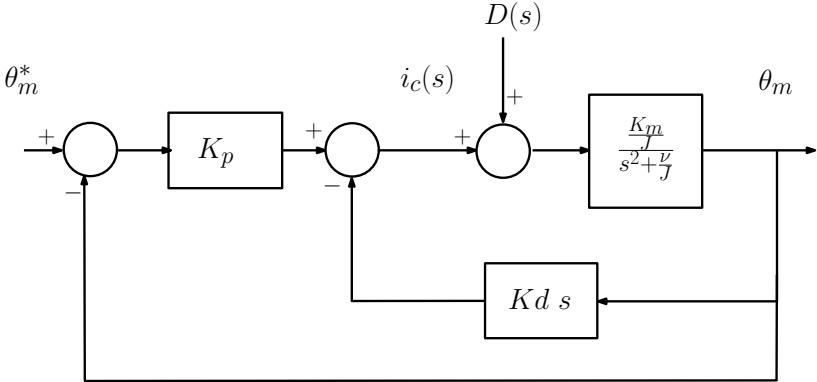


Fig. 4.3 P with velocity feedback motor position controller.

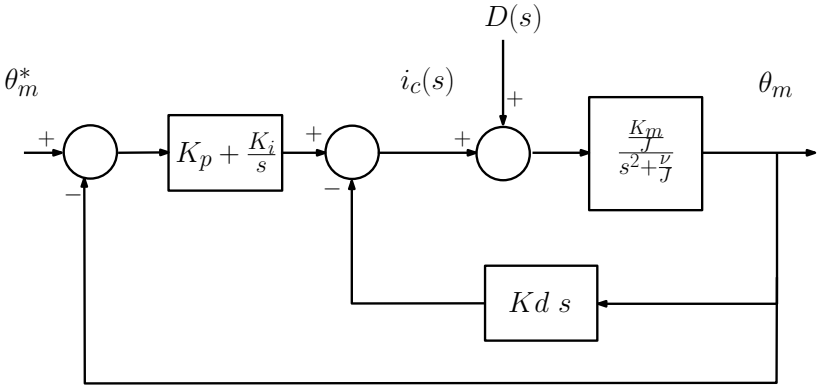


Fig. 4.4 PI with velocity feedback motor position controller.

**The two-degree-of-freedom controller** , developed in the previous section, solves all the problems mentioned above by decoupling the response of the system to reference signals  $\theta_m^*$  from the response of the system to disturbances  $D(s)$ . In this case the two zeros present in the relation  $\frac{\theta_m(s)}{\theta_m^*(s)}$  can be canceled with two poles without making the system unstable (Condition 3 and Condition 4). Furthermore, the presence of a multiplicative term 's' in the numerator of the transfer function  $\frac{D(s)}{\theta_m^*(s)}$  ensures a zero steady state error due to static/Coulomb disturbances.

The main characteristics of each controller are summarized in Table 4.3.

Table 4.2 Controller design: comparison of characteristics.

| Desirable characteristic                                    | PD controller | P with velocity feedback controller | PI with velocity feedback controller | Two-degree-of-freedom controller |
|-------------------------------------------------------------|---------------|-------------------------------------|--------------------------------------|----------------------------------|
| Absence of steady-state error owing to reference signals    | Yes           | Yes                                 | Yes                                  | Yes                              |
| Absence of steady-state error owing to Coulomb disturbances | No            | No                                  | Yes                                  | Yes                              |
| Absence of overshoot                                        | Yes           | Yes                                 | No                                   | Yes                              |
| Absence of 'set-point kick' phenomenon                      | No            | Yes                                 | Yes                                  | Yes                              |

To conclude this chapter, we present the experimental validation of all the controller discussed above. Fig. 4.5 shows the time responses of the closed loop systems when a step command of 2 degrees is applied. In these experiments, the controllers were tuned so that the poles of the closed loop system were in  $p_i = -30$ . Table 4.3 summarizes the performance of each controller in the experiments in terms of overshoot, steady state error and settling time. These results corroborate the observations made in Table , in which the two-degree-of-freedom motor position controller was the only capable of providing zero overshoot and zero steady state error. In addition to these characteristics it presents a reasonable settling time comparable with all the other control schemes. Given the clear superiority of the two-degree-of-freedom motor controller, it will be considered in the subsequent chapters.

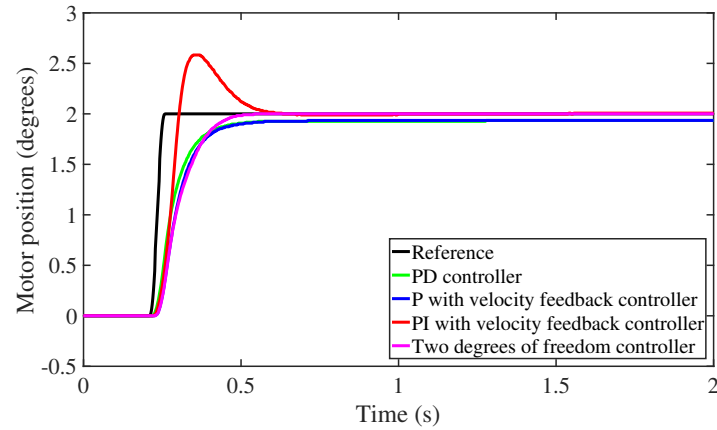


Fig. 4.5 Validation - motor position controller: comparison of the time response to a step command.

Table 4.3 Validation - motor position controller: comparison of performance to a step command.

| Desirable characteristic | PD controller | P with velocity feedback controller | PI with velocity feedback controller | Two-degree-of-freedom controller |
|--------------------------|---------------|-------------------------------------|--------------------------------------|----------------------------------|
| Overshoot                | 0 %           | 0 %                                 | 29.2 %                               | 0 %                              |
| Steady state error       | 0.065 degrees | 0.065 degrees                       | 0 degrees                            | 0 degrees                        |
| Settling time            | 0.47 s        | 0.49 s                              | 0.56 s                               | 0.54 s                           |



# Chapter 5

## Position control based on feedback linearization

In this chapter, we address the trajectory tracking problem of the end-effector of a single-link flexible arm moving under gravity and in the presence of joint friction. The design procedure considers a double loop cascade control scheme used to deal with the joint friction: the inner loop is responsible for the motor positioning in the presence of the non-linear joint friction, while the outer loop implements an input-state feedback linearization-based control to suppress the vibrations on the link and achieve the end-effector trajectory. The design of the inner loop controller was presented in Chapter 4. The present chapter is devoted to the design of the outer loop controller and an evaluation of the performance of the overall controller considering when considering the duraluminum link. A comparative study based on simulations is presented in order to highlight the superiority of the properties of our approach when compared to other methodologies. An experimental validation of the controller is also presented.

The results presented in this chapter have been partly published in

- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Feedback linearizing controller for a flexible single-link arm under gravity and joint friction,” in *ROBOT2013: First Iberian Robotics Conference*, (Madrid, Spain), pp. 169–184, Springer, 2014
- J. C. Cambera and V. Feliu-Batlle, “Input-state feedback linearization control of a single-link flexible robot arm moving under gravity and joint friction,” *Robotics and Autonomous Systems*, vol. 88, pp. 24–36, Elsevier, 2017

## 5.1 Introduction

One of the most relevant approaches used to deal with the problem of trajectory tracking in non-linear systems is the feedback linearization technique. The idea behind this technique is to convert, by means of exact state transformation and feedback, the nonlinear system dynamics into a fully or partly linear one, so that the linear control principles can be applied [42].

Although the feedback linearization control is a well-established technique in the control of rigid link manipulators, it is not as well known in the case of flexible link manipulators since its adaptation has not proven to be simple. The main inconvenience is the non-minimum phase characteristic of the system due to the non-collocation of actuators and outputs (tip position). In this respect, applying directly the input-output feedback linearization technique implies an unstable internal dynamics, which, in turn, results in that the local stability of the control scheme cannot be guaranteed. Several alternative solutions have been proposed in the bibliography. In [43], De Luca and Siciliano redefined the joint position as the output of the system in a way that resulting system is a minimum phase. This made possible the implementation of the input-output feedback linearization but it was not an effective strategy to suppress the oscillations in the flexible arm. In [44], De Luca et al. stated that there is a continuous set of points, starting at the motor position and along the flexible link that can be considered as outputs, in a manner that the system is minimum phase, and therefore, input-output linearizable. This study proved, through simulations, that selecting a point which is near to the tip position, and still in the minimum phase set, results in a better trajectory tracking of the tip position but implies very large torque demands. In [45], Lucibello and Di Benedetto used simulations to prove that, at least for periodic end-effector velocities, exact trajectory tracking might be achieved with bound internal dynamics despite the non-minimum phase characteristics of the plant. Other researches have also selected alternative outputs for the control of flexible link manipulators, but motivated by other control techniques. Wang and Vidyasagar [46] selected an alternative output that they called as the *reflected tip position* which, in those cases in which the link is sufficiently rigid, allows the system to satisfy the property of passivity.

In the following section, we apply the feedback linearization approach to a minimum phase dynamic model describing a flexible link robot arm, which has only one significant vibrational mode. Unlike the methods commented on above, in this approach there is no need to define auxiliary outputs in order to establish an input-output minimum phase relationship that ensures the stability of the controller. An approach based on a single lumped mass dynamic model and input state feedback linearization was carried out by Castillo in [41] to tackle the position control problem 2 degree-of-freedom antenna sensor moving under grav-

ity. Although both developments started from a dynamic based on a single lumped mass, the work here present is different in several aspects. (1) In the Castillo's work, the dynamic model assumes that the link mass is negligible with regards to the payload mass, whereas in our work we considered a modified single lumped mass dynamic model (see Section 3.1.4) that takes into account those cases in which the link mass is not negligible in comparison to the payload mass, and that at the same time maintains the minimum phase property. (2) Both in Castillo's work as in our developments a double loop cascade approach in combination with input state linearization was considered. However, in the former the closed loop dynamic of the motor was compensated by an approximation of its inverse dynamics, so it could be neglected in the design of input-state feedback linearization control law. Since the inversion of the closed loop motor dynamics was not realizable, the dynamic inversion was approximated by multiplying the inverse dynamics by a low pass filter. This has to be carried out very carefully, since from the control theory it is known that this undoubtedly produces a delay in the response of inner system, which at the same time affects negatively the performance and, above all, the stability of the overall system. In our approach, we eliminated the problem from the root by explicitly considering the closed loop dynamics of the motor in the design of the input state control law. The resulting control law did not significantly increase the computational burden in comparison with the former approach.

## 5.2 Controller design

In this chapter, we apply the same design methodology as that presented in [47], in which a double loop cascade control scheme was considered. This methodology divides the control problem into two simpler ones with specific restrictions. The inner loop handles the motor position control in a manner that the issues related to the joint friction are solved independently from the restrictions imposed by the flexible link. The tip position control is addressed in the outer loop. By following this strategy, the gains of the inner loop can be substantially high with the purpose of removing the friction effects, without any restriction being imposed by the non-minimum phase characteristic of the flexible link arm. Fig. 5.1 shows a schematic representation of the general control scheme adopted. The inner loop implementation corresponds to the two degree of freedom algebraic controller explained in Chapter 4. This section presents the design of the outer loop controller based on feedback linearization for the outer loop.

Let us consider the inner loop transfer function (4.23), and let us then combine (3.26) and (3.27) by eliminating  $\Gamma_{coup}$  in order to obtain a single equation that describes the flexible link dynamics. If  $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$  is considered to be the state vector, where  $x_1 = \theta_t$ ,  $x_2 = \dot{\theta}_t$ ,

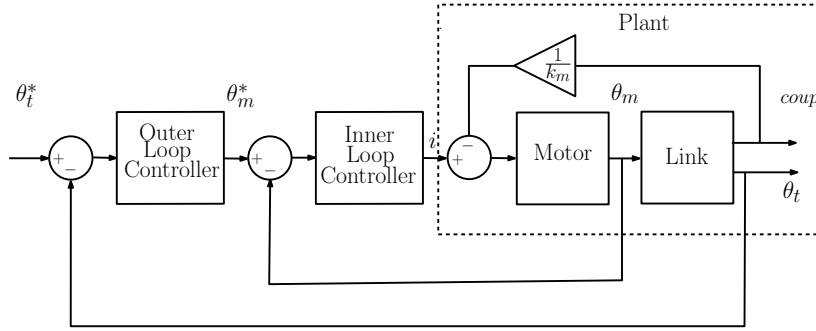


Fig. 5.1 Controller design: general schematic representation of the control algorithm for the tip positioning.

$x_3 = \theta_m$ ,  $x_4 = \dot{\theta}_m$ , and the system input is defined as  $u = \theta_m^*$ , the space state representation of the model can be written as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}) u = \begin{bmatrix} x_2 \\ \frac{3 E I_z}{m_a l^3} (x_3 - x_1) - \frac{m_g g}{m_a l} \cos(x_1) \\ x_4 \\ -\frac{1}{\alpha^2} x_3 - \frac{2}{\alpha} x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\alpha^2} \end{bmatrix} u \quad (5.1)$$

$$\dot{\mathbf{y}} = \mathbf{h}(\mathbf{x}) = x_1 \quad (5.2)$$

The non-linear system above is said to be input-state linearizable if, and only if, there exists a closed connected region  $\Omega$  in  $\mathbf{R}^n$  such that its vector fields  $\mathbf{f}(\mathbf{x})$  and  $\mathbf{g}(\mathbf{x})$  hold the conditions of controllability and involutivity [42]. These conditions are defined in terms of the Lie bracket of  $\mathbf{f}$  and  $\mathbf{g}$  ( $ad_{\mathbf{f}}\mathbf{g}$ ) and its repeated Lie brackets ( $ad_{\mathbf{f}}^i\mathbf{g}$ ) as:

- Controllability condition. All vectors in the controllability matrix  $[\mathbf{g}, ad_{\mathbf{f}}\mathbf{g}, ad_{\mathbf{f}}^2\mathbf{g}, ad_{\mathbf{f}}^3\mathbf{g}]$  are linearly independent in  $\Omega$ .
- Involutivity condition. The set  $[\mathbf{g}, ad_{\mathbf{f}}\mathbf{g}, ad_{\mathbf{f}}^2\mathbf{g}]$  is involutive in  $\Omega$ .

The substitution of  $\mathbf{f}(\mathbf{x})$  and  $\mathbf{g}(\mathbf{x})$  in the controllability matrix leads to

$$\begin{bmatrix} 0 & 0 & 0 & \frac{-3 E I_z}{m_a \alpha^2 l^3} \\ 0 & 0 & \frac{3 E I_z}{m_a \alpha^2 l^3} & \frac{6 E I_z}{m_a \alpha^3 l^3} \\ 0 & \frac{-1}{\alpha^2} & \frac{-2}{\alpha^3} & \frac{-3}{\alpha^4} \\ \frac{1}{\alpha^2} & \frac{2}{\alpha^3} & \frac{3}{\alpha^4} & \frac{4}{\alpha^5} \end{bmatrix} \quad (5.3)$$

The controllability condition is easily checked by considering that the matrix above is full rank for  $EI > 0$  and  $0 < m, l, \alpha < \infty$ . With regard to the involutivity condition, since

the vector fields  $[g, ad_f g, ad_f^2 g]$  are constant, this condition is trivially satisfied. In this case, the input-state linearization of the system is global as long as both conditions, controllability and involutivity, are satisfied everywhere and, therefore, for any region  $\Omega$ .

The next step is to find the state transformation and the input transformation in order to achieve the input-state linearization. To find the state transformation  $\mathbf{z}(\mathbf{x}) = [z_1, z_2, z_3, z_4]^T$ , the following constraints on the transformed state  $z_1$  should be considered

$$\nabla_{z_1} ad_{\mathbf{f}}^k \mathbf{g} = 0 \quad k = 0, 1, 2, \quad (5.4)$$

$$\nabla_{z_1} ad_{\mathbf{f}}^3 \mathbf{g} \neq 0. \quad (5.5)$$

These constraints are satisfied when  $z_1 = x_1 = \theta_l$ . The whole state  $\mathbf{z}(\mathbf{x})$  transformation can be computed in term of the Lie derivatives of  $z_1$  with respect to  $\mathbf{f}$  ( $L_{\mathbf{f}} z_1$ ) and its repeated derivatives ( $L_{\mathbf{f}}^i z_1$ ) as follows

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ L_{\mathbf{f}} z_1 \\ L_{\mathbf{f}}^2 z_1 \\ L_{\mathbf{f}}^3 z_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \frac{3 E I_z}{m_a l^3} (x_3 - x_1) - \frac{m_g g}{m_a l} \cos(x_1) \\ (\frac{m_g g}{m_a l} \sin(x_1) - \frac{3 E I_z}{m_a l^3}) x_2 + (\frac{3 E I_z}{m_a l^3}) x_4 \end{bmatrix} \quad (5.6)$$

The transformed system is, therefore:

$$\dot{\mathbf{z}} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ z_4 \\ -\frac{3 E I_z}{m_a l^3} (\frac{1}{\alpha^2} z_1 + \frac{2}{\alpha} z_2 + z_3) - \frac{1}{\alpha^2} z_3 - \frac{2}{\alpha} z_4 \\ (\frac{m_g g}{m_a l}) [(z_2^2 - \frac{1}{\alpha^2}) \cos(z_1) + (\frac{2}{\alpha} z_2 + z_3) \sin(z_1)] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{3 E I_z}{m_a l^3 \alpha^2} \end{bmatrix} u \quad (5.7)$$

The input transformation, or linearizing control law, to be applied has the following form  $u = \psi_1(\mathbf{z}) + \psi_2(\mathbf{z})v$ , where  $v$  is the new input of the linearized system, and  $\psi_1(\mathbf{z})$  and  $\psi_2(\mathbf{z})$  are

$$\psi_1(\mathbf{z}) = -\frac{L_{\mathbf{f}}^4 z_1}{L_{\mathbf{g}} L_{\mathbf{f}}^3 z_1}, \quad (5.8)$$

$$\psi_2(\mathbf{z}) = \frac{1}{L_{\mathbf{g}} L_{\mathbf{f}}^3 z_1}. \quad (5.9)$$

The input  $u$  can be written explicitly as

$$u = z_1 + 2\alpha z_2 + \left(\alpha^2 + \frac{m_a l^3}{3 E I_z}\right) z_3 + \left(\frac{2 m_a l^3 \alpha}{3 E I_z} z_4\right) + \frac{m_g g l^2}{3 E I_z} (1 - \alpha^2 z_2^2) \cos(z_1) \quad (5.10)$$

$$- \frac{m_g g l^2 \alpha}{3 E I_z} (2 z_2 + \alpha z_3) \sin(z_1) + \left(\frac{m_a l^3 \alpha^2}{3 E I_z}\right) v$$

The substitution of the equation (5.10) into (5.7) leads to the following linear system

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v \quad (5.11)$$

Bearing in mind that the link tip position  $z_1 = x_1 = \theta_t$  should track a predefined trajectory  $z_1^*$ , the following linear control law is proposed

$$v = z_1^{*(4)} - k_3(z_1^{(3)} - z_1^{*(3)}) - k_2(\ddot{z}_1 - \ddot{z}_1^*) - k_1(\dot{z}_1 - \dot{z}_1^*) - k_0(z_1 - z_1^*) \quad (5.12)$$

If  $e_1 = z_1 - z_1^*$ , the tracking error dynamics can be expressed as

$$e_1^{(4)} + k_3 e_1^{(3)} + k_2 e_1^{(2)} + k_1 \dot{e}_1 + k_0 e_1 = 0 \quad (5.13)$$

The dynamics above is exponentially stable if positive constants  $k_i$  are chosen adequately. These constants can be obtained by equating the characteristic polynomial of (5.13) in the Laplace variable  $s$  to the desired closed loop polynomial

$$s^{(4)} + k_3 s^{(3)} + k_2 s^{(2)} + k_1 s + k_0 = (s - p_o)^4, \quad (5.14)$$

where all the desired closed loop poles for the outer loop are placed in the same position  $p_o$ .

Before this controller is implemented on the physical platform described in Chapter 2, the non-linear control law (5.10) has to be adapted to the available sensor measurements. The only measurements available on our experimental platform are the motor position ( $\theta_m$ ) and the coupling torque ( $\Gamma_{coup}$ ), which are measured using the strain gauges placed at the base of the link. If these signals and equation (3.27) are considered, the state vector  $\mathbf{x}$  can be fully estimated as

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \begin{bmatrix} \hat{\theta}_t \\ \hat{\dot{\theta}}_t \\ \theta_m \\ \hat{\dot{\theta}}_m \end{bmatrix} = \begin{bmatrix} \theta_m - \Gamma_{\text{coup}} \frac{l}{3 E I_z} \\ \dot{\theta}_m - \dot{\Gamma}_{\text{coup}} \frac{l}{3 E I_z} \\ \theta_m \\ \dot{\theta}_m \end{bmatrix} \quad (5.15)$$

where states  $\hat{x}_2$  and  $\hat{x}_4$  are obtained by means of the numerical differentiation of states  $\hat{x}_1$  and  $\hat{x}_3$ , respectively. A diagram of the outer loop control scheme is shown in Fig. 5.2, all the equations considered in the control algorithm are summarized.

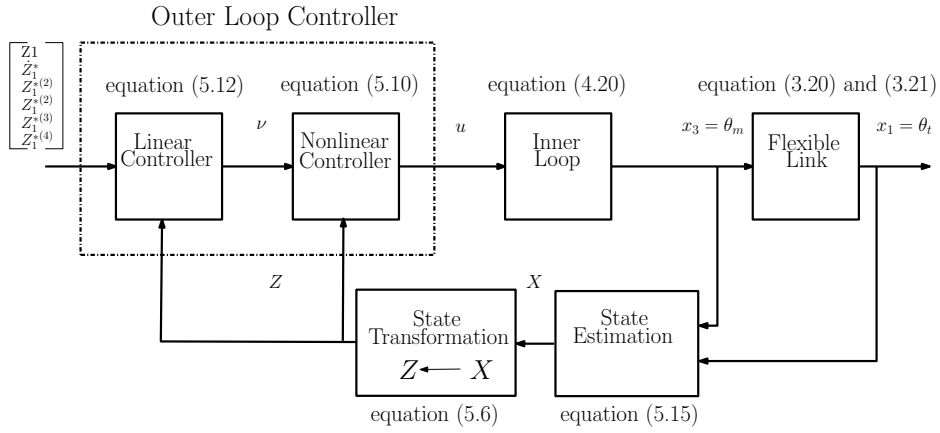


Fig. 5.2 Controller design: detailed schematic representation of the control algorithm for the tip positioning.

## 5.3 Simulations

In this section, two studies based on simulations are presented in order to assess the performance of the proposed controller. The first section presents a comparative study in which two approaches other than that proposed in this thesis are considered. This first study explore the effects of Coulomb friction on the stability of the approaches in the context of a task in which payload mass uncertainty exists (e.g., the pick and place task). The second study quantifies the effects of applying the proposed controller flexible arms whose mass is comparatively significant with in relation to the payload mass.

### 5.3.1 Effects of Coulomb friction and payload mass changes

The static/Coulomb friction is not taken into account by most of the approaches of controller design for flexible link arms. This apparent slight assumption adversely affects the

performance of the system. In this section, we compare three control approaches based on feedback linearization with the goal of evaluating the advantages of our general control algorithm scheme (two nested loop control) over others in the presence of joint friction. The control schemes are: (a) the single control loop scheme, in which no inner loop is used to control the motor position [44]; (b) the single loop approach with an integrative action as an attempt to cope with some of the negative effects of the joint friction of the previous approach (steady state errors); and (c) two nested loop control scheme proposed in this thesis. Before discussing the results obtained in the simulations, we present a brief summary of the design methodology followed for the single loop controller (a), and the single loop controller with integrative action (b).

The first two controllers, the single loop controller (a) and the single loop controller with integrative action (b), were designed taking into consideration an equivalent procedure to the one described in Section 5.2 for the double loop approach (c). The most significant difference is the dynamic model considered as starting point. In all cases, the same minimum phase dynamic model of the flexible link was considered. However, in the cases (a) and (b), it was combined with the open loop dynamic equation of the motor instead of the dynamics of the motor controlled in position, as in the approach (c). The dynamic model of the link corresponds to equations (3.26) and (3.27), while the open loop dynamics equation of the motor is provided by (3.1). If we consider the states  $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$  as the state vector, where  $x_1 = \theta_t$ ,  $x_2 = \dot{\theta}_t$ ,  $x_3 = \theta_m$ ,  $x_4 = \dot{\theta}_m$ , and upon defining the system input as the armature current ( $u = i$ ), the state equation can be written as

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x}) u = \begin{bmatrix} x_2 \\ \frac{3 E I_z}{m_a l^3} (x_3 - x_1) - \frac{m_g g}{m_a l} \cos(x_1) \\ x_4 \\ -\frac{\nu}{J} x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_m n}{J} \end{bmatrix} u \quad (5.16)$$

As in the procedure shown in Section 5.2, it can be found a state transformation  $\mathbf{z}(\mathbf{x})$  and a linearizing control law  $u(\mathbf{z})$ . Its substitution in (5.16) results into a full fourth order linear system. The single loop feedback linearization (a) takes into consideration the same linear control law in (5.12), in which the parameters  $k_i$  are defined in terms of the desired closed loop poles  $p_o$ , as in (5.14). On the other hand, the single loop feedback linearization with integrative action (b) takes the following linear control law into consideration

$$v = \ddot{z}_1^{*(4)} - k_4(\dot{z}_1^{(3)} - \dot{z}_1^{*(3)}) - k_3(\ddot{z}_1 - \ddot{z}_1^*) - k_2(\dot{z}_1 - \dot{z}_1^*) - k_1(z_1 - z_1^*) - k_0 \int (z_1 - z_1^*) dt, \quad (5.17)$$

where the parameters  $k_i$  are obtained by equating the corresponding characteristic polynomial to the desired closed loop polynomial

$$s^{(5)} + k_4 s^{(4)} + k_3 s^{(3)} + k_2 s^2 + k_1 s + k_0 = (s - p_o)^5, \quad (5.18)$$

and where all the closed loop poles are placed in the same position  $p_o$ .

In these simulations, the poles for the inner loop  $p_i$  (only applicable in (c)) and the poles for the outer loop  $p_o$ , were placed to the same position:  $p_i = p_o = -30$ . An abrupt fourth order reference trajectory was taken into account in order to replicate an an step like response of the system and make evident the differences in the performance of the controllers. It is important to note, from equations (5.12) and (5.17), that in order to guarantee finite control signals the references need to be at least least fourth order differentiable. The trajectory was designed following the procedure indicated in [48], and goes from  $\theta_{tip}^* = 90$  to  $\theta_{tip}^* = 135$  degrees in 0.1 s.

The discussion about the performance of the controllers considers the case of the Duraluminum link and is focused on four indicators: steady state error, residual vibrations, settling time, and maximum overshoot. The simulations consider the context of a pick and place task, in which the payload is constantly changing and its mass is, in the most general case, unknown. In order to emulate this situation, we took the mass  $m_{tip-d2}$  defined in Table A.1 in Appendix A as the nominal payload mass, i.e., the payload mass used to tune the control algorithms. Then, we studied and compared the performance of the control algorithms in the presence of payload mass variations within the range between  $\frac{1}{3} m_{tip-d2}$  and  $3 m_{tip-d2}$ . These results are presented in terms of the ratio between the real and the nominal payload mass  $\frac{m_{real}}{m_{nom}}$ .

In the case in which the actual payload mass is equal to the nominal payload mass, the single loop controller (a) had two problems as consequence of the joint friction: a non negligible steady state error in the tip positioning (2.74 mm), and a certain residual vibration (0.56 mm peak-peak), shown in Fig. 5.3. The use of an additional integrative action in the case of the single loop control (b) eliminates the steady state positioning error and improves the residual vibrations. However, it seriously undermines the robustness of the system in the case of payload mass uncertainty. Note the deterioration of the performance in Figs. 5.4 and 5.5. The double loop scheme, meanwhile, shown a zero steady-state with

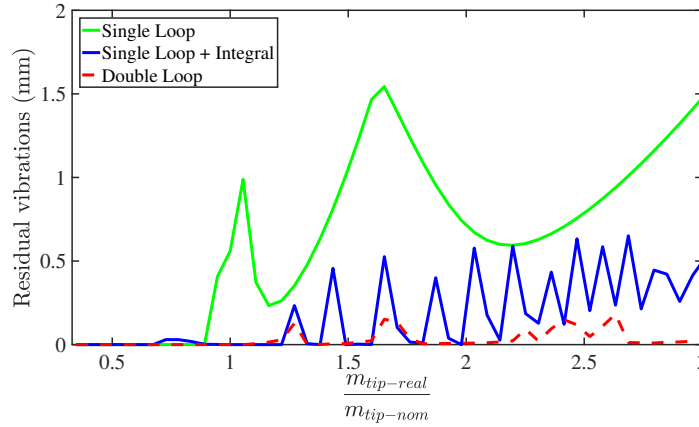


Fig. 5.3 Simulation - duraluminum link: comparison of the residual vibrations.

an insignificant residual vibration without the other disadvantages of the integrative action in the non-nominal case. The characteristics of the performance of the controllers in the non-nominal case will be discussed in the following paragraph.

In the case of non-nominal payload mass, the single loop (a) and the double loop (c) controllers show a similar performance for all the indicators except for the residual vibration, where the double loop (c) proved to be, on average, one order of magnitude below (see Figs. 5.3). Moreover, the single loop with integrative action (b) performed very poorly as regards the maximum overshoot and the settling time (see Figs. 5.4 and 5.5). These results show that it has a strong propensity toward instability when the real payload mass surpasses the nominal value, and is, therefore, a non-viable alternative for a pick and place task. It is true that the controller (b) is capable of correcting the steady state errors in the presence of payload mass uncertainty, however, given the drawbacks, it results in a non-viable alternative to the problem. Chapter 6 proposes an alternative controller that is capable of correct the problems associated with payload mass uncertainty without the necessity of considering an integrative action.

Among these approaches compared, the double loop scheme stood out. It performed similarly to or better than other options in the case of all the indicators. This comparative study underlined its robustness to the static/Coulomb friction effects and payload mass uncertainties. It is important to highlight that, in practice, the static/Coulomb frictions are difficult to model, and, therefore, to pre-compensate, since they might change over time with the operational conditions, as it was stated in [49] and [50].

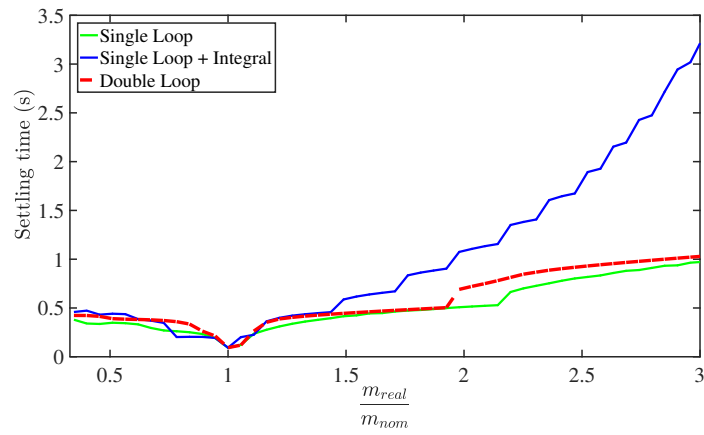


Fig. 5.4 Simulation - duraluminum link: comparison of the settling time.

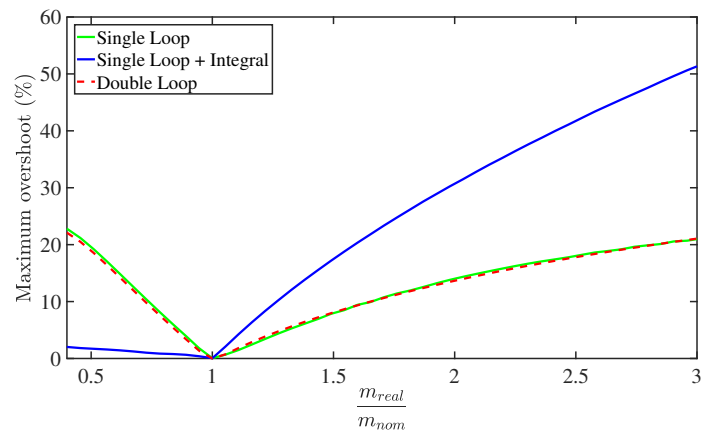


Fig. 5.5 Simulation - duraluminum link: comparison of the maximum overshoot.

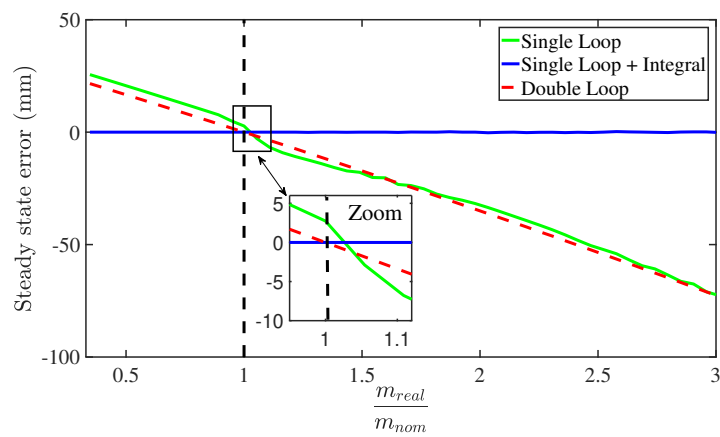


Fig. 5.6 Simulation - duraluminum link: comparison of the steady state error.

### 5.3.2 Validity of the control design method in the case of non-negligible link mass

In this section, we examine how well the proposed controller performs when it is considered on a system on which the mass of the flexible link is very important in comparison with the payload mass. In Section 3.3.2 we showed that the dynamic model on which the controller is based on (modified single lumped mass model), loses accuracy as the link mass to payload ratio increases. In the case of our physical platform with the duraluminum link, we showed that the accuracy provided by the dynamic model is reasonable for the payload mass range considered in our experiments ( $m_{tip-d1}$ ,  $m_{tip-d2}$  and  $m_{tip-d3}$ ), and that it could even be considered on case on which the link mass to payload ratio is even greater. In this section, we evaluate how the losses of accuracy due to changes on the link mass to payload ratio lead to a deterioration in the performance of the proposed controller.

The performance study was based on coosimulations with Matlab/Simulink and MSC Adams and was limited only to the duraluminum link. The control algorithm was simulated in Matlab/Simulink, whereas the dynamics of the link was computed with MSC Adams by considering 100 discrete flexible elements. The simulations were performed considering only the smaller payload mass ( $m_{tip-d1}$ ), as it represents the worst case scenario, i.e., the situation in which the link-mass-to-payload-mass-ratio is the greatest. The variations of the link mass are the same as those considered in the validation of the link mass assumption in Section 3.3.2:  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 1, 2 and 4 times the actual link mass value on our platform. The corresponding link-mass-to-payload-ratios ( $\frac{m_{link}}{m_{tip-d1}}$ ) are 0.14, 0.27, 0.55, 1.08 and 2.16, respectively. The controller was redesigned for each case and the parameters  $EI$ ,  $m_{link-a}$  and  $m_{link-g}$  considered for each link-mass-to-payload-ratio were, therefore, taken from Table 3.5 in Section 3.3.2; the remaining parameters are indicated in Table A.1 in Appendix A. For the purposes of these simulations, in all cases the closed loop poles for the inner loop  $p_i$  and the outer loop  $p_o$  were allocated to the same position:  $p_i = p_o = -30$ . As trajectory reference, we considered a fourth order trajectory that goes from 90 to 135 degrees in 0.4 seconds, a very steep trajectory for the physical platform.

The performance was evaluated in the same terms as in the previous simulations: steady state error, residual vibrations, settling time, and maximum overshoot. The settling time was computed as the time needed to enter and stay within the 1% settling band. The performance indicators are summarized in Table 5.1.

The results show that link mass variations in the range considered have no great effect on the performance and stability of the system. As might be expected, the system performance is diminished as the link mass increases due to the dynamic model accuracy loses, but it only has a minor impact. In fact, in all cases the system remained stable, the overshoot

Table 5.1 Simulation - duraluminum link: performance of the controller associated with different link-mass-to-payload-ratio.

| Link mass<br>to payload<br>mass ratio<br>( $\frac{m_{link}}{m_{tip}}$ ) | Link<br>mass           | Steady<br>state<br>error<br>(mm) | Residual<br>vibration<br>(mm) | Settling<br>time<br>(s) | Maximum<br>overshoot<br>(%) |
|-------------------------------------------------------------------------|------------------------|----------------------------------|-------------------------------|-------------------------|-----------------------------|
| 0.14                                                                    | $\frac{1}{4} m_{link}$ | 0.06                             | 0                             | 0.36                    | 0.23                        |
| 0.27                                                                    | $\frac{1}{2} m_{link}$ | 0.14                             | 0                             | 0.36                    | 0.32                        |
| 0.55                                                                    | $m_{link}$             | 0.27                             | 0                             | 0.36                    | 0.48                        |
| 1.08                                                                    | $2 m_{link}$           | 0.44                             | 0                             | 0.36                    | 0.77                        |
| 2.16                                                                    | $4 m_{link}$           | 0.86                             | 0                             | 0.62                    | 1.19                        |

was below 1.2 %, the residual vibrations stayed at zero, and the settling times were not different, except in the case in which the link mass was  $4 m_{link}$ . In this case, the settling time was almost double that of the other cases. The most significant impact occurred in the deterioration of the steady state error as the link mass increased, which is congruent with the observations made in Section 3.3.2 as regards the deflection errors. In summary, depending on the accuracy requirements, these results indicate that the control algorithm is still applicable for other platforms on which the link mass is significant in relation to the payload mass as long as the other assumptions of the model are met.

## 5.4 Experiments

In this section, we present the experiments carried out to evaluate the nominal performance of the input-state feedback linearization controller proposed in Section 5.2. In order to highlight the benefits of our control algorithm, we compared its performance with a joint position controller, i.e., the two degrees-of-freedom motor position controller presented in Chapter 4.

In these experiments, we considered our duraluminum flexible arm holding the payload mass  $m_{d2}$  and tracking a fourth order trajectory that goes from 110 and 150 degrees in 1 second. According to (5.12), the reference trajectories have to be at least four times differentiable in order to ensure bounded control signals. With regard to the inner and outer loop design, their parameters were chosen in such a way that closed loop poles for the inner loop and the outer loop poles were placed at  $-30$ , in other words,  $p_i = p_o = -30$  in (4.12) and (5.14).

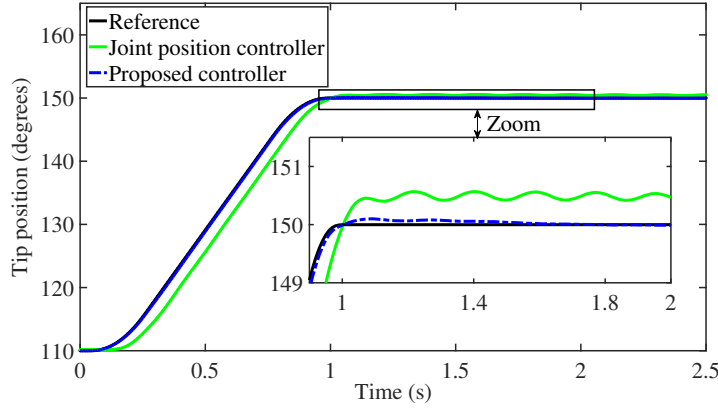


Fig. 5.7 Validation - duraluminum link: comparison of the tip position measurements.

Fig. 5.7 depicts a comparison of the time response of the system to the aforementioned trajectory. The signals shown in this figure were obtained from the external camera system, described in Section 2.2.2. In this figure, it is evident that the proposed controller performed significantly better than the scheme in which only the joint motor position control was considered. The proposed controller achieved a fast and almost oscillation free tip positioning with very little overshoot (0.26 %). In this case, the steady states errors were in the same order of magnitude as the precision of the camera tracking system (error of about 0.5 mm) and could not be computed. By comparison, the joint position controller tracked the trajectory slowly, there was a low-damped residual vibration in the tip positioning (2.3 mm peak-peak) and a significant steady state error (7.79 mm) for the same trajectory. These results can be appreciated more clearly in Fig. 5.8, in which the tip position errors are depicted. This figure also shows the similarity between the tip position measurements, obtained from the external camera system, and the tip position estimates calculated from the motor position (encoder measurements) and the coupling torque measurements (strain gauge measurements), as in (5.15).

It is important to notice that the time responses measured by the external camera system, Fig. 5.7, did not show in any of the cases the characteristic undershoot of the non-minimum phase system. Since the flexible link arms have a non-minimum phase nature, this undershoot definitely exists, but its amplitude might be below the precision of the camera system and, therefore, has an almost negligible amplitude. This observation, in addition to the experimental results previously presented, suggests that the minimum phase approximation of the flexible arm has, in principle, been suitable for our experimental setup.

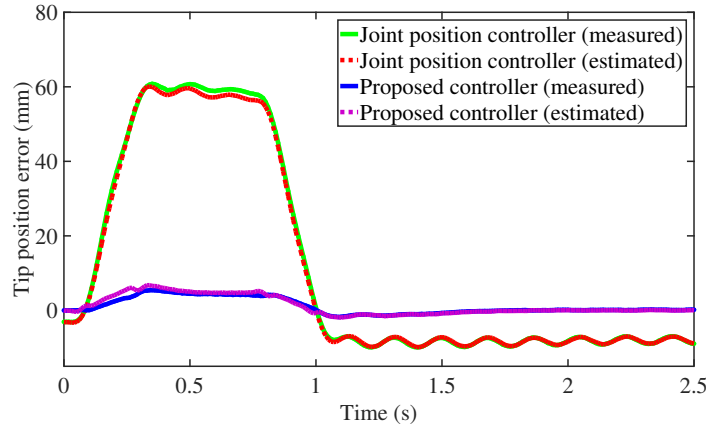


Fig. 5.8 Validation - duraluminum link: comparison of the tip position error.

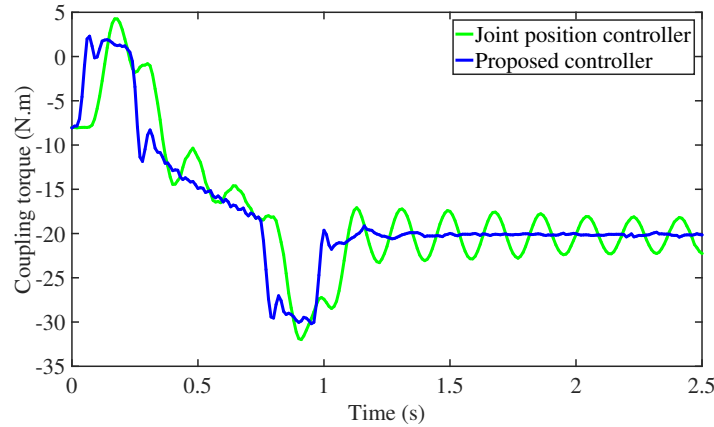


Fig. 5.9 Validation - duraluminum link: comparison of coupling torque.

## 5.5 Concluding remarks

In this chapter, we have developed a controller based on input/state feedback linearization to undertake the trajectory tracking tip position control of a single link flexible arm moving under gravity. The dynamic of the plant was accurately described by a single lumped-mass, which reproduces the first and most significant vibrational mode of the system. In the resulting dynamic model, and unlike other modeling alternatives, there exists no internal dynamics in the relation between the tip position and the motor position, which is, therefore, minimum phase. This is a very convenient characteristic, and a requirement, for the full linearization of the plant achieved by means of the input-state linearization technique.

The general control design followed a two nested loop control scheme. In the inner loop, a two-degree-of-freedom linear controller was considered to achieve a precise tracking of the commanded trajectories for the actuator in presence of the static/Coulomb friction dis-

turbances. In the outer loop, an input-state feedback linearization controller was used to set the desired closed-loop dynamics of the overall system to make the tip position to track fourth order trajectory references, whereas the vibrations of the flexible link are suppressed. The two nested control scheme was considered as an alternative to a model based compensation of the static/Coulomb friction on the actuator. The static/Coulomb friction is difficult to model and normally changes over time depending on the operating conditions of the actuator.

We carried out simulation-based studies to show the superiority of the two-nested loop control approach over the single-loop control approach for the case of a single link flexible arm. In these studies, we found that the single-loop approach was highly sensitive to the existence of static/Coulomb friction, which normally implied steady errors and significant residual vibrations. These problems can be partially overcome by considering an additional integrative control action. However, we have shown that this alternative impairs the steady state response and the stability of the system under conditions of payload mass uncertainty. The two-nested loop control scheme presented in this thesis overcame all these issues achieving a precise trajectory tracking.

# Chapter 6

## Adaptive Position Control of a Single Link Flexible Arm

### 6.1 Introduction

Pick and place is one of the most common tasks performed by robotic manipulators. In the most general case, the payload mass held by the end-effector during this task regularly changes to unknown values. These payload mass changes severely impact on the dynamics of the flexible link manipulators, thus altering the vibrational modes, and also on the bending of the flexible link and, therefore, the tip positioning. In the field of flexible link manipulators, an important challenge is to design control algorithms that are capable of canceling the vibrations and achieving an accurate positioning of the end-effector, even when there is uncertainty in the payload mass. In this chapter, we propose an adaptive controller with which to improve the behavior of our system under conditions of payload mass uncertainty.

Adaptive control is a field that is devoted to the design of controllers capable of modifying their behavior in response to changes in the dynamics of the plant. An adaptive controller consists of a controller with adjustable parameters and an algorithm that is used to adjust those parameters. Depending of the way in which the algorithm adjusts the parameters, it can be classified as *direct* or *indirect*. In the *direct approaches*, also referred to as implicit adaptive control, the controller parameters are computed directly without the explicit computation of the parameters of the plant. In the *indirect approach*, referred to as explicit adaptive control, the parameters of the system are computed explicitly and then used to compute the controller parameters. In the following two paragraphs the most relevant works dealing with payload mass uncertainty in flexible link robots are presented.

There is certain research work in the *direct approach* classification that is worth to mentioning. Meldrum [51] studied the applicability of the Model Reference Adaptive Control (MRAC) to a flexible link arm moving on a horizontal plane. Here, it was pointed out that although the MRAC can often be successfully applied to systems with collocation of actuators and sensors, it is very difficult to control flexible link manipulators on which the actuation is at one end of a flexible element, and the variable to be controlled (or the sensing) is at the other end. This results from the fact that positive realness conditions, which are necessary to ensure the stability of a MRAC controller, cannot be very easily satisfied. Siciliano in [52] applied the MRAC technique to the control of a single link flexible arm moving horizontally. In this case, the goal of using an adaptive control law was to compensate for the errors in the model caused by the linearization of a more complex dynamic model. In order to avoid the problems associated with the positive realness condition, Siciliano selected the motor position as system output. The simulation results showed the superiority of this approach to an alternative pure optimal control in the suppression capabilities. However, it emphasized that these results cannot be easily extended to the control of multiple link arms. In [53], Yuh applied a discretized version of the MRAC to the same problem. This approach is very relevant since, unlike that which occurs in the continuous case, the positive realness is no longer a limiting condition [51] in the discrete time version of the MRAC. The simulation results of Yuh's work showed the feasibility of the discrete time MRAC. However, it also showed that the higher the number of flexible modes considered, the larger the control input demanded, which is an important practical limitation.

In the indirect approach, the same control algorithm applied for the position control of flexible link robots under nominal conditions was applied in combination with an online identification algorithm in order to adjust its parameters. The most commonly used techniques were PD [54], PID[55], LQR[56] and input shaping[57]. The determinant factor as regards the performance of the overall controller was the choice of the algorithm for the parameter estimation of the plant. These algorithms were divided into the frequency domain methods and the time domain method. In the frequency domain, Tzes and Yurkovich in [58] proposed an on-line recursive computation of the frequency spectrum of the flexible link robot using the Time-Varying Transfer Function Estimation (TTFE). This technique is very precise but, given its convergence times, it is only applicable to very large trajectories. With regard to the time domain methods, most of the techniques used correspond to some variant of Least Mean Squares (LMS) algorithm. Rovner and Franklink([59]) and Rovner and Cannon ([56]) considered the standard LMS and the RLS, respectively. Both methods use numerical differentiation and, therefore, especial emphasis was placed on the importance of prefiltering the input signals so as to avoid the amplification of high frequency noise. In

[54], Feliu proposed a very fast and simple estimation method that consisted of integrating the differential equation describing the dynamics of a single link flexible arm with only one natural resonant frequency. Feliu et al. ([34]) generalized the previous work to the case of multiple natural resonant frequencies. This methodology, which is based on the integration of the differential equation, although simple, is valid only when the parameter estimation starts from the arm in a static condition. However, taking into consideration that the dynamics of the flexible link robots is lightly damped, reaching a perfectly static condition after picking up or dropping an object might not be immediate and may delay the execution of the following trajectory. Yang et al.([55]) pointed out the existence of residual vibrations in flexible link robots in pick and place tasks, especially after dropping a load. Becedas et al. ([60]) proposed a very fast identification method based on Algebraic identification. This method is also based on the integration of the differential equation describing the dynamics, but it handles the problem of the initial conditions effectively, i.e., the robot is not required to start from a static position.

Among the online identification algorithms previously mentioned, the least well known is the Algebraic Identification algorithm and, therefore, deserves a special mention. The Algebraic identification is a methodology proposed by Fliess and Sira-Ramírez in [61] for the online estimation of parameters, which represents a competitive alternative to more classical methods such as Least Mean Squares and Maximum Likelihood. This methodology is fundamentally different from other approaches in some basic respects: (a) it does not require any statistical knowledge of the noise corrupting the signals and the classical Gaussian noise assumptions are not, therefore, necessary; (b) it does not need to compute iterative time derivatives of noise corrupted signals; (c) it is not an asymptotic approach; (d) it is not necessary to persistently excite inputs in order to make the system identifiable; (e) it can be very easily designed to be robust to a large variety of additive perturbations [62].

In this chapter, we propose an adaptive controller with which to deal with the payload mass changes in a single-link flexible manipulator. More specifically, we consider an indirect approach in which we first estimate the payload mass of the manipulator with an Algebraic identifier, and we then use this information to update a controller designed using input-state feedback linearization, similar to that proposed in Chapter 5. This work takes as its starting point the adaptive control presented by Becedas et al. in [60], which was applied to the control of a single link flexible robot and also considered the Algebraic Identification framework. There are significant differences between the approach presented here and the work carried out by Becedas. Becedas's work has the following characteristics: (a) the flexible link model contemplated was linear, which was only possible owing to the fact that its movements were constrained to a horizontal plane and the validity of the assumption of

small deflections; (b) the control technique considered corresponds to a Generalized Proportional Integral controller that is, in principle, only applicable to linear models; (c) the parameter estimated was the first vibrational frequency of the link; (d) control law updates took place only once the Algebraic Identifier had completely converged. The adaptive controller presented in this chapter differs from the previous work in the following aspects: (a) the flexible link arm movements are constrained to a vertical plane and in the presence of the force of gravity, and it is, therefore, necessarily represented by a non-linear dynamic model; (b) the control technique considered is an input-state feedback linearization, which is a technique for the control of non-linear systems; (c) the parameter to estimate was a payload mass and (d) the control law is continuously updated from the moment that the Algebraic Identifier reaches a rough estimate of the parameters, which is normally a better estimate than the nominal parameters, and stops when the Algebraic Identification fully converges.

The organization of this chapter is as follows. Section 6.2 introduces the design of the Algebraic Identification algorithm for the online payload mass identification. Section 6.3 presents the experimental results of the online estimation algorithm and a comparison with the Least Mean Squares technique. while Section 6.4 is devoted to the design of the adaptive control law. In Section 6.5, the local stability of the controlled system is studied and finally, in Section 6.6, the experimental validation of the adaptive controller is presented.

The results presented in this chapter have been partly published in

- J. C. Cambera, A. San-Millan, and V. Feliu-Battle, “Payload mass identification of a single-link flexible arm moving under gravity: an algebraic identification approach,” *Shock and Vibration*, vol. 2015, Hindawi Publishing Corporation, 2015
- A. S.-M. Rodriguez, J. C. Cambera, and V. Feliu-Battle, “Online algebraic identification of the payload changes in a single-link flexible manipulator moving under gravity,” in *International Federation of Automatic Control (IFAC) World Congress 2014*, vol. 47, (Cape Town, South Africa), pp. 8397–8402, Elsevier, 2014

## 6.2 Algebraic identification algorithm

In this section, we apply the algebraic methodology, proposed by Fliess and Sira-Ramírez in [61], to design an estimation algorithm to perform an online estimation of the payload mass of a flexible link robot moving under gravity. The methodology assumes that the parameters to be estimated are constant or piecewise constant throughout time, as in the case of payload mass changes during a pick and place task. For the purposes of clarity, the design methodology is described in three stages: (Stage I) the algebraic reformulation of

the problem; (Stage II) the parameter calculation; (Stage III) the resetting and switching-off considerations. Before discussing these stages, let us to establish the dynamic model considered in the subsequent sections.

In this chapter, we have complemented the original dynamic model of the flexible link subsystem, expressed by equations (3.26) and (3.27), to include a linear model of damping in the link. Specifically, we have introduced the term  $(\mu_c \dot{\theta}_t)$  into equation (3.26), where  $\mu_c$  represents the damping coefficient. The resulting dynamic model is given by

$$\begin{aligned}\Gamma_{coup} &= (m_{tip} + m_{link-a}) l^2 \ddot{\theta}_t + (m_{tip} + m_{link-g}) g l \cos(\theta_t) + \mu_c \dot{\theta}_t \\ &= (m_a) l^2 \ddot{\theta}_t + (m_g) g l \cos(\theta_t) + \mu_c \dot{\theta}_t.\end{aligned}\quad (6.1)$$

$$\Gamma_{coup} = \frac{3 E I_z}{l} (\theta_m - \theta_t). \quad (6.2)$$

Our experiments enabled us to observe that considering an approximate linear model of the damping improves the performance of the Algebraic Identifier without adding substantial complexity to the identification algorithm. The dynamic model presented above is assumed to be valid in the small interval of time in which the identification takes place. The addition of more accurate damping models was avoided, since these typically rely on complex non-linear formulations (see [65, 66]), and would make the online identification unnecessarily complicated. Although the damping was considered in the online identification algorithm, it was ignored in the dynamics considered in the control algorithm for the following reasons: (a) the linear model of the damping does not describe the phenomenon in a global manner; (b) the flexible link arm is very lightly damped and, therefore, the contribution of the damping to the dynamics is very small; this is confirmed by observing the open loop response of the system in Figs 3.6 and 3.7; (c) the controller suppresses the vibrations with a dynamics that is much faster than the damping; (d) the damping does not constitute a threat to the stability of the closed loop and, on the contrary, makes the system more stable and supports the main goal of the controller, which is the suppression of the vibrations on the link.

The Algebraic Identification algorithm identifies the parameters  $m_{tip}$  and  $\mu_c$  from equation 6.1. In order to do this, the coupling torque  $\Gamma_{coup}$  and the angular tip position  $\theta_t$  need to be known. The sensory system of our robot provides the angle of the motor ( $\theta_m$ ) and the coupling torque ( $\Gamma_{coup}$ ) at the base of the link. The angular tip position ( $\theta_t$ ) is not measured but can be estimated from equation (6.1) as follows

$$\hat{\theta}_t = \theta_m - \Gamma_{coup} \frac{l}{3 E I_z}. \quad (6.3)$$

Here, it is important to note that the estimation of the tip position,  $\hat{\theta}_t$ , is completely independent from the payload mass and damping coefficient, whose values are, in principle, unknown.

### 6.2.1 First stage: algebraic reformulation of the problem

Performing an online parameter estimation from equation (6.1) implies, for some identification techniques, numerical differentiations of a noise corrupted signal  $\hat{\theta}_t$ . As is well known, these operations amplify the noise in the resulting signals, and have an adverse affect on the performance of the identification algorithms. The Algebraic Identification provides a methodology with which to avoid this issue. In this section, we formulate a new mathematical relationship between the signals measured and the parameters of the model by applying standard operational calculus and algebraic operations. This new expression is, when compared with the original model, purposefully formulated to be independent of time derivatives and initial conditions. The procedure is as follows.

First, let us apply the Laplace transform to equation (6.1)

$$\Gamma_{coup}(s) = m_a l^2 [s^2 \theta_t(s) - s\theta_t(t_0) - \dot{\theta}_t(t_0)] + \mu_c [s\theta_t(s) - \theta_t(t_0)] + m_g g l \chi(s), \quad (6.4)$$

where  $\theta_t(t_0)$  and  $\dot{\theta}_t(t_0)$  represent unknown initial condition and  $\chi(s)$  is the Laplace transform of  $\cos(\theta_t(t))$ . Upon taking two times derivatives with respect to  $s$ , the initial conditions are eliminated

$$\frac{d^2 \Gamma_{coup}(s)}{ds^2} = m_a l^2 \left[ 2\theta_t(s) + 4s \frac{d\theta_t(s)}{ds} + s^2 \frac{d^2 \theta_t(s)}{ds^2} \right] + \mu_c \left[ 2 \frac{d\theta_t(s)}{ds} + s \frac{d^2 \theta_t(s)}{ds^2} \right] + m_g g l \frac{d^2 \chi(s)}{ds^2}. \quad (6.5)$$

In order to eliminate the derivatives in the time domain (positive power of  $s$ ), we multiply both sides by  $s^{-2}$  to obtain

$$\begin{aligned}
s^{-2} \frac{d^2 \Gamma_{coup}(s)}{ds^2} = & m_a l^2 \left[ 2s^{-2} \theta_t(s) + 4s^{-1} \frac{d\theta_t(s)}{ds} + \frac{d^2 \theta_t(s)}{ds^2} \right] + \\
& \mu_c \left[ 2s^{-2} \frac{d\theta_t(s)}{ds} + s^{-1} \frac{d^2 \theta_t(s)}{ds^2} \right] + \\
& m_g g l s^{-2} \frac{d^2 \chi(s)}{ds^2}.
\end{aligned} \tag{6.6}$$

Equation (6.6) can be written in the time domain by applying the inverse Laplace transform<sup>1</sup>. The resulting equation is given by<sup>2</sup>

$$\begin{aligned}
\int^{(2)} t^2 \Gamma_{coup}(t) = & m_a l^2 \left[ 2 \int^{(2)} \theta_t(t) - 4 \int t \theta_t(t) + t^2 \theta_t(t) \right] \\
& \mu_c \left[ -2 \int^{(2)} t \theta_t(t) + \int t^2 \theta_t(t) \right] + \\
& m_g g l \int^{(2)} t^2 \cos(\theta_t(t)),
\end{aligned} \tag{6.7}$$

and, in a more compact form, as

$$q(t) = m_a l^2 \beta(t) + m_g g l \xi(t) + \mu_c \eta(t). \tag{6.8}$$

Functions  $q(t)$ ,  $\beta(t)$ ,  $\xi(t)$ , and  $\eta(t)$  can be expressed as the outputs of the following time-varying linear and unstable filters in a perturbed Brunovsky's canonical form.

$$\begin{aligned}
q(t) &= z_1, & \xi(t) &= z_5, \\
\dot{z}_1 &= z_2, & \dot{z}_5 &= z_6, \\
\dot{z}_2 &= t^2 \Gamma_{coup}(t), & \dot{z}_6 &= t^2 \cos(\theta_t(t)), \\
\beta(t) &= z_3 + t^2 \theta_t(t), & \eta(t) &= z_7, \\
\dot{z}_3 &= z_4 - 4t \theta_t(t), & \dot{z}_7 &= z_8 + t^2 \theta_t(t), \\
\dot{z}_4 &= 2\theta_t(t), & \dot{z}_8 &= -2t \theta_t(t).
\end{aligned} \tag{6.9}$$

<sup>1</sup>There are a few properties that are especially useful for this problem. If we denote the inverse Laplace transform as  $\mathcal{L}^{-1}$ , then it is well known that  $\mathcal{L}^{-1}s(\cdot) = d/dt(\cdot)$ ,  $\mathcal{L}^{-1}1/s(\cdot) = \int_0^t (\cdot)(\sigma) d\sigma$ , and  $\mathcal{L}^{-1}d^v/ds^v(\cdot) = (-1)^v t^v(\cdot)$

<sup>2</sup>We use  $(\int^{(j)} \phi(t))$  to denote the integral expression  $\int_0^t \int_0^{\sigma_1} \dots \int_0^{\sigma_{j-1}} \phi(\sigma_j) d\sigma_j \dots d\sigma_1$  with the definition of  $(\int \phi(t))$  as  $(\int_0^{(1)} \phi(t)) = \int_0^t \phi(\sigma_1) d\sigma_1$

The implementation of this algorithm requires the following inputs: the coupling torque measurements ( $\Gamma_{coup}$ ) and the estimation of the tip position ( $\hat{\theta}_t$ ), presented the equation (3.29). Note that, even when the original expression (6.1) and the resulting equation (6.8) have the same form, the latter does not depend on time numerical differentiations.

### 6.2.2 Second stage: parameter calculation

In order to identify the parameters of a system, we need at least the same number of linearly independent equations as unknown parameters. In Algebraic Identification, the definition of these equations must be defined from the equation resulting from the algebraic reformulation of the problem, in our case from equation (6.8). In this context, several approaches with which not only define these equations, but also enhance the performance of the identifier in the presence of noise in the measurements have been proposed. In the following paragraphs, we first briefly describe the most relevant approaches, and then detail the procedure followed in this chapter. The approaches we mention are not necessary related to our specific application, but they could easily be adapted to our problem.

In [67], the equation system employed to determine the parameters consists of: (a) the equation resulting from the algebraic reformulation of the problem evaluated at the present time, in our case, equation (6.8), and (b) some additional equations defined by means of multiple definite integration of both sides of the same expression. The number of definite integrals depends, in general, on how many parameters need to be identified. Once this equation system has been solved, and in those cases in which high frequency noise is present in the measurements, a low pass filtering can be used to improve the signal to noise ratio of the parameter estimation.

Depending on the nature of the measurements considered, the additional equations, defined in the approach described above, might not be completely linearly independent all the time. When this happens, a local loss of the identifiability may occur. In [68], this drawback is solved by using invariant non-linear filtering.

Another approach with which to define an equation system, that allows us to compute the parameters, is to use the resulting equation of algebraic reformulation of the problem and to evaluate it at different times, in our case, equation (6.8). With this approach, an overdetermined system equation is normally defined by taking a large number of points in time. This action allows us to achieve the identification of the parameters, whereas the noise present in the measurements is properly dealt with. The greater the number of points in time considered, the greater the low pass effect on the measurements. The overdetermined equation system is normally solved by considering a least square fitting based method. In [69], the least square method in connection with Algebraic Identification was proposed to iden-

tify the parameters for induction motors. In order to increase the computational efficiency of this stage, in [70] the standard recursive formula for least square algorithm was used to identify the two main vibration modes of a flexible structure. In [71], the adjustment of the overdetermined system was carried out by using recursive least-squares. This identifier was designed in order to determine the parameter of a servo model. In this chapter, we use the standard recursive formula for least-squares algorithm. A detailed description is presented below.

The goal of the following steps is to determine the unknown parameters  $m$  and  $\mu_c$  from equation (6.8). However, at the end of this section we shall focus our attention only on the value of the parameter  $m_{tip}$ , while the value of the parameter  $\mu_c$  will be discarded. The parameter  $\mu_c$ , associated with viscous friction, was introduced in the model only to improve the behavior of the estimator. According to our observations, an approximate model of the damping enhances the performance in terms of precision. Having made this clarification, we define the following cost function from equation (6.8)

$$\varepsilon(t) = \int_0^t \left\{ \begin{bmatrix} l^2 \beta(\tau) + gl\xi(\tau) & \eta(\tau) \end{bmatrix} \cdot \begin{bmatrix} m_{tip} \\ \mu_c \end{bmatrix} - \check{q}(\tau) \right\}^2 d\tau, \quad (6.10)$$

where  $\check{q}(\tau) = q(\tau) - m_{link-a} l^2 \beta(\tau) - m_{link-g} l \xi(\tau)$ . Note that in these formulas the contribution of the tip mass and the links mass to the dynamics were considered separately, which means that  $m_a = m_{tip} + m_{link-a}$  and  $m_g = m_{tip} + m_{link-g}$ . The minimization of the cost function leads to

$$\begin{bmatrix} \hat{m}_{tip}(t) \\ \hat{\mu}_c(t) \end{bmatrix} = \left[ \int_0^t \begin{bmatrix} l^2 \beta(\tau) + gl\xi(\tau) \\ \eta(\tau) \end{bmatrix} \begin{bmatrix} l^2 \beta(\tau) + gl\xi(\tau) \\ \eta(\tau) \end{bmatrix}^T d\tau \right]^{-1} \cdot \int_0^t \begin{bmatrix} l^2 \beta(\tau) + gl\xi(\tau) \\ \eta(\tau) \end{bmatrix} \check{q}(\tau) d\tau. \quad (6.11)$$

This can be efficiently implemented in a recursive manner as follows. Let us to rewrite equation (6.11) as

$$\begin{bmatrix} \hat{m}_{tip}(t) \\ \hat{\mu}_c(t) \end{bmatrix} = \mathbf{A}_{alg}^{-1}(t) \mathbf{B}_{alg}(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix}^{-1} \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} \quad (6.12)$$

where

$$\begin{aligned}
a_{11}(t) &= \int_0^t (l^2 \beta(\tau) + gl \xi(\tau))^2 d(\tau) \\
a_{12}(t), a_{21}(t) &= \int_0^t (l^2 \beta(\tau) + gl \xi(\tau)) (\eta(\tau)) d(\tau) \\
a_{22}(t) &= \int_0^t (\eta(\tau)^2) d(\tau) \\
b_1(t) &= \int_0^t (l^2 \beta(\tau) + gl \xi(\tau)) q(\tau) d(\tau) \\
b_2(t) &= \int_0^t \eta(\tau) q(\tau) d(\tau)
\end{aligned} \tag{6.13}$$

If the functions  $\beta(t)$ ,  $\xi(t)$ ,  $\eta(t)$  and  $q(t)$  are sampled with a sampling time denoted by  $t_s$  and the function  $\phi_{\text{alg}}^T[k] = \begin{bmatrix} (l^2 \beta[k] + gl \xi[k]) & \eta[k] \end{bmatrix}$  is defined, then the matrices  $A$  and  $B$  can be computed recursively in discrete time as follows:

$$\begin{aligned}
\mathbf{A}_{\text{alg}}[k] &= \mathbf{A}_{\text{alg}}[k-1] + \phi_{\text{alg}}[k] \phi_{\text{alg}}^T[k] \\
\mathbf{B}_{\text{alg}}[k] &= \mathbf{B}_{\text{alg}}[k-1] + \phi_{\text{alg}}[k] \check{q}[k] \\
\mathbf{A}_{\text{alg}}[0] &= \mathbf{0}_{[2 \times 2]} \\
\mathbf{B}_{\text{alg}}[0] &= \mathbf{0}_{[2 \times 1]}
\end{aligned} \tag{6.14}$$

The parameters  $m_{tip}$  and  $\mu_c$  are computed from these matrices as

$$\begin{bmatrix} m_{tip}[k] \\ \mu_c[k] \end{bmatrix} = \mathbf{A}_{\text{alg}}^{-1}[k] \mathbf{B}_{\text{alg}}[k] \tag{6.15}$$

### 6.2.3 Third stage: resetting and switching-off considerations

In the algorithm presented above, we considered that the uncertain parameters remain constant throughout time. However, this condition is not sufficiently broad to solve the payload identification problem of the typical pick and place tasks. In this application, the payload mass value changes abruptly to a new value as the manipulator takes or drops an object. In order to continuously detect payload changes, we require the algorithm to be reinitiated once it has achieved an estimate of the current parameters. But the instant at which the parameter is accurately computed is, in principle, unknown and we, therefore, require a method with which to determine it.

We detect when the identification algorithm has converged by considering the strategy proposed in [72]. In this strategy, the convergence time is expressed in terms of the moving

average and the moving standard deviation of the parameter to identify. In our case, if we define  $m[k]$  as the payload estimate for the sample  $k$ , its moving average ( $E[m_{tip}[k]]$ ) and its moving standard deviation ( $\sigma[m_{tip}[k]]$ ) are defined as follows:

$$E[m_{tip}[k]] = \frac{1}{M} \sum_{i=0}^{M-1} m_{tip}[k-i] \quad (6.16)$$

$$\sigma[m_{tip}[k]] = \sqrt{\sum_{i=0}^{M-1} \frac{(m_{tip}[k-i] - E[m_{tip}[k]])^2}{M}} \quad (6.17)$$

where the length of the window ( $M$ ), defines the set of samples considered in the calculations. The payload estimated,  $m_{tip}[k]$ , is said to have converged when the following criterion is fulfilled:

$$\frac{\sigma[m_{tip}[k]]}{E[m_{tip}[k]]} \leq \Delta \quad (6.18)$$

The constant  $\Delta$  is the tolerance parameter, and should be set up according the application's requirements, taking into account that there is a trade-off between speed and precision. If the application needs high precision, a small  $\Delta$  should be provided. On the other hand, if the application requires a fast estimation, a greater  $\Delta$  should be considered.

There are some other alternatives to the procedure presented above. In [68], the convergence time was computed in terms of an artificial parameter, denominated as the sentinel parameter. This parameter emulates, in terms of time of convergence, the behavior of the remaining system parameters, but unlike the others, its value converges to a known arbitrary value explicitly defined beforehand.

On a final note, we would like to point out that for those applications in which the parameters never change, and only a unique estimate of the parameters is necessary, it is advisable to switch off the algorithm once it has converged. The unstable nature of the filters involved in the identification may produce an arithmetic overflow when a long time has passed. Given the fast nature of the Algebraic Identification, the algorithm is expected to be reset long before any numerical problems might arise.

## 6.3 Experimental validation of the Algebraic Identifier

In this section, the Algebraic Identification algorithm is experimentally validated and compared with a more standard alternative for the payload mass identification: a recursive implementation of the Least Mean Squares algorithm. This validation covers the duraluminum

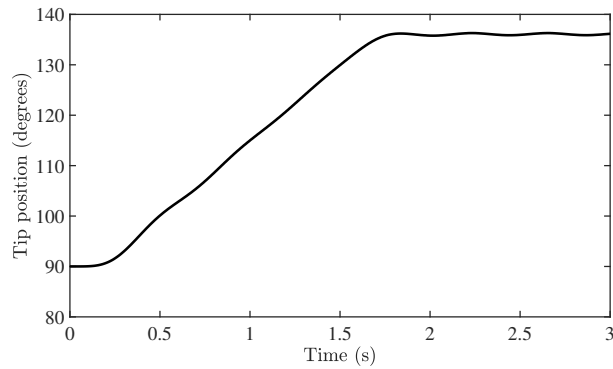
and composites link setup and considers their corresponding payload masses defined in Tables A.1 and A.2. The signals measured used in the identification algorithms are the coupling torque ( $\Gamma_{coup}$ ) and the motor position ( $\theta_m$ ). These signals are noisy and, as might be expected, their time numerical derivatives are even noisier and should not be used in the identification algorithm. To illustrate this, Figure 6.1 shows the signals relevant for the identification algorithms for an arbitrary experiment with the composite link : (a) the estimation of the tip position computed as was shown in equation (6.3); (b) and (c) the first and second order time numerical differentiation of the estimation of the tip position, which depend on the time numerical differentiation of the motor position and the coupling torque; (d) the coupling torque. The Algebraic Identifier considers only the estimation of the tip position and the coupling torque as input signals, whereas the Least Mean Squares algorithm requires all of them.

### 6.3.1 Experiments definition

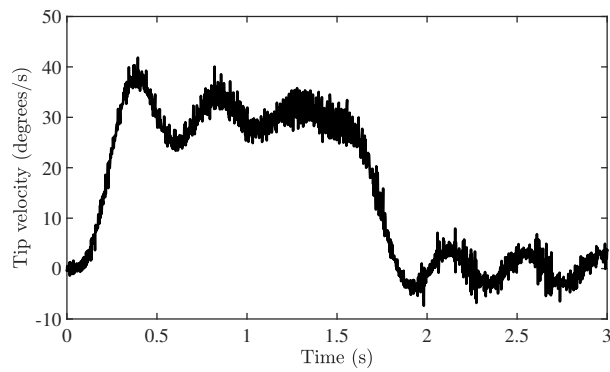
The Algebraic Identifier was validated for the typical operations of a robotic manipulator that is involved in a pick and place task. In the validation experiments, the motor position was controlled by using the two-degrees-of-freedom controller presented in Chapter 4, whereas no outer loop controller was considered to cancel vibrations. It is noteworthy that the Algebraic Identifier presented does not depend on the control algorithm used for the motor position. In fact, it considers only the dynamics of the flexible link. The experiments considered are the following

**Experiment type 1:** In these experiments, the motor position ( $\theta_m$ ) follows a fourth order trajectory that goes from 90 degrees, the vertical pose and 135 degrees in 2 seconds. This experiment represents a pick and place task, in which the robot takes from one position to another following a defined trajectory. This trajectory represents the worst case scenario in which, despite starting from a resting state, the gravity component of the dynamics is zero and is not, therefore, useful to estimate the payload mass.

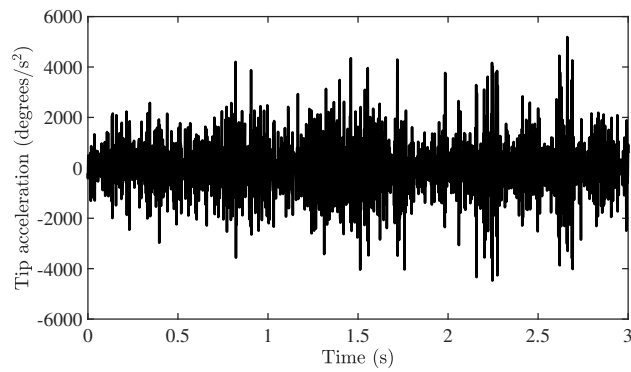
**Experiment type 2:** The motor position ( $\theta_m$ ) tracks a 1 Hz sinusoidal trajectory. In the case of the duraluminum link, the trajectory amplitude is of 10 degrees peak-to-peak, whereas in the case of the composite link it is of 2 degrees of amplitude. This experiment is used to prove the stability of the algorithm when a persistent input is considered, in addition to validating the capability of the algorithm to effectively estimate the payload mass without the need to reach a resting state, which might be the case when a payload mass is released or grasped in the middle of a trajectory. The



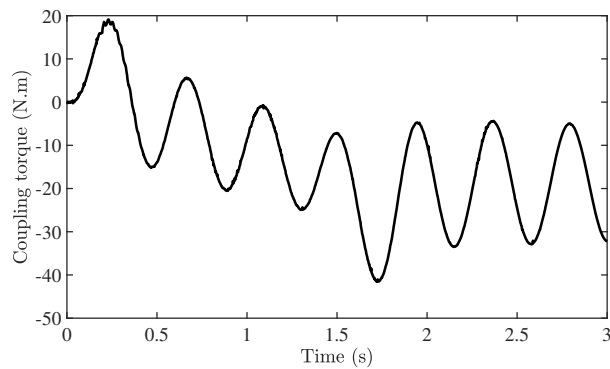
(a) Angular position of the tip.



(b) Angular velocity of the tip.



(c) Angular acceleration of the tip.



(d) Coupling torque.

Fig. 6.1 Online identification - composite link: input signals to be considered in the identification algorithms.

amplitudes of the trajectories considered were selected in order to avoid the saturation of the motor control signal.

**Experiment type 3:** The motor is blocked at  $\theta_m = 45$  degrees and the link is gently hit on its tip. This experiment validates the capability of the Algebraic Identifier to estimate the payload mass when the motor position is static but there are some vibrations present along the link. In flexible link robots, some vibrations might appear immediately after the action of picking up or placing objects, when the motor can be considered approximately static. In [55], Yang et al. called this effect the "ring effect" and noted that it is especially important after the load has been released.

Before proceeding with the following section, we depict the motor position ( $\theta_m$ ) and coupling torque ( $\Gamma_{coup}$ ) measurements for each experiment type, considering the composite link and the payload mass  $m_{tip-c12}$  (Figures 6.2-6.4). The plots corresponding to the other payload masses and the duraluminum link are similar and do not add any value to the discussion.

### 6.3.2 Least Mean Squares algorithm

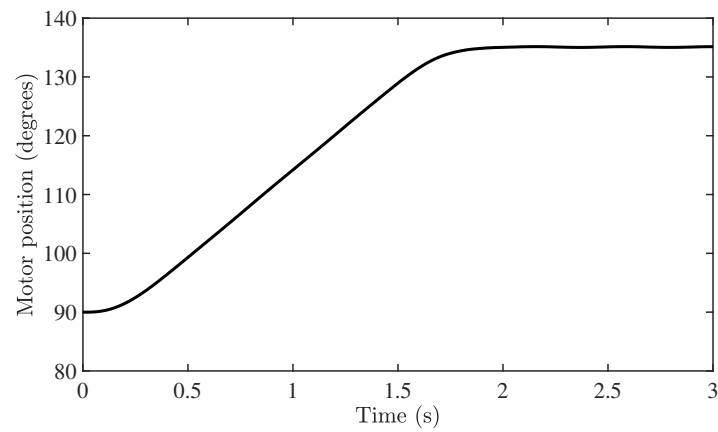
The Algebraic Identifier developed in this chapter is compared with a recursive implementation of the Least Mean Squares algorithm. The implementation presented here is similar to that used in the second stage of the Algebraic Identifier (Section 6.2.2). In this case, the algorithm requires the first and second order time numerical differentiation of the tip position, which are very noisy and consequently damage the accuracy and speed of convergence of the estimation. A filtered version of the Least Mean Squares algorithm improves its performance, as was indicated by Rovner in [56] and [59], and is, therefore, considered in this section. The procedure is as follows.

First, let us to consider equation (6.1) filtered at both sides by a second-order low pass Butterworth filter represented by the operator  $F_{lp}[]$

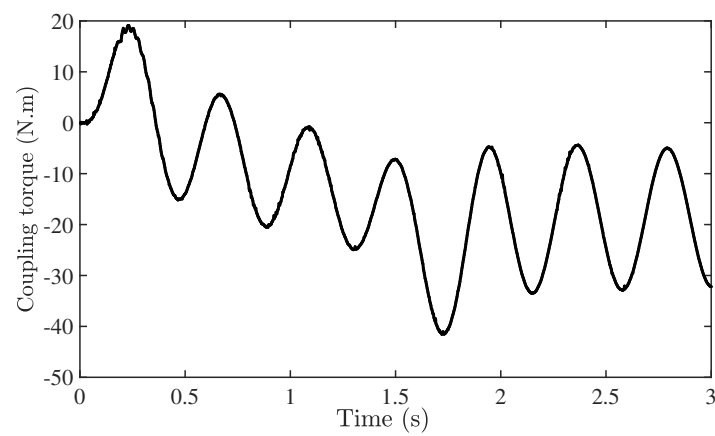
$$F_{lp}[\Gamma_{coup}] = F_{lp}[(m_a) l^2 \ddot{\theta}_t](t) + F_{lp}[(m_g) g l \cos(\theta_t(t))] + F_{lp}[\mu_c \dot{\theta}_t(t)] \quad (6.19)$$

If  $\Gamma_{coup}[k]$ ,  $\theta[k]$ ,  $\dot{\theta}[k]$ ,  $\ddot{\theta}[k]$  represent the signals  $\Gamma_{coup}(t)$ ,  $\theta(t)$ ,  $\dot{\theta}(t)$ ,  $\ddot{\theta}(t)$  sampled every  $t_s$  seconds, and taking into consideration that  $m_a = m_{tip} + m_{link-a}$  and  $m_g = m_{tip} + m_{link-g}$ , the following functions are defined

$$\phi_{lms}^T[k] = [F_{lp}[l^2 \ddot{\theta}[k] + g l \cos(\ddot{\theta}[k])], F_{lp}[\dot{\theta}[k]]], \quad (6.20)$$

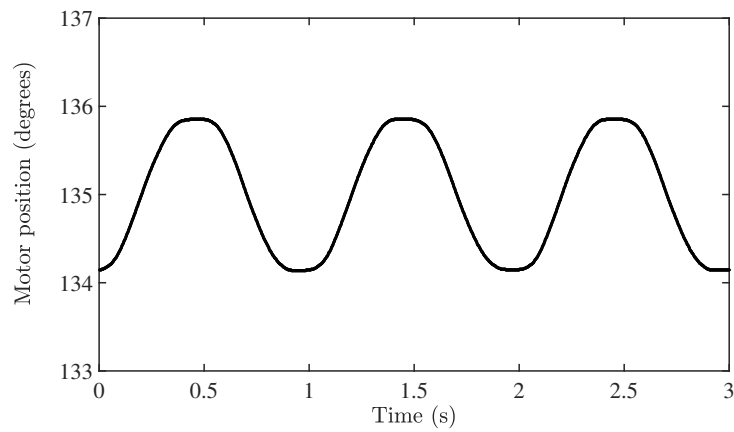


(a) Angular position of the motor.

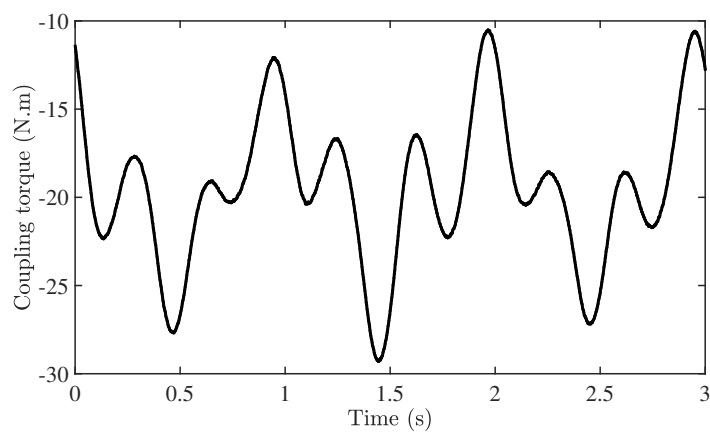


(b) Coupling torque.

Fig. 6.2 Online identification - composite link: experiment type 1 - measurements for the payload mass  $m_{tip-c12}$ .

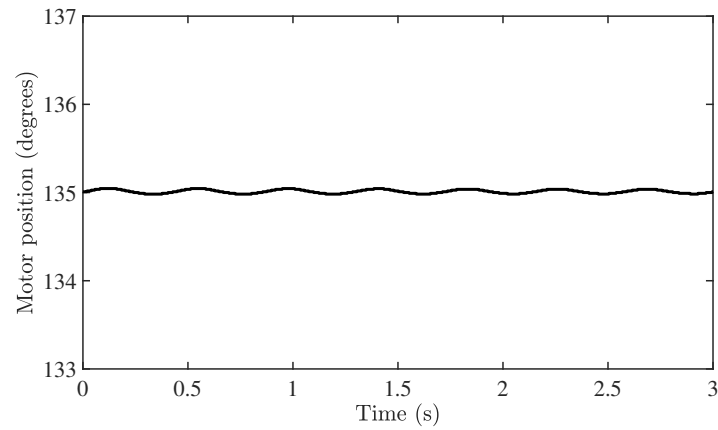


(a) Angular position of the motor.

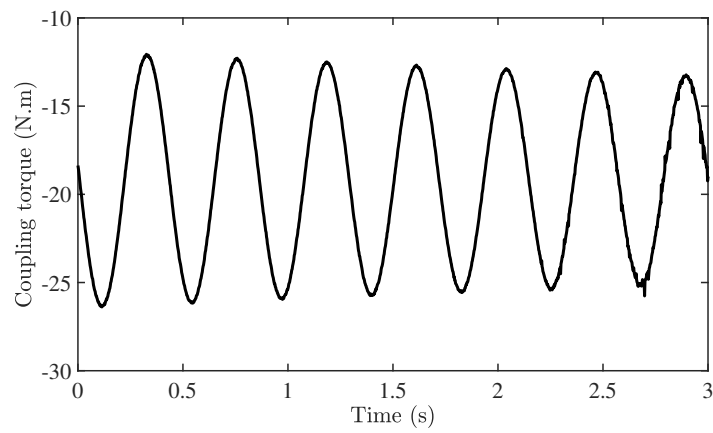


(b) Coupling torque.

Fig. 6.3 Online identification - composite link: experiment type 2 - measurements for the payload mass  $m_{tip-c12}$ .



(a) Angular position of the motor.



(b) Coupling torque.

Fig. 6.4 Online identification - composites link: experiment type 3- measurements for the payload mass  $m_{tip-c21}$ .

$$\ddot{q}_{lms}[k] = F_{lp}[\Gamma_{coup}[k] - m_{link-a} l^2 \ddot{\theta}_t[k] - m_{link-g} g l \cos(\theta_t[k])]. \quad (6.21)$$

The parameters  $m_{tip}$  and  $\mu_c$  can be estimated in terms of these functions as

$$\begin{bmatrix} \hat{m}_{tip}[k] \\ \hat{\mu}_c[k] \end{bmatrix} = \mathbf{A}_{lms}^{-1}[k] \mathbf{B}_{lms}[k] \quad (6.22)$$

where

$$\begin{aligned} \mathbf{A}_{lms}[k] &= \mathbf{A}_{lms}[k-1] + \phi_{lms}[k] \phi_{lms}^T[k] \\ \mathbf{B}_{lms}[k] &= \mathbf{B}_{lms}[k-1] + \phi_{lms}[k] \ddot{q}_{lms}[k] \\ \mathbf{A}_{lms}[0] &= \mathbf{0}_{[2 \times 2]} \\ \mathbf{B}_{lms}[0] &= \mathbf{0}_{[2 \times 1]} \end{aligned} \quad (6.23)$$

In order to stretch the performance of the Least Mean Squares algorithm to the limit, it is necessary to optimize the cut-off frequency of the filter considered. The procedures followed in our case consisted of finding the cut-off frequency that reduced the time it takes the algorithm to reach the 5% band error in the most challenging experiment: the trajectory tracking task (Experiment type 1). In particular, we limit this procedure to the case of the intermediate payload mass, i.e., the mass  $m_{tip-d2} = 2.16$  Kg in the case of the duraluminum link and the mass  $m_{tip-c12} = 1.20$  Kg in the case of the composites link. The optimum cut-off frequencies were 7.67 Hz for the duraluminum link and 4.90 Hz for the composite link, as can be seen in Figs 6.5 and 6.6. The curves in these figures suggest that adjusting the cut-off frequency of the filter is a trade-off between filtering the noise of the sensors and retaining the right amount of information regarding the system, which is essentially concentrated in the low frequencies.

### 6.3.3 Duraluminum link results

Tables 6.1 and 6.2 summarize the performance of the Algebraic Identification algorithm and the Least Mean Squares algorithm in terms of convergence times. In these tables, we present the convergence times for the three types of experiments presented in Section 6.3.1 and considering the time it takes the algorithm to reach and stay in the bands of 10%, 5% and 2% of the real payload mass value.

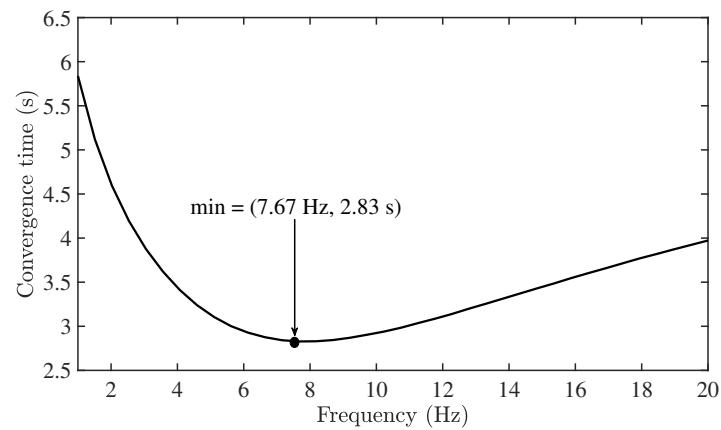


Fig. 6.5 Online identification - duraluminum link: convergence times vs. cut-off frequency of the low pass filter for the least mean squares algorithm.

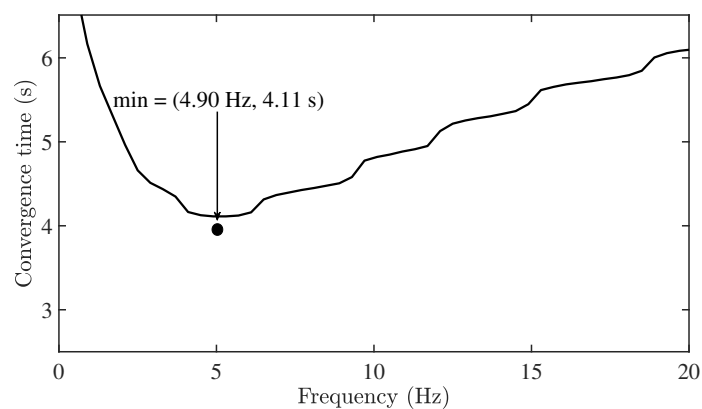


Fig. 6.6 Online identification - composites link: convergence times vs. cut-off frequency of the low pass filter - for the least mean squares algorithm.

Table 6.1 Online algebraic identification - duraluminum link: convergence times for the different experiments and payloads.

| Errors               | Experiment type 1 |        |        | Experiment type 2 |        |        | Experiment type 3 |        |        |
|----------------------|-------------------|--------|--------|-------------------|--------|--------|-------------------|--------|--------|
|                      | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     |
| Payload $m_{tip-d1}$ | 0.20 s            | 0.45 s | 0.86 s | 0.20 s            | 0.27 s | 0.41 s | 0.12 s            | 0.13 s | 0.18 s |
| Payload $m_{tip-d2}$ | 0.18 s            | 0.23 s | 0.63 s | 0.22 s            | 0.23 s | 0.24 s | 0.12 s            | 0.19 s | 0.25 s |
| Payload $m_{tip-d3}$ | 0.31 s            | 0.33 s | 0.36 s | 0.12 s            | 0.27 s | 0.32 s | 0.09 s            | 0.10 s | 0.26 s |

Table 6.2 Online least mean squares identification - duraluminum link: convergence times for the different experiments and payloads.

| Errors               | Experiment type 1 |        |        | Experiment type 2 |        |        | Experiment type 3 |        |        |
|----------------------|-------------------|--------|--------|-------------------|--------|--------|-------------------|--------|--------|
|                      | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     |
| Payload $m_{tip-d1}$ | 2.05 s            | 2.81 s | 4.30 s | 0.48 s            | 0.54 s | 1.48 s | 0.18 s            | 0.31 s | 0.57 s |
| Payload $m_{tip-d2}$ | 2.06 s            | 2.83 s | 4.36 s | 0.48 s            | 0.56 s | 1.45 s | 0.12 s            | 1.14 s | 0.15 s |
| Payload $m_{tip-d3}$ | 2.05 s            | 2.82 s | 4.34 s | 0.47 s            | 0.55 s | 1.43 s | 0.11 s            | 1.12 s | 0.19 s |

Upon comparing the convergence times in these tables, it will be observed that the Algebraic Identifier performs much faster than the Least Mean Squares algorithm. The difference in performance is quite significant for the trajectory tracking (experiment type 1), which is the essence of a pick and place application. For this task, the Algebraic Identifier algorithm required an average of 0.34 s seconds to reach the 5% margin of error, whereas the Least Mean Squares algorithm obtained the same result in 2.82 seconds. This signifies that the Algebraic Identifier was about 8 times faster than the Least Mean Squares for this specific task. It is important to state that the amount of time spent by the Algebraic Identifier represents only 17 % of the total duration of the trajectory, and that the same number in the case of the Least Mean Squares algorithm is larger than the total duration of the trajectory, which means that this latter alternative is not useful for this type of task. The speed of converge of the payload mass identification algorithms in experiment types 2 and 3 was far superior to that in experiment type 1. This is a consequence of considering trajectories that start from an intermediate orientation, in which the gravitational torque is very significant and in which the signal/noise ratio of the torque measurements is quite high. Even in these cases, the Algebraic Identifier outperformed the Least Mean Squares Identifier. In the tracking of persistent inputs (experiment type 2), the Algebraic Identifier performed about twice fast as the Least Mean Squares algorithm as regards reaching the 5% error, which is still a reasonable improvement. In experiment type 3, in which the robot was initially at rest and was then perturbed, the Algebraic Identifier outperformed the Least Mean Squares, reaching the

Table 6.3 Online algebraic identification - duraluminum link: performance of the convergence time detection algorithm in the trajectory tracking (experiment type 1).

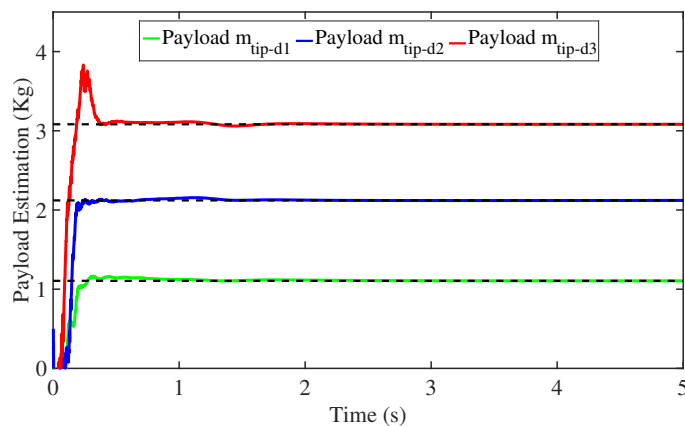
|                      | Convergence time | % of the trajectory duration | Error  |
|----------------------|------------------|------------------------------|--------|
| Payload $m_{tip-d1}$ | 0.78 s           | 39.0 %                       | 2.58 % |
| Payload $m_{tip-d2}$ | 0.72 s           | 35.9 %                       | 1.54 % |
| Payload $m_{tip-d3}$ | 0.83 s           | 41.3 %                       | 0.97 % |

same 5% band in about an average of one third of the time. These trends were approximately maintained for the 2 % and 10 % band errors.

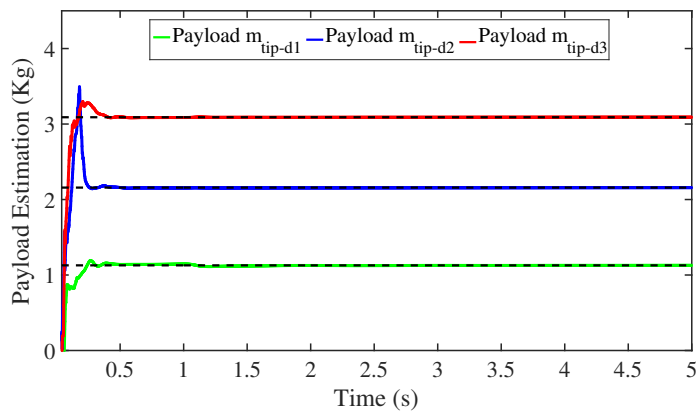
Figs 6.7 and 6.8 depict the time response of the Algebraic Identifier and the Least Mean Squares identifier, respectively. Note that wrong estimates of the parameters are obtained at the very beginning of the execution of the Algebraic Identification algorithm. These inaccurate estimates are the consequences of having a very limited number of samples of the inputs which are corrupted by noise. Once this phase is over and the Algebraic Identification has a reasonable number samples available, the effects of the noise are very quickly reduced and the estimation rapidly approaches the actual payload mass value. From the point of view of the adaptive control, knowing when the Algebraic Identifier has converged to acceptable values is an important information that is required in order to update the control law properly. On the one hand, it indicates when to discard or consider the parameter estimates, and on the other, it allows us to reinitialize the identifier so as to detect new parameter changes or to stop the identifier algorithm because no more changes are going to take place. In Section 6.2.3, an algorithm for detecting the convergence time of the identifier was presented. In the following, we present the results of implementing that algorithm for detecting the convergence time in the tracking of a trajectory that goes from 90 degrees (vertical position) to 135 degrees in 2 seconds (experiments type 1). Here, it is assumed that it is desirable to perform acceptable payload estimates in a period of time shorter than one half of the trajectory (1 second). As was seen in Table 6.1, this is feasible when considering estimation errors of about 2%. This design condition is met when considering a tolerance parameter ( $\Delta$ ) equal to 0.0075 and a windows size ( $M$ ) of 250 samples (0.5 seconds). A description of the performance of this algorithm is presented in Table 6.3

### 6.3.4 Composite link results

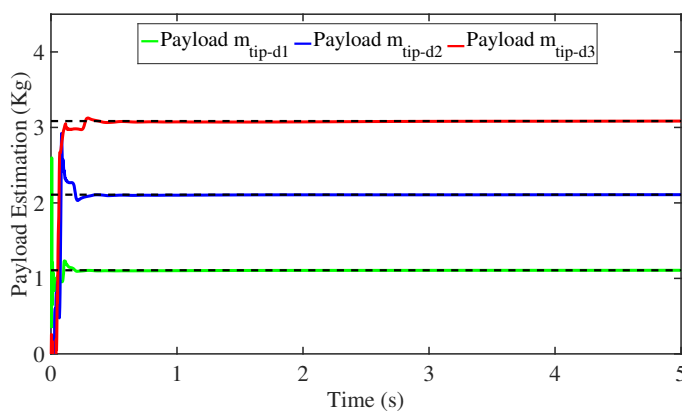
The performance of the Algebraic Identification algorithm and the Least Mean Squares algorithm are summarized in Tables 6.4 and 6.5, respectively. These tables, specifically,



(a) Experiment type 1.

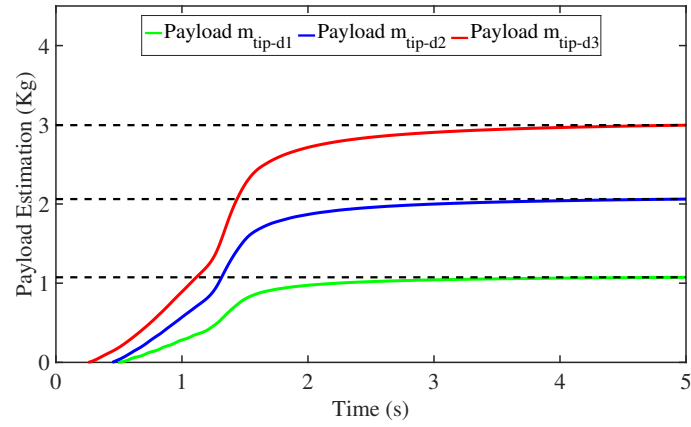


(b) Experiment type 2.

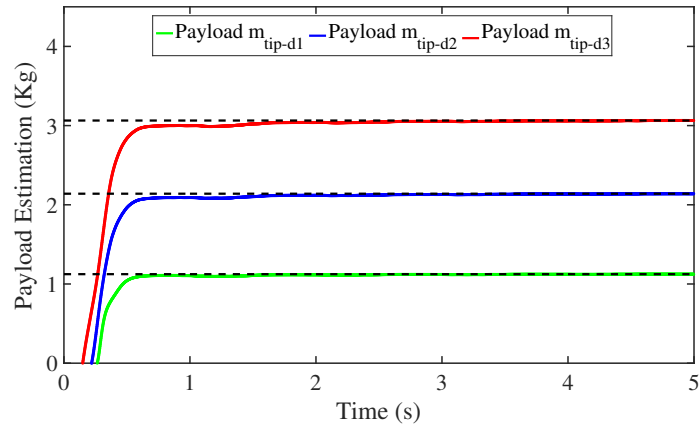


(c) Experiment type 3.

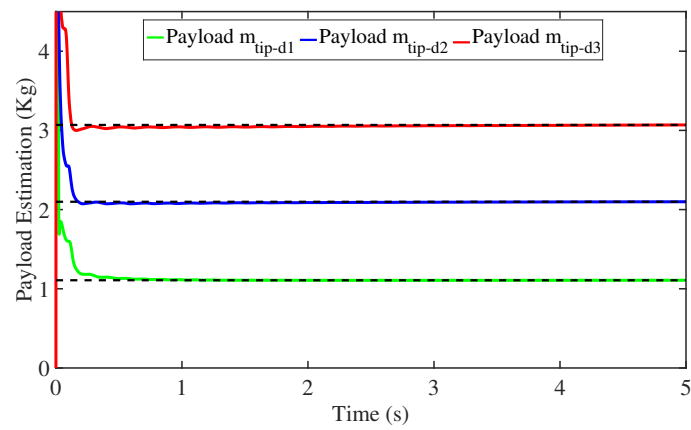
Fig. 6.7 Online algebraic identification - duraluminum link: time response.



(a) Experiment type 1.



(b) Experiment type 2.



(c) Experiment type 3.

Fig. 6.8 Online least mean squares identification - duraluminum link: time response.

present the time spent by the identification algorithms to reach and stay in the 10 %, 5% and 2% band error for the three types of experiments detailed in Section 6.3.1.

Table 6.4 Online algebraic identification - composite link : convergence times for the different experiments and payloads.

| Errors                | Experiment type 1 |        |        | Experiment type 2 |        |        | Experiment type 3 |        |        |
|-----------------------|-------------------|--------|--------|-------------------|--------|--------|-------------------|--------|--------|
|                       | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     |
| Payload $m_{tip-c11}$ | 0.41 s            | 0.75 s | 1.55 s | 0.05 s            | 0.68 s | 0.76 s | 0.29 s            | 0.31 s | 0.36 s |
| Payload $m_{tip-c12}$ | 0.62 s            | 0.97 s | 1.85 s | 0.32 s            | 0.44 s | 0.55 s | 0.08 s            | 0.11 s | 0.43 s |
| Payload $m_{tip-c13}$ | 0.70 s            | 0.73 s | 1.15 s | 0.41 s            | 0.44 s | 0.45 s | 0.20 s            | 0.27 s | 0.46 s |

Table 6.5 Online least mean squares identification - composite link: convergence times for the different experiments and payloads.

| Errors                | Experiment type 1 |        |        | Experiment type 2 |        |        | Experiment type 3 |        |        |
|-----------------------|-------------------|--------|--------|-------------------|--------|--------|-------------------|--------|--------|
|                       | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     | 10%               | 5%     | 2%     |
| Payload $m_{tip-c11}$ | 3.16 s            | 4.51 s | 6.39 s | 0.11 s            | 0.48 s | 0.70 s | 0.13 s            | 0.19 s | 0.28 s |
| Payload $m_{tip-c12}$ | 2.81 s            | 4.11 s | 6.09 s | 0.46 s            | 0.49 s | 0.98 s | 0.17 s            | 0.18 s | 0.42 s |
| Payload $m_{tip-c13}$ | 2.95 s            | 4.32 s | 6.18 s | 0.47 s            | 0.77 s | 1.18 s | 0.28 s            | 0.42 s | 0.77 s |

The results in the tables indicate that the Algebraic Identifier performs better than the Least Mean Squares in the trajectory tracking experiment (experiment type 1), which is the most fundamental situation in a pick and place task. If we consider the 5% error band, the Least Mean Squares required an average of 4.31 seconds to identify the payload mass, in contrast with the Algebraic Identifier which achieved it in an average of only 0.82 seconds. This represents an improvement of about 5 times in the case of this specific task. In proportion to the total duration of the trajectory, the time spent by the Algebraic Identifier represents 41% of the time, whereas the time spent by the Least Mean Squares exceeds the total duration and is not, therefore, acceptable for the pick and place application. In relation to the other tasks (experiment types 2 and 3), the Algebraic Identifier performed slightly better than the Least Mean Squares algorithm. In the tracking of persistent inputs (experiment type 2), the Algebraic Identifier required an average of 0.52 seconds to reach the 5 % error band, whereas the Least Mean Squares algorithm took 0.58 seconds. With regard to experiment type 3, in which the robot is initially at rest and is then perturbed, the

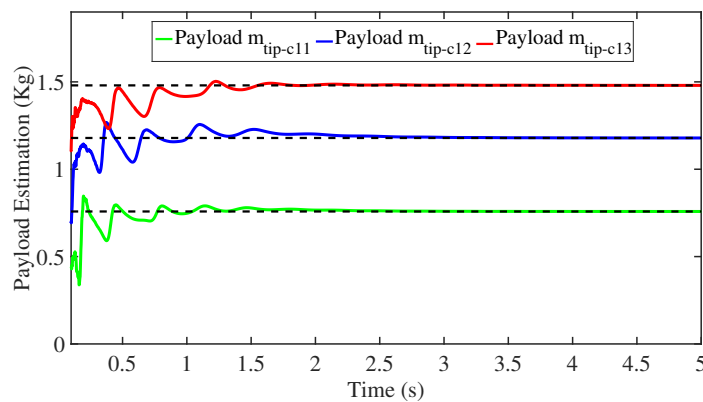
Table 6.6 Online algebraic identification - composite link: performance of the convergence time detection algorithm in the trajectory tracking (experiment type 1).

|                       | Convergence time | % of the trajectory duration | Error  |
|-----------------------|------------------|------------------------------|--------|
| Payload $m_{tip-c11}$ | 0.97 s           | 48.6 %                       | 4.20 % |
| Payload $m_{tip-c12}$ | 0.97 s           | 48.7 %                       | 6.60 % |
| Payload $m_{tip-c13}$ | 0.99 s           | 49.7 %                       | 4.27 % |

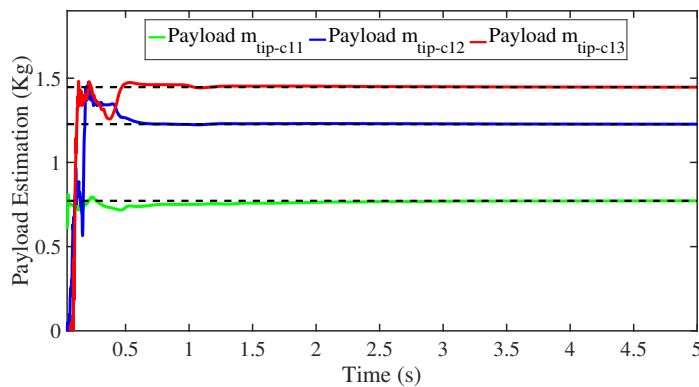
Algebraic Identification and the Least Mean Squares algorithm needed 0.23 and 0.26 seconds, respectively, to enter and stay into the 5% error band. The trends observed in the 5 % band were consistent with the results observed in the 5 % and 10 % bands. Figs 6.9 and 6.10 depict the time response of the Algebraic Identifier and the Least Mean Squares identifier, respectively.

Similarly to the case of the duraluminum link, the algorithm for the detection of the convergence time, presented in Section 6.2.3, was set up and tested in the tracking of trajectories that go from 90 degrees (vertical position) to 135 degrees in 2 seconds (experiments type 1). The condition for adjusting the algorithm was obtaining acceptable payload estimates in a period of time shorter than one half of the trajectory (1 second). Table 6.4 shows that this is achievable with errors in the order of 5%. This condition is met by configuring the algorithm with a tolerance parameter ( $\Delta$ ) equal to 0.04 and a window size ( $M$ ) of 200 samples (0.4 seconds). A description of the performance of this algorithm is presented in Table 6.6.

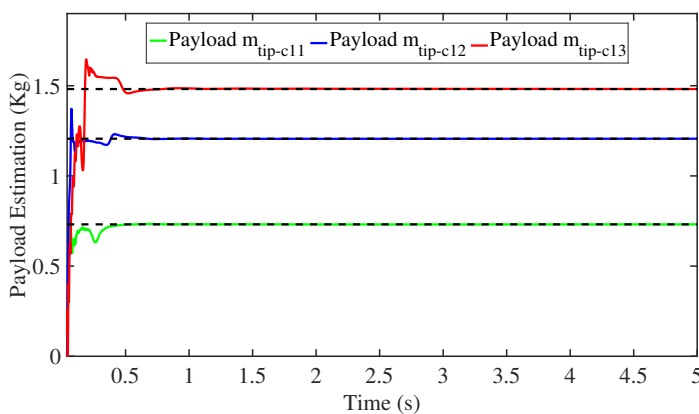
One interesting aspect of the overall experimental results is that the payload estimations were faster in the case of the duraluminum link than in the case of the composite link. This trend was not only present in the Algebraic Identification, in which the proportion was 2.13 times, but also in the case of the Least Mean Squares where the proportion was 1.22 times. We suggest that this may be the result of two circumstances: (1) the setup with the duraluminum link has, on average, a faster dynamics than the setup with the composite link, i.e., a higher fundamental vibrational frequency and it could, therefore, be expected that more information per unit of time is obtained by the experiments with the duraluminum link; (2) the composite link has a second vibrational mode that is not included in the dynamic model in the identification algorithm and it is, therefore, reasonable to expect that this second vibrational mode acts as a disturbance in the identification process. An argument against the first hypothesis is that if the speed of the dynamics determines the speed of convergence of the identification algorithm, the same trend should be noticeable across the payload masses



(a) Experiment type 1.

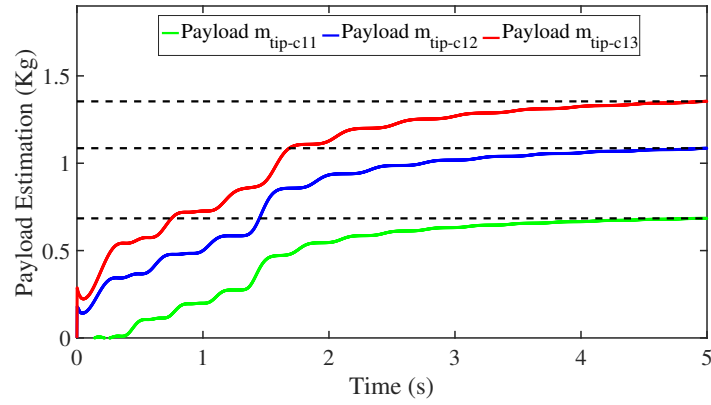


(b) Experiment type 2.

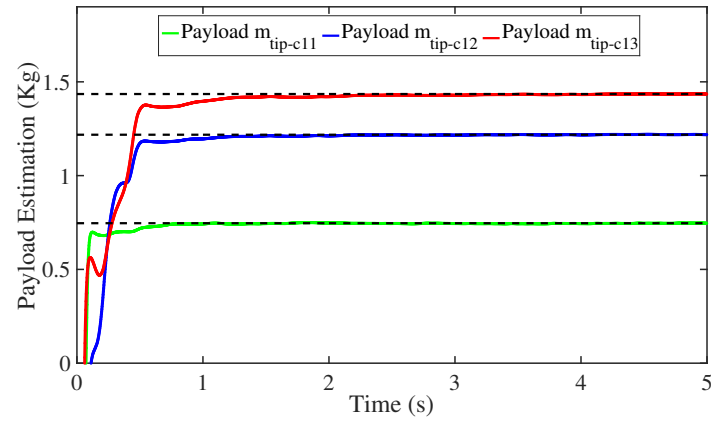


(c) Experiment type 3

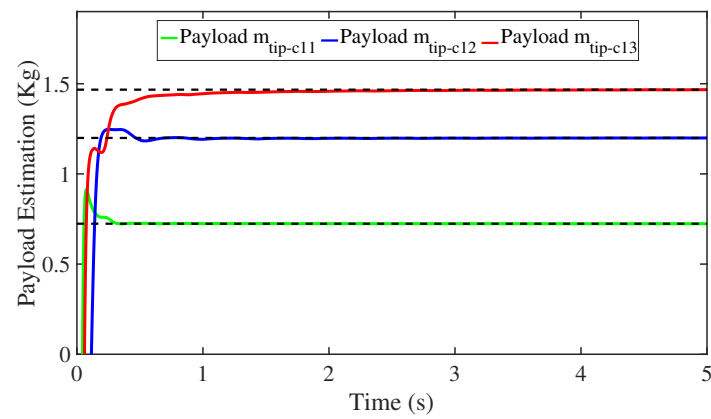
Fig. 6.9 Online algebraic identification - composite link: time response.



(a) Experiment type 1.



(b) Experiment type 2.



(c) Experiment type 3.

Fig. 6.10 Online least mean squares identification - composite link: time response.

in each link setup. However, no clear trend is observed in the tables. One possible explanation for this could be that only one experiment was performed for each payload mass and experiment type more experiments, therefore, need to be carried out to reveal the actual pattern. With regard the second hypothesis, we were unable to establish any arguments, but likewise, we believe that further experiments considering a wider range of link payload masses are required in order to reach a definitive conclusion.

## 6.4 Design of the adaptive controller

In this section, we propose an explicit adaptive control algorithm for a single link flexible arm moving under gravity. The controller combines the algebraic identifier, presented in the previous sections, with the input-state feedback linearizing controller, developed in Chapter 5. The primary goal of the algorithm is to achieve the flexible link tip positioning under conditions of payload mass uncertainty. In Chapter 5, we concluded that the mismatch between the nominal and actual payload mass may produce significant increases of the settling time and the maximum overshoot in the transient response of the system, in addition to residual vibrations and constant errors in the steady state. The adaptive control algorithm presented in this section considerably reduces these effects by updating the control law with the payload mass estimates computed online.

The most interesting starting situation for the adaptive control algorithm is when the payload mass has recently been changed and is, therefore, unknown. In this situation, the payload mass could be trivially computed from the torque measurements and the equation (3.26) if the starting position of the robot is completely static and is away from the vertical configuration. However, this does not correspond to the most general case. In the case of flexible link robots, having a perfect static condition after placing or picking up a payload is not normally immediate owing to the lightly damped dynamics of the flexible link, which commonly results in residual vibrations. These vibrations are especially significant after releasing a payload, as stated by Yang et al. in [55]. The Algebraic Identifier allows us to compute a fast estimation of the payload for the most general case, i.e., with total independence of the initial orientation and without requiring the robot to be initially static.

The following is a general description of the operation of the adaptive control algorithm. Fig 6.11 shows a schematic representation of the overall adaptive control algorithm, including the citations of the equations considered. The execution of the adaptive control algorithm is divided into three subsequent phases, detailed below.

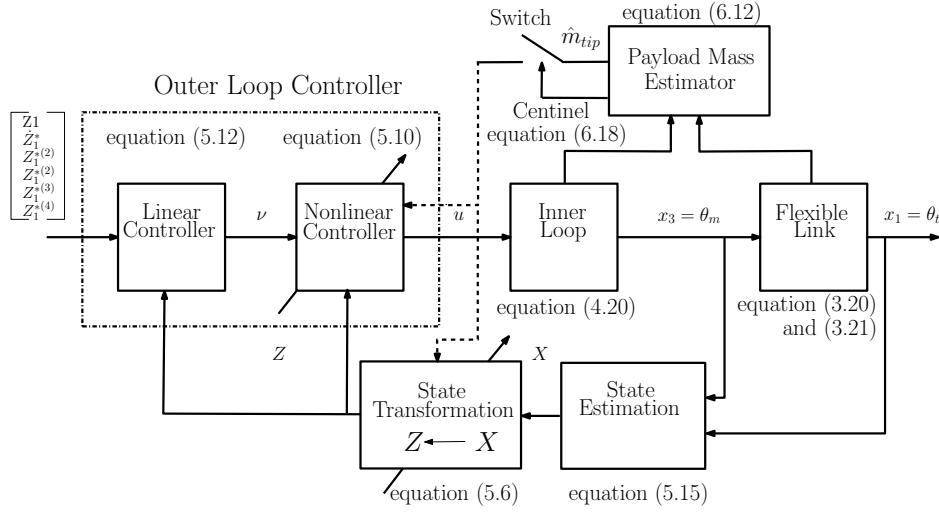


Fig. 6.11 Adaptive tip position controller design: schematic representation.

**First phase:** In this phase, it is assumed that the payload has been recently changed. The robot moves considering the regular non-adaptive position controller, the input-state feedback linearizing controller developed in Chapter 5, that takes into account an initial guess of the payload mass. It is assumed that even in presence of the mismatch between the nominal and the real payload mass, the systems behaves stable and the trajectory is tracked with a small but non negligible error. Immediately after the payload changes, the online Algebraic Identifier initiates but its estimates are not taken into account yet in the control law. Since the algebraic identifier does not provide reliable estimates during a certain period of time, the switch in Fig 6.11 stays open so the resulting estimates are discarded.

**Second phase:** this begins when the tolerance parameter of the Algebraic Identifier  $\Delta$  is lower than a certain value  $\Delta_1$ , which means that the payload mass estimates have approximately reached a certain predefined error band. Once this condition is met, the estimation of the payload mass is assumed to be reliable, therefore, the switch in Fig 6.11 is closed and the payload estimates, therefore, start to be used to update the *State Transformation* and *Nonlinear Controller* blocks. From this point forward, the payload estimation error typically decreases and the estimates approach the payload mass very quickly.

**Third phase:** this starts when the tolerance parameter  $\Delta$  is lower than a certain value  $\Delta_2$ . When this occurs, the switch  $S_1$  id again opened again, thus stopping the update of the controller blocks, and the Algebraic Identifier is stopped. By reaching the tolerance parameter  $\Delta_2$  the payload estimates are approximately in a very small band error in

which no significant variations in the estimates are observed. As mentioned in Section 6.2.3, since the Algebraic Identifier is composed of unstable filters, it is advisable to stop the Algebraic Identifier once the payload mass has converged so as to avoid the arithmetic overflow when a long time has passed.

The adaptive control algorithm presented here took as its starting point the adaptive controller designed by Becedas in [60] for the position of the control of a single-link flexible arm. There are some significant differences between the work carried out by Becedas and that presented in this chapter. The most relevant differences are listed below.

1. The controller proposed by Becedas was applied to a single-link flexible arm moving in a horizontal plane, which was described by a linear model. The single link flexible arm considered throughout this thesis is constrained to a vertical plane and under force of gravity and is necessarily described by a non-linear model.
2. In Becedas' work, the resulting Algebraic Identifier was combined with a Generalized Proportional Integral controller (GPI), whereas, in the work presented in this chapter, the resulting Algebraic Identifier was combined with an input-state feedback linearizing controller. The generalized proportional controller, as presented in Beceda's work, is only applicable to a linear system. The feedback linearization control is fundamentally a non-linear technique and therefore, more suitable for our system.
3. The differences between our non-linear model of the flexible link arm and the linear model considered by Becedas implied some changes in the design methodology of the Algebraic Identifier, which was still applicable since the mathematical expression considered, equation(6.1), was linear in the parameters.
4. The Algebraic Identifier used by Becedas was designed for the estimation of the first natural frequency of the flexible link arm, whereas the Algebraic Identifier considered in the present work was used to compute the payload mass and the viscous friction of the model. Since a linear model with no viscous friction was considered in Becedas' work, the system could be successfully normalized in terms of only one parameter, the natural frequency of the arm. Upon doing this, all the possible parameter variations in the model were captured by the identification of this single parameter. In our case, the non-linearity in the model prevented us from achieving a similar parameter reduction. Although it is true that a reduction in the number of parameters is a desirable condition that would allow us to compensate for errors, the only parameter with a significant variation in a pick and place task is the payload mass held by the robot.

5. Another difference as regards the Algebraic Identifier is that in Becedas's work the estimates originating from the identifier are not considered in the control law until it has completely converged. This extends the period of time in which the robot is controlled by a regular non-adaptive control law with a set of nominal parameters. In our work, the parameter estimates start to be considered earlier, specifically when the estimates reach the 10 % band error with respect to the final value. The identification algorithm is stopped when the estimation reaches the 1 % band error. By following this approach, the control algorithm considers, on average, better estimates of the payload mass which reduces the tracking error during the trajectory execution.

## 6.5 Study of local stability

In the section, we study the stability of the controlled system in terms of the payload mass variations. The aim is to define the range of payload mass uncertainty that the adaptive control algorithm can tolerate without becoming unstable. Describing the stability of our controlled system in detail is a very challenging problem given the time varying characteristics of the controlled system and the non-linear nature of the platform. In this section, it is not our intention to carry out an exhaustive analysis of the stability of the system, but rather at least provide a preliminary study. The results presented here are limited to a local stability analysis of the system when there is a certain mismatch between the nominal and the actual payload mass. The analysis considers the two lumped mass dynamic model presented in Section 3.2.3, although the controller design considered a single lumped mass dynamics model. The two lumped mass dynamic model reproduces the two most significant vibrational modes of the system, and allows us to evaluate the impact of the higher order dynamics that was not considered in the controller designed, which is commonly referred to in research literature as *spill-over*.

For the stability analysis, we configured the platform with the composite link, which has two significant vibrational modes in the lowest part of the frequency spectrum (see Fig 3.4b). With regard to the control algorithm, the poles of the inner loop (joint position control) were allocated at  $p_i = -12$  and the poles of the outer loop (tip position control) were located in  $p_o = -15$ . The poles were positioned to guarantee the stability of the system and ensure the absence of *spill-over* effects when the flexible link supports a payload mass of  $m_{tip-c12} = 1.20$  Kg. These parameters for the control algorithm will be considered in the experimental validation of the controller, in Section 6.6. The local stability analysis was carried out around an intermediate point of equilibrium that satisfies that the motor position is  $\theta_{m-0} = 45$  degrees. As commented on Section 3.2.3, it is important to remember that frequency

response of the system does not significantly change with regard to the equilibrium point. In other words, the linearization about other equilibrium points leads to similar linear models, and the local stability analysis is practically equivalent.

The procedure considered for the analysis of the local stability is simple but still extensive. In the following lines, we summarize the steps followed without going into detail. We first considered the two lumped mass dynamic model as the plant model (equations (3.47) and (3.48)); we substituted the payload mass constant  $m_{tip}$  for the constant  $m_{tip-real}$ , which denoted the actual payload mass supported by the manipulator, and made all the derivatives in the model zero, in order to determine the state vector of the plant and of the measurable outputs (coupling torque) that satisfy the equilibrium condition. We then replaced the payload mass constant  $m_{tip}$  with its nominal value, indicated as  $m_{tip-nom}$ , in the equations of the non-adaptive controller, including the control law (equations (5.10) and (5.12)) the state estimator (equation (5.15) and the state transformation (equation 5.6). We subsequently linearized the plant and controller, and obtained a transfer function between the tip position reference and the actual tip position. The numerator and the denominator of the resulting transfer function both correspond to a six order polynomial. The system is said to be stable, in the bounded-input-bounded-output sense, when the poles of the transfer function are all in the left half of the complex plane. The Routh-Hurwitz stability criterion is typically used to determine the conditions in the parameters that satisfy this stability condition. However, in our case it does not provide very practical rules with which to establish the range of payload mass uncertainty that the system could tolerate. This is because of the complexity of the system dynamics, the control algorithm, and the number of parameters involved. Instead, we preferred to compute numerically the combination of the nominal and real payload masses for which the stability of the system is guaranteed. The combination of parameters that satisfy the stability of the system defines what we call the stability region, and is depicted in Fig 6.12.

Graphically defining the stability region is a interesting information, but is not very practical from the point of view of a user. It is more useful to define, for instance, the range of payload masses that the robot can manage without incurring stability problems. We define the area that satisfies this latter condition as the feasibility region, which is also shown in Fig 6.12. The differences between the stability and feasibility regions is illustrated as follows.

Let us pose the question concerning whether or not it is correct to affirm that the robot can handle payload masses of between 2 Kg and 3 Kg. The correct answer is that it cannot, and the reason for this is explained with the following example. Let us suppose that the robot starts functioning at a stable point at which the real payload mass and the nominal

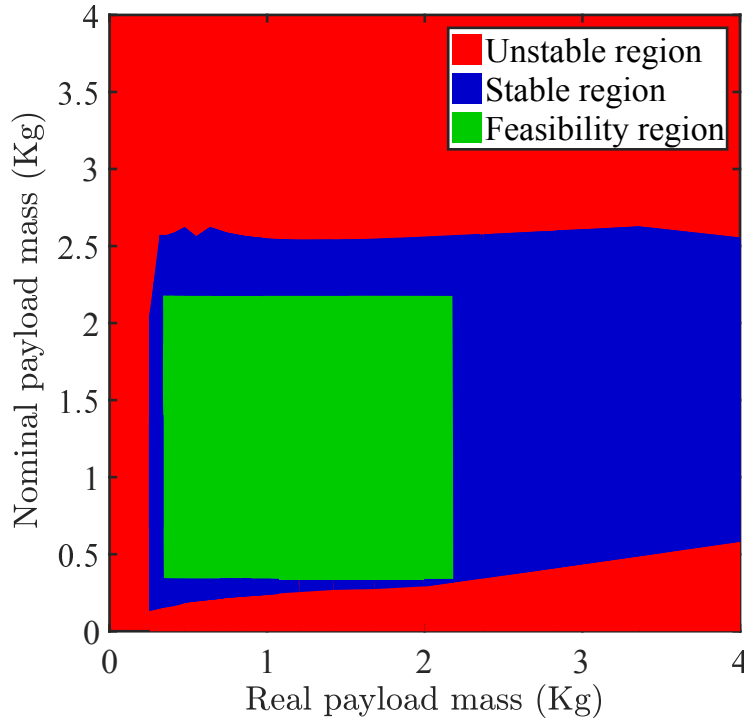


Fig. 6.12 Adaptive tip position controller: local stability study in terms of payload mass uncertainty.

payload mass are equal to 2 Kg. During a manipulation task, the robot picks up a payload of 3 Kg (real payload mass) immediately after releasing the initial payload of 2 Kg (nominal payload mass). As Fig 6.12 shows the system remains stable, but only for a short period of time. Once the online payload identifier estimates the actual payload mass (3 Kg), the estimates are used to update the nominal payload mass and the system leaves the stability region and becomes unstable. If we extend this reasoning, it is not correct to state that the robot can handle payload masses between 2 Kg and 3 Kg, because only some combinations of nominal and real payloads satisfy the stability condition.

Unlike the stability region, the feasibility region defines the range of payload masses that the system can handle in the context of an adaptive controller, considering that the values of nominal payload masses and the real payload masses constantly change. The feasibility region is confined to a maximum square area within the stability region, which is symmetrical about a line with a unitary slope passing through the origin. Given this configuration, the minimum and maximum positions of the vertex in any of the axes define the range of payload masses that guarantee, at least locally, the stability of the system. Fig 6.12 shows that the range runs from 0.34 Kg to 2.18 Kg.

The results presented in Fig 6.12 allow us only to ensure the conditions of local stability. However, it is also interesting to describe the positions of the poles in terms of the payload mass variation. For demonstration purposes, in Fig 6.13 we depict the root contour diagram of the system for the case in which the real payload is set to  $m_{tip-c12} = 1.20$  Kg and the nominal payload mass varies between its stability limits, 0.25 Kg and 2.53 Kg. The zeros were omitted in the root contour diagram since they are not required for the stability analysis. Likewise, only the upper part of the root contour diagram is shown since the lower part is symmetrical about the real axis. Several aspects are worthy of examination of this figure. First, in the case in which the nominal and the real payload masses are equal, the closed loop poles are not exactly situated at the design location  $p_o = -12$ , but tend to be cluster around it. This was to be expected, since the dynamic model considered to describe the system was not the same as the dynamic model considered to develop the control algorithm. In the controller design, we considered the one lumped mass dynamic model that is described by 2 poles, whereas in the evaluation of the stability we considered the two lumped mass dynamic model which has 4 poles. Another important observation in the same figure is that the root contour diagram is concentrated in two regions: a region that is far from the X-axis and the region near to the X-axis. Note that the region that is far from the X-axis is represented by a branch which is almost parallel to it, situated at a distance in the Y-axis that is about 270 rad/s  $\approx$  43 Hz, which is the value of the second vibrational frequency of the arm for that payload mass. Based on this observation, we confirm that this region is undoubtedly linked to the high order dynamics and, therefore, determine the *spill-over* effects in the system, whereas the region which is closer to the X-axis is related to the low order dynamics. In relation to the payload mass variations, the same figure shows that the nominal payload mass approaches the lower limit of stability, the low order poles move towards the Y-axis and dominate the response of the system, while the high order poles become more stable. In contrast, when the nominal payload comes near the upper stability boundary, some low order poles tend to approach the Y-axis but are at the high order poles that dominate the response of the system. Although these observations may not hold in the more general case, they serve to partially show two things: first, that our system remains stable for a certain range of mismatch between the nominal and the real payload mass, and, second, that ignoring higher order dynamics in the control design does not necessarily imply instability. The *spill-over* is not only a source of instability of the system, but may also affect the precision of the tip positioning in those situations in which the system remains stable, as will be observed in the experiments presented in the following section.

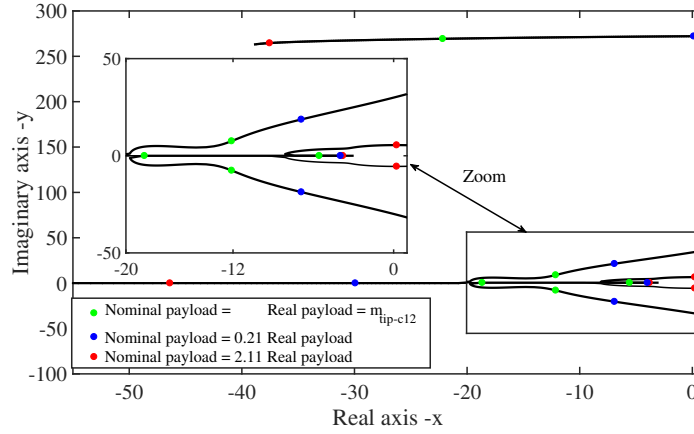


Fig. 6.13 Adaptive tip position controller - composite link: root contour diagram of the linearized adaptive control system when the robot holds a payload masses  $m_{tip-real} = m_{tip-c12}$  and the nominal payload mass varies.

## 6.6 Experimental validation of the adaptive controller

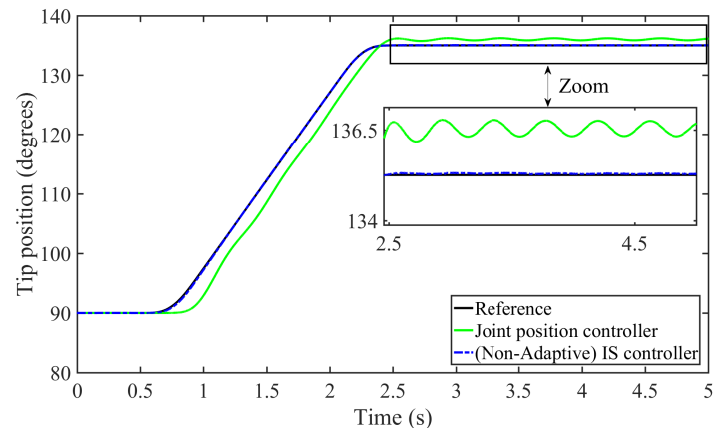
In these experiments, we examine the performance of the adaptive controller. The configuration of the system was as follows. We considered the link made of composite material and the payload mass  $m_{tip-c12} = 1.20$  Kg. With regard to the controller, the poles of the inner loop (joint position control) were allocated at  $p_i = -15$ , while the pole of the outer loop (tip position control) were located in  $p_o = -14$ . In relation to the Algebraic Identifier, we considered a windows size (M) of 250 samples equivalent to 0.5 seconds. The estimates started to be considered for the update of the controller when they reached a band of error of approximately 10 % with respect to the final value of the estimates ( $\Delta_1 = 0.015$ ), whereas the Algebraic Identifier was stopped once the estimate reached the band of error of 1% ( $\Delta_2 = 0.005$ ). During the experiments, the controllers were commanded to track a fourth order trajectory that went from 90 to 135 degrees in 2 seconds. As mentioned in Chapter 5, the input-state feedback linearizing controller requires a trajectory that is at least fourth order differentiable in order to maintain the control signal finite. The trajectory was designed by following the guidelines explained in [48].

Before evaluating the adaptive controller, we shall evaluate the non-adaptive version of the controller. First, we prove that the controller performs correctly in nominal payload mass conditions. Fig 6.14 compares the response of the flexible arm when only the joint position is controlled, as opposed to the case in which the flexible link tip position is controlled. The response of the Joint Position Controller represents the behavior of the system when it is controlled as a rigid arm. A joint position controller would be a viable control option in the case of a single rigid arm, in which the angular position of the motor corresponds to the

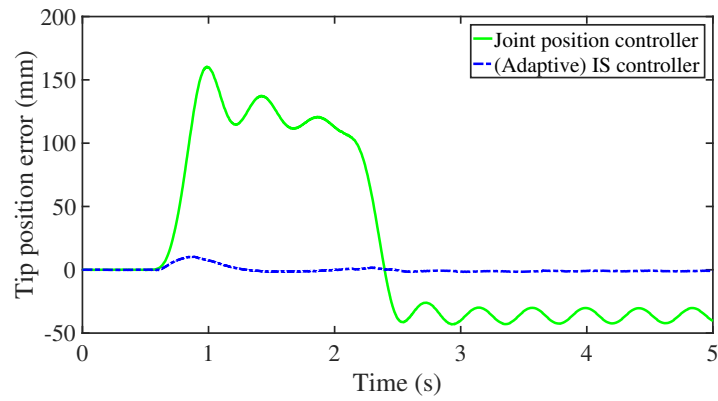
angular position of the tip point. In the case of a flexible link arm, there is certain dynamics in the relationship between these two variables.

The Joint Position Controller considered was designed by following the procedure of the two degrees-of-freedom actuator controller presented in Chapter 4. The controller for the tip position corresponds to the Input-State Feedback Linearizing controller proposed in Chapter 5. As the figure shows, the Input-State Feedback Linearizing controller is clearly superior to the joint position controller. The Input-State Feedback Linearizing controller reduced the maximum tracking error from 160 mm to only 10 mm, the steady state error from 36 mm to 1 mm, and the residual vibrations from 12.9 mm to a 0.1 mm. These last two quantities were calculated by employing the estimation of the tip position, (equation (6.3)) since its amplitude was in some cases below the accuracy of the camera tracking system. Although it is almost completely negligible in these experiments, the coupling torque signal evidences some frequency components of 45 Hz, which correspond to the seconds vibrational frequency of the flexible link. The existence of these frequency components in the response of the system is known as the *spill-over* effect, and is a consequence of not having taken into account the higher vibrational modes of the flexible link arm in the control design. Given the amplitudes of the frequency components in the response of the system, they do not endanger the stability of the system.

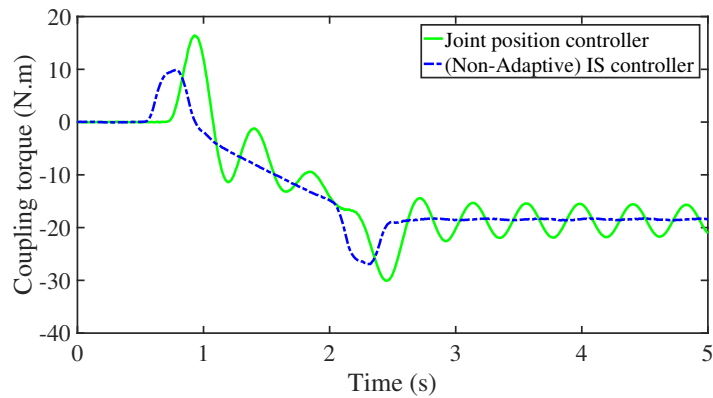
To continue with the evaluation of the non-adaptive version of the controller, we shall now assess the impact of the payload mass uncertainty on the performance of the system. Fig 6.15 compares the experimental results obtained when considering  $m_{tip-c12} = 1.20$  Kg as the real payload mass, and nominal payload masses which are one fourth of, equal to and twice the real payload mass. In comparison to the nominal case, the mismatch between the nominal and the real payload mass produces very significant increases in the tracking error and steady state error. The maximum tracking error increased from 10 mm, in the nominal case, to 277 mm and 133 mm for the cases in which the nominal payload was one fourth of and twice the real payload respectively. The steady state error increased from 1 mm, in the nominal case, to 184 mm and 93 mm, respectively. In all the cases, the first vibrational model of the arm was effectively canceled. However, there was a certain amount of increase in the residual vibrations that went from 0.1mm, in the nominal cases to 0.9 mm and 0.7 mm owing to the effects of the *spill-over*. Since the system considered payload mass mismatches that were around the limit of the stability of the system, the *spill-over* effects increased in both non-nominal payload mass situations. This was slightly greater in the case in which the nominal payload mass was one fourth of the real payload mass. The root contour diagram presented in Fig 6.13, might lead one to expect that the poles corresponding to the higher order dynamics would become more damped as the nominal payload mass increases with



(a) Tip position.



(b) Absolute value of the tracking error.



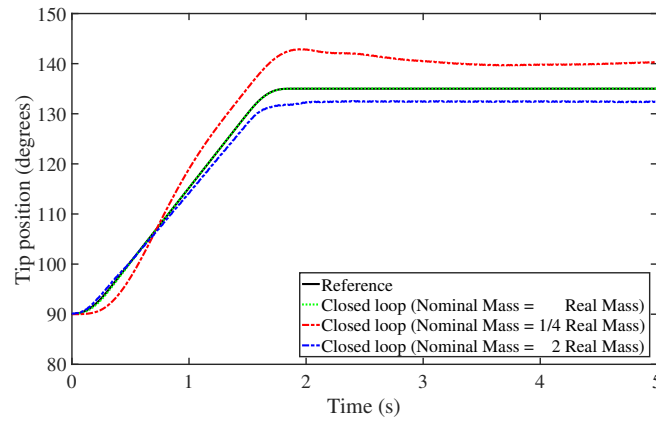
(c) Coupling torque.

Fig. 6.14 Validation - composite link: comparison of the time response of the system with the Joint Position Controller (without outer controller) vs. the Input State Feedback Linearizing Controller (IS Controller) for the tip positioning under nominal conditions.

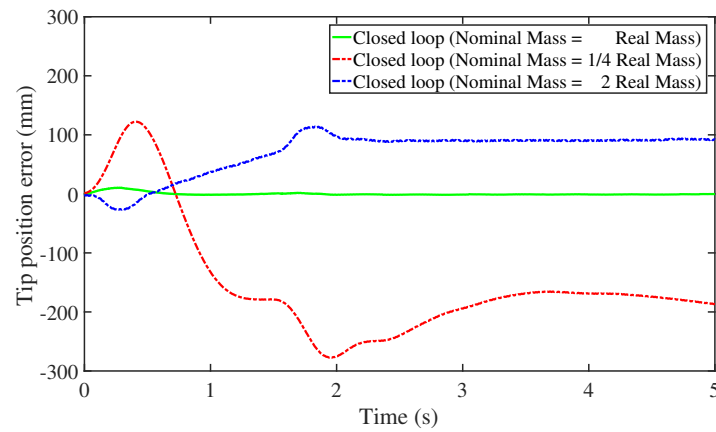
regard to the real payload masses, and less damped in the opposite case. However, it does not entirely describe these experimental observations, in which the case of the nominal payload mass being twice the real payload mass still has *spill-over*. While it is true that the root contour diagram has been useful to provide guiding principles regarding the design of the system and its stability, since it is based in the linearized system, it cannot predict certain phenomena. In future research work, it will be necessary to study the stability of the non-linear dynamics, including the higher order vibrational modes, in greater depth.

In summary, the experiments have shown that in the non-adaptive controller the mismatch between the nominal and the real payload mass increases the tracking error, in the transient and steady state, and may produce some residual vibrations. The objective of the adaptive controller is to improve system performance in these terms. Figs. 6.15 and 6.16 compare the responses of the non-adaptive controller with the adaptive controller for cases in which the nominal payload mass is one fourth of and twice the real payload mass. These figures allow us to observe that the algebraic identifier started to update the controller in 0.9 and 1.1 seconds, when about one-half of the trajectory had passed. The tracking errors of the adaptive controller in comparison to the non-adaptive controller immediately improved after the update of the payload. The improvements of the adaptive controller were very significant in terms of the tracking error, and the results show that the maximum tracking errors were reduced from 277 mm to 85 mm in the first case, and from 113 mm to 49 mm in the second case. In the steady state, the reduction in the tracking error was from 184 mm to 1.4 mm in the first case, and from 93 mm to 1.3 mm in the second case. With regard to the residual vibrations, it can also be seen that the adaptive controlled system has a barely noticeable residual vibration, below 0.1 mm, on average, for both cases studied. These residual vibrations were mainly caused by the *spill-over* effects, which were maintained approximately constant in both cases. In the case of the non-adaptive controller the *spill-over* was always larger than in the adaptive controller, once the payload mass had started to be updated in the controller.

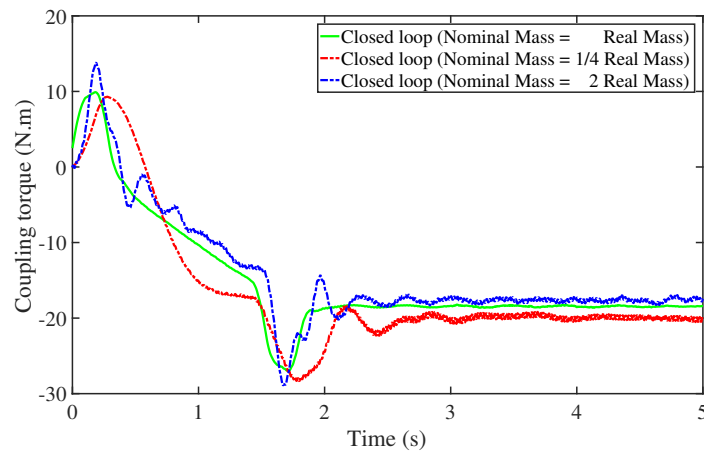
With regard to the payload mass estimation based on the algebraic identifier, Fig. 6.18 presents the response in each case. This figure shows that considering different initial payload mass guesses slightly affects the speed of convergence of the identifier in the execution of a specific task, but these differences are not very significant. Upon observing these figures, it is clear that the selection of the criterion required to start considering payload mass estimates from the algebraic identifier is very conservative, and there is certain room for improvement in the experimental results obtained for the previous controller. The selection of this criterion is a key in the implementation of this adaptive algorithm; on the one hand, it could significantly reduce the maximum tracking error, whereas in the other, it could also



(a) Tip position.



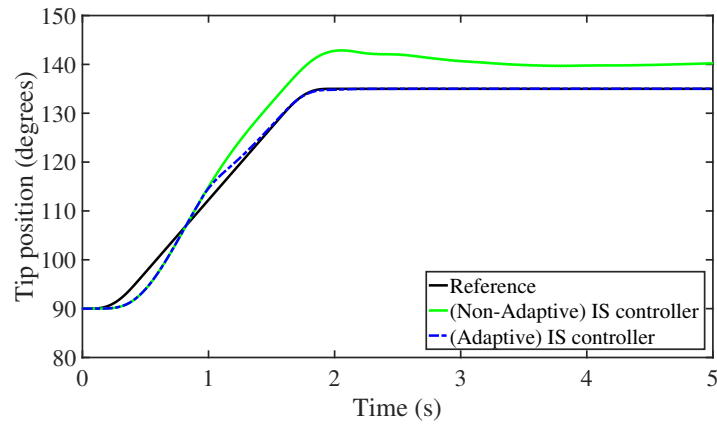
(b) Absolute value of the tracking error.



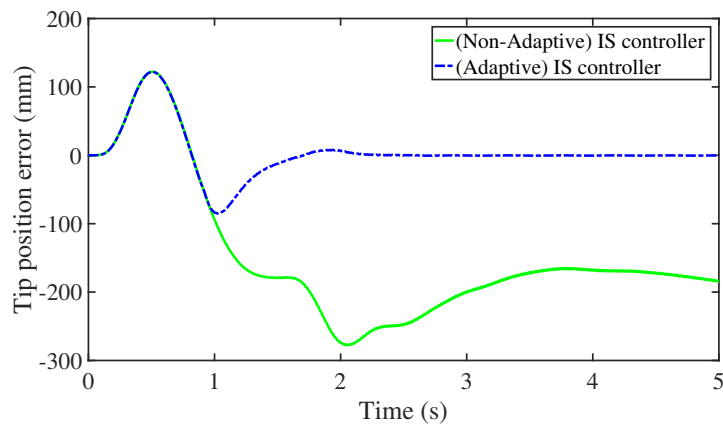
(c) Coupling torque.

Fig. 6.15 Validation - composite link: comparison of the time response of the Non-Adaptive Input State Feedback Linearizing Controller (IS Controller) for the tip positioning under conditions of nominal and non-nominal payload mass.

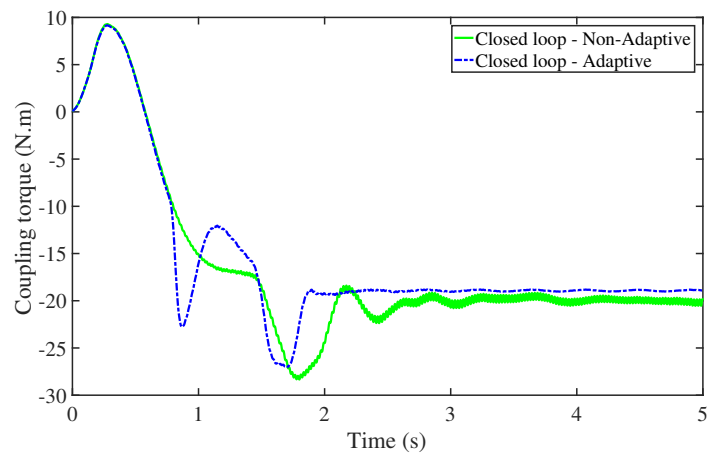
open the possibility of considering payload mass estimates that are worse estimates than the initial payload mass guess and that can change abruptly during the estimation. It is necessary to be particular careful about how this could affect the stability of the system the latter condition mentioned.



(a) Tip position.

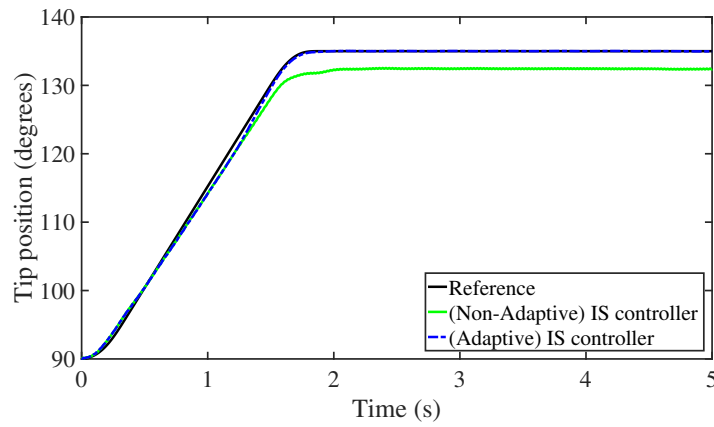


(b) Absolute value of the tracking error.

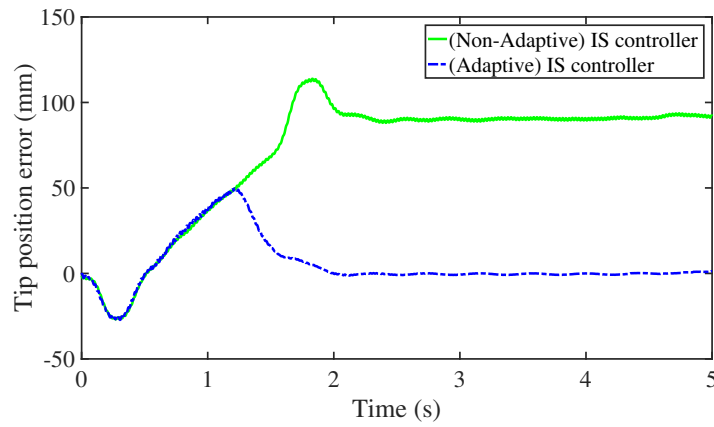


(c) Coupling torque.

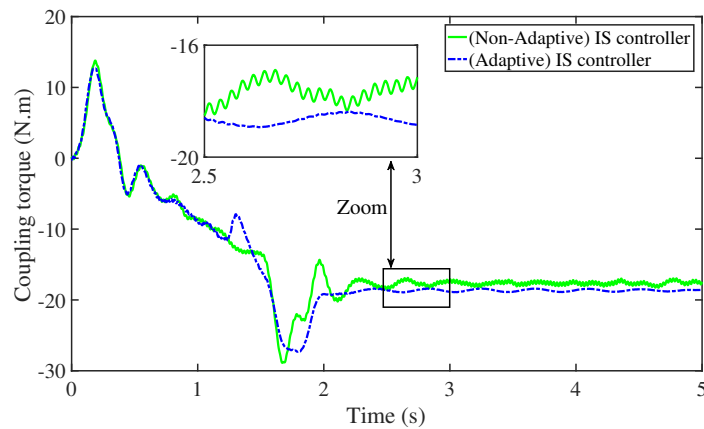
Fig. 6.16 Validation - composite link: comparison of the time response of the Non-Adaptive vs. Adaptive Input-State Feedback Linearizing Controller when the nominal payload mass is equal to one-fourth of the real payload mass.



(a) Tip position.

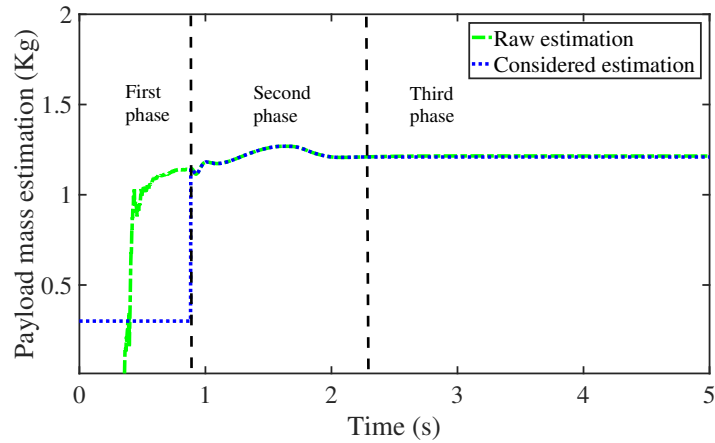


(b) Absolute value of the tracking error.

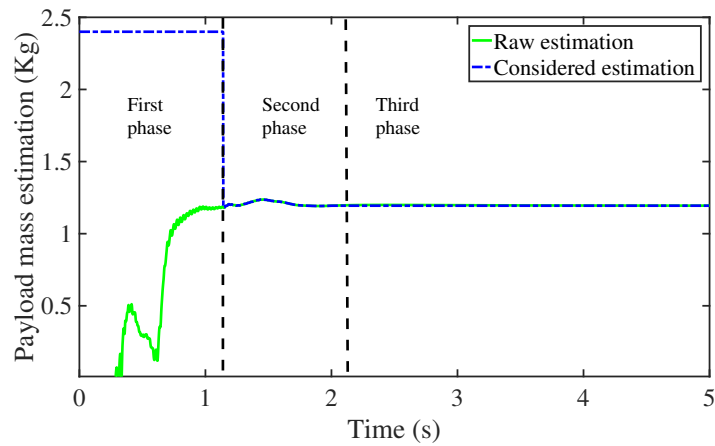


(c) Coupling torque.

Fig. 6.17 Validation - composite link: comparison of the time response of the Non-Adaptive vs. Adaptive Input-State Feedback Linearizing Controller (IS Controller) when the nominal payload mass is twice as big as the real payload mass.



(a) Nominal payload mass equals one fourth of the real payload mass.



(b) Nominal payload mass equals twice the real payload mass.

Fig. 6.18 Validation - composite link: payload mass estimations during the execution of the Adaptive Input-State Feedback Linearizing Controller.



# **Chapter 7**

## **Passive Gravity Compensation of a Single Link Flexible Arm**

### **7.1 Introduction**

A considerable amount of the force/torque available at the actuators in a robot manipulator is used to compensate the effects of the gravity acting on the structure of the robot. The objective of a passive gravity compensation mechanism is, as its name suggests, is to completely or partially cancel these effects on the actuator by considering mechanical parts that are capable of storing energy. At best, when the perfect cancellation is achieved, the robot can remain static in any configuration without the need for the actuators to exert any force/torque. The advantages of these mechanisms are reductions in the time required to perform tasks and improvements in the payload capacity. They also leave open the possibility of using less powerful motors, which are normally smaller, lighter and less expensive. Gravity compensation mechanism approaches can normally be divided into counterweight balanced mechanisms and spring-balanced mechanisms.

With regard to the counterweight balanced mechanisms, there is an exchange of the gravitational potential energy between the structure of the manipulator and the counterweights, which results in constant potential energy in the case of a perfectly balanced mechanism. In simple terms, when a manipulator's link is lifted, a counterweight goes down, and vice versa, compensating the torque induced by the gravity at the robot's joints. This strategy allows the designer to easily enhance the energy efficiency and torque requirements of the system in those cases in which the manipulator moves at relatively slow speeds/accelerations. However, as long as it implies the addition of significant masses (counterweights), in the circumstances of very quick maneuvers, an important portion of the torque decreases derived from

the gravity compensation is wasted on accelerating the counterweights. Several authors have worked on the design of a gravity balanced mechanism by considering counterweights. Chung and Cho in [73] examined the effects of using an active counter-balancing mechanism to compensate the gravity loadings of the first three links of a PUMA-760 manipulator. As an additional consequence of the approach, they also observed a deterioration in the precision of the robot owing to an increase in its vibration characteristics. Another approach is worth mentioning although it does not represent a passive gravity compensation mechanism. In [74], Agrawal et al. proposed and experimentally tested an interesting approach for the active gravity balance of a planar robot, which involves two steps. In the first step, a mechanism denominated as "auxiliary parallelograms" is added to the manipulator and used to physically identify the location of the center of mass of the whole structure. In the second step, a counterweight is actively moved above the center of mass of the augmented robot in order to balance the system. This is not a very practical alternative, considering that it requires the active coordination of the movements between the manipulator and the counterweights, in addition to the construction of the auxiliary parallelogram mechanism, along with a structure with which to support the counterweights and the corresponding actuator for active gravity compensation.

In the case of the spring-balanced mechanism, the potential energy of the system is divided into two parts: the gravitational potential energy stored in the structure of the manipulator, and the elastic potential energy stored in the springs. Depending on the configuration of the robot, the potential energy is redistributed into these two parts with the equivalent effect of reducing the torque at the joints. The main benefit of using springs-based mechanism instead of counterweights-based mechanism is that the former does not significantly affect the inertia of the robotic manipulator. The design of these mechanisms is, however, normally not simple. The greatest challenge of the spring-based gravity compensation mechanism is that gravitational torque is pose-dependent and, therefore, non-linear. The most common design strategy considers linear springs organized in geometric arrangements that produce a non-linear compensation torque by altering the length of the moment arm for each pose. Important research effort has been directed toward finding spring arrangement with which to solve this problem. Sharan and Mahalingam [75] proposed one of the first mechanisms based on linear springs that reduce the gravity-induced torque in the forearm of an industrial serial robot. The industrial manipulator considered was the T3R3 model manufactured by Cincinnati Milacron. Since the spring arrangement proposed could perfectly balance the robot in only one configuration, the authors suggested an optimal solution oriented toward reducing the imbalance during the execution of a specific maneuver. Although the results significantly improved the torque demands for the maneuver, the approach proved

very impractical because the optimum spring configuration changed depending on the maneuver and, therefore, needed to be changed depending on the task in which the robot was involved. Ulrich and Kumar [76] proposed a mechanism based on wires, linear springs, and a customized pulley to compensate the gravity of serial manipulators. This method requires a very careful design of the pulleys and is applicable to multiple link manipulators with specific geometries in which all joint axes are perpendicular to the direction of the gravitational force, as in the case of the Unimation PUMA and the Cincinnati Milacron manipulators. Experiments based on this method showed compensation errors of 1 % of the maximum gravity-induced torque for a two-joint and two-link manipulator. Gopalswamy et al. [77] introduced a parallelogram linkage configuration in combination with torsional springs to compensate the gravity-induced torque in a two degree-of-freedom manipulator. In this case, even when the exact gravity compensation for all the poses of the manipulators was not achieved, it significantly reduced the joint torque in comparison with the counter-balanced parallelogram linkage used in the MIT Direct-Drive Arm II. Rahman et al. [78] proposed a very simple method that exactly compensates the gravity-induced torque for an articulated manipulator of one link or  $n$ -links connected in series with each joint with two degrees-of-freedom. The solution for the one link case consisted of only linear springs and wire ropes, whereas the extension to  $n$ -links case required considering a four-bar mechanism for each link. Morita et al. in [79] drawn the attention to the fact that the four-bar mechanism is not an appropriate design for an articulated manipulator since the range of motion is restricted by the contact between the links. Instead, an alternative mechanism, denominated as a "pseudo-parallelogram mechanism", with a greater range of motion, the same exact gravity compensation characteristic and composed of pulleys and belts was proposed. Although no experimental results were shown in the mentioned paper, the effectiveness of the method was proved in simulations for a two-link two-degree-of-freedom manipulator. Agrawal and Fattah [80] proposed a passive gravity compensation mechanism for spatial manipulators capable of balancing the manipulator regardless of the direction of the gravity vector, signifying that the base of the manipulator could be oriented in any direction with respect to the gravity. This gravity compensation was divided into two steps. The first step was to locate the center of mass of the overall structure by considering the aforementioned auxiliary parallelogram mechanism, considered in [74]. The second step consisted of applying a spring based compensation force directly to the center mass, which made the potential energy invariant with respect to the configuration of the manipulator and the orientation of its base.

One of the main benefits of flexible link manipulators over rigid manipulators is the reduction in the link masses, which directly results in lower torque requirements. However,

in the case of serial flexible link manipulators, even when the link masses are considerably lower, an important part of the torque in the motor is used to counteract the payload at the tip and the mass of the successive actuators when they are placed directly at the joints. Thus, gravity compensation mechanisms represent great potential benefits for flexible link manipulators, to further lower the motor torque requirements. There are some published research works that study the use of mechanisms based on counterweights for flexible link manipulators. In [13], Yohsikawa et al. developed a model and a control algorithm of a flexible link manipulator with three joints and two links, with a counterweight mounted at the base of the first link. In [2], Freese et al. introduced the Gryphon robot, a mobile demining robot with a robotic manipulator on board which included a gravity compensation system based on counterweights. We did not find published research dealing with the use of the spring based compensation mechanism for flexible link arm.

The main goal of the two research projects within this thesis is the development of mobile robotics platforms with flexible link robots on board (see 1.1). In this context, the potential advantages of the spring based gravity compensation mechanisms are even greater: (1) it lowers the static torque demands of the actuators of the manipulator, without the penalty of a greater dynamic torque as in the case of counterweight-based compensation; (2) the reduced torque demands allow the use of less powerful actuators, which also translates into smaller, lighter characteristics and a more affordable price; and (3) the use of less powerful actuators implies lower energy consumption, a very significant improvement considering that it is a limited resource on mobile applications.

In this chapter, we have concerned ourselves with the modeling, identification, and control of a single link flexible link manipulator with a spring-based gravity compensation mechanism. The structure of this chapter includes the following sections. The second section introduces the design and construction details of the gravity compensation mechanism. The third section is focused on the development of a dynamic model for the gravity compensated system. The fourth and fifth sections are oriented toward the identification and validation of the dynamic model, respectively. The sixth and seventh sections are devoted to the design and experimental validation of an input-state feedback linearization controller, while the last section presents the conclusions and outlines some recommendations for further research.

The results presented in this chapter have been partly published in

- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, "Modeling and identification of a single link flexible arm with a passive gravity compensation mechanism," in *Robotics and Automation (ICRA), 2018 IEEE International Conference on*, (Brisbane, Australia), IEEE, 2018

- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Feedback linearizing controller for a single link flexible arm with a passive gravity compensation mechanism,” in *Intelligent Robots and System (IROS), IEEE International Conference on*, (Madrid, Spain), IEEE, 2018
- J. C. Cambera, D. Feliu-Talegon, S. Gharab, and V. Feliu-Batlle, “Flexible link arm with a passive gravity compensation mechanism: Design and modelling,” in *XXXIX Jornadas de Automática*, (Extremadura, Spain), CEA/IFAC, 2018

## 7.2 Design of the gravity compensation mechanism

### 7.2.1 Gravity compensation of a single link rigid arm - preamble

The gravity compensation of a single-link rigid arm was considered as the starting point for the design of the compensation mechanism developed in this chapter. The simplest configuration with which to achieve this considers only one spring installed as presented in Fig 7.1.

The rigid link arm is considered to be perfectly balanced when it remains static in any configuration without requiring any torque from the actuator. This situation can be mathematically expressed in terms of the sum of all the torques about the motor axis,  $\sum \Gamma_{motor-axis}$ , as follows

$$\begin{aligned} \sum \Gamma_{motor-axis} = 0 = & \Gamma_m^0 + k_{spring} \left( 1 + \frac{(l_s - l_1)}{\sqrt{l_s^2 + l_1^2 - 2 l_s l_1 \sin(\theta_m)}} \right) l_s l_1 \cos(\theta_m) \\ & - \left( \frac{m_{link}}{2} + m_{tip} \right) g l \cos(\theta_m) \end{aligned} \quad (7.1)$$

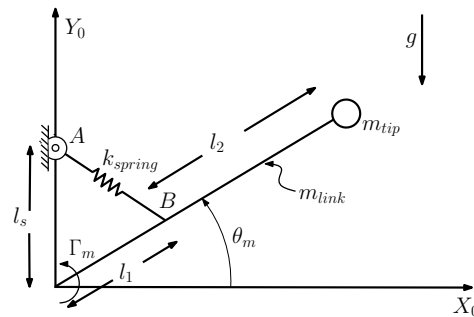


Fig. 7.1 Experimental setup: diagram of a gravity compensation mechanism for 1 link rigid arm.

where  $k_{spring}$  represents the spring constant;  $l$  defines the total distance of the link;  $l_s$  and  $l_1$  are the distances between the motor rotation axis (the origin) and the attachment points of the spring along the axis  $Y_0$  and along the link, respectively;  $g$  is the gravitational constant, and  $m_{link}$  and  $m_{tip}$  denote the masses of the link and the payload, respectively. Two conditions are derived from this equation. The first condition defines the spring constant in terms of the other parameters of the system

$$k_{spring} = \frac{(\frac{m_{link}}{2} + m_{tip}) g l}{l_s^2} \quad (7.2)$$

The second condition will be referred to as the "zero free length condition", and states that the attachment points of the spring (points A and B) must be at the same distance with respect to the motor axis, such that  $l_s = l_1$ . This condition implies that the length of the spring should be zero when the link is vertically oriented and this is, therefore, as is evident from the properties of the springs, not physically possible. However, it is possible to circumvent this problem by arranging the spring outside the line A-B. Two building alternatives have been introduced in [79], and are presented in Fig 7.2.

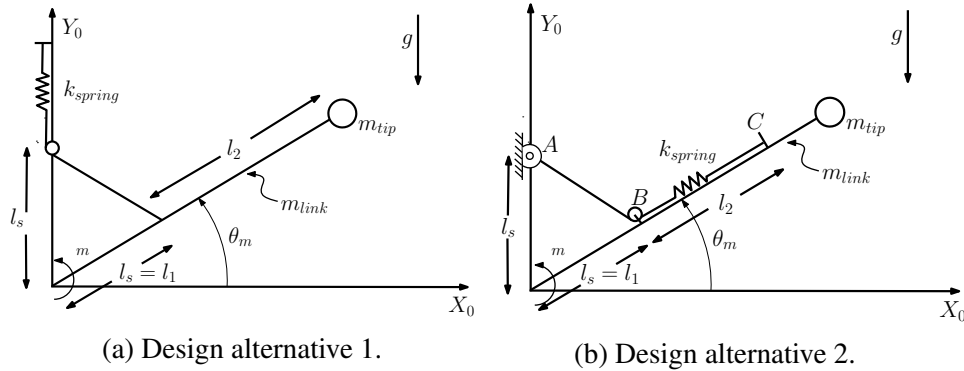


Fig. 7.2 Experimental setup: diagram of construction alternatives for the gravity compensation mechanism for single-link rigid arm.

### 7.2.2 Description of the gravity compensation mechanism

The gravity compensation mechanism we propose corresponds to the implementation of the first alternative presented in Fig 7.2 in the case of a single link flexible arm. The first design alternative was selected rather than the second for following reasons: (1) ease of construction, (2) to prevent the introduction of some additional and unnecessary bending of the link owing to the tangential force applied to the point C by means of the rope. The bending could be very significant if we take into consideration the fact that point C is far from the

base of the link; however this could be corrected by considering a duplicated compensation mechanism, i.e., two identical springs rather than one, both arranged symmetrically around the flexible link, and (3) to avoid the mass of the spring significantly affecting the dynamic of the system. The spring mass is not very significant. However, in our case, it is still comparable to the flexible link mass and will, therefore, need to be included in the model as a moving mass whose position depends on the orientation of the link. It adds unnecessary complexity to the model, without providing a relevant contribution to the overall system. Fig 7.3 shows a photograph of the single link flexible arm platform with the gravity compensation mechanism installed. The main characteristics of the gravity compensation mechanism are detailed below.



Fig. 7.3 Experimental setup: photograph of the single link flexible robot with the passive gravity compensation system.

Fig. 7.4 shows a CAD view of the overall platform with the gravity compensation mechanism. The interrelation between the parts of the gravity compensation mechanism is as follows. The two support rods (part 1) are vertically arranged and fixed to the motor support and on both sides of the flexible link. In the upper part of the rods, the springs are hooked onto two fixed supports (part 2). The wire ropes (part 3) are attached to the lower part of the springs and are maintained parallel to the rods until they cross the wire rope guides (part 4). The wire rope guides redirect the wire ropes toward the link, where they are attached to the

link by means of the attachment connector (part 5). Fig. 7.4 presents a general overview of the of the overall system.

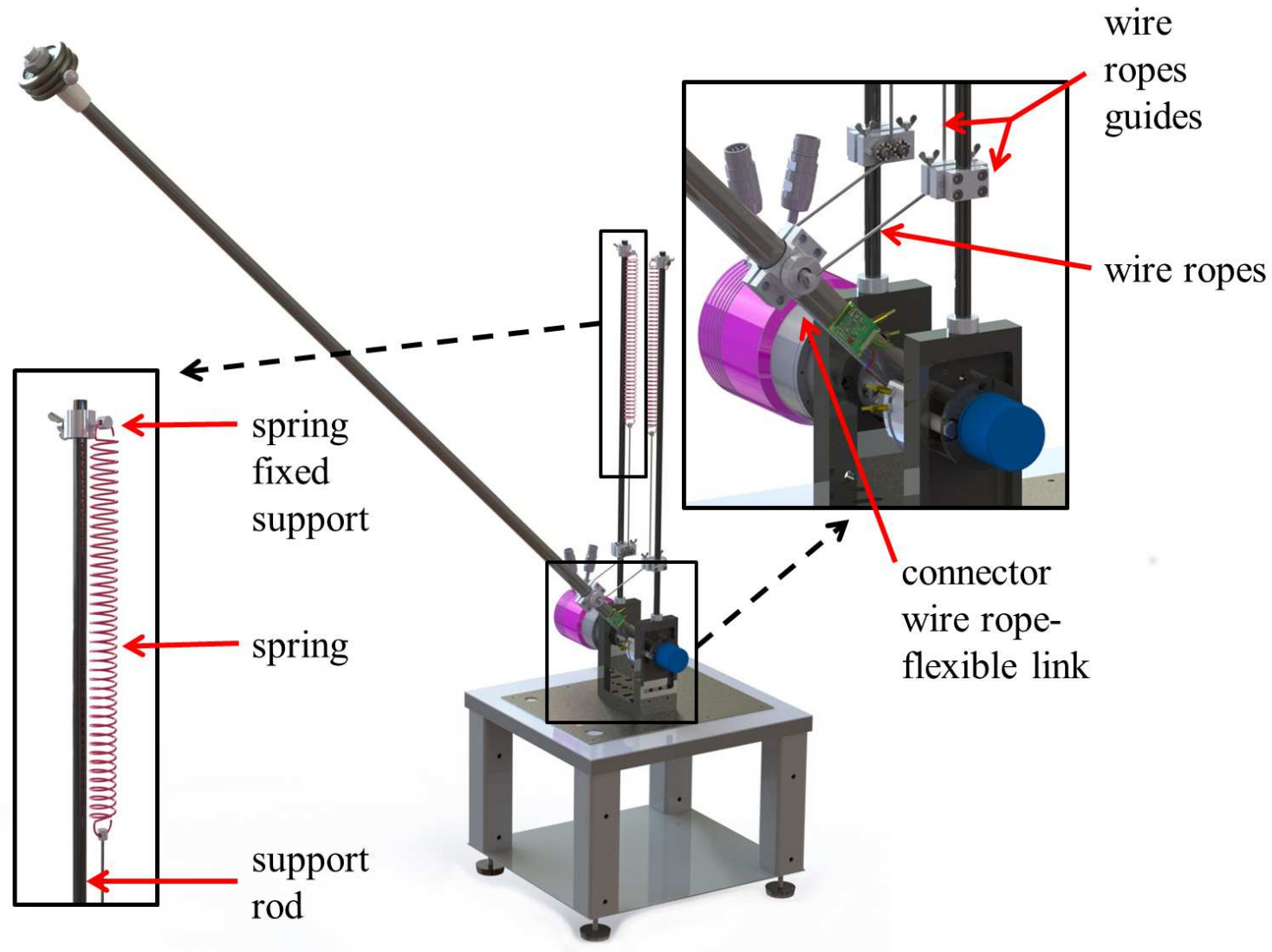


Fig. 7.4 Experimental setup: CAD representation of the 1-DOF flexible link arm with the passive gravity compensation system (perspective view).

The upper support of each spring (part 2) consists of a ring with two screws, as shown in Fig 7.6. The first screw (the grub screw) is used to fix the piece to the rod and adjust the height at which the upper part of the spring is set. The second screw, together with the two nuts, act as a supporting surface for the spring hook and ensure that the spring hook does not slip and keep it at a safe distance from the rods and the flexible link.

The wire rope guides (part 4), detailed in Fig 7.7, consist of two rollers located side by side that change the direction of the wire ropes with a minimum amount of friction and simultaneously maintain the wire rope trapped during any maneuver carried out by the robot. As an additional adjustment possibility, the two butterfly screws allow the distance between the wire rope and the supporting rod to be configured.

The attachment connector (part 5), described in Fig 7.8, permits the connection between the wire rope and the flexible link without having to drill the flexible link. In addition, it enables the alignment of the wire rope with the lower part of the wire rope guides. Drilling the composite link may cause the material to delaminate and, therefore, generate undesired changes in the mechanical properties of the link. The alignment of the wire rope at the attachment point avoids the accumulation of unnecessary tension in the springs that could impair the correct compensation. The attachment connector consists of a clamping collar that connects it to the flexible link, and two shafts that rotate freely and permit the automatic alignment of the wire rope.

In an ideal case, the perfect gravity compensation of the single-link flexible arm is not attainable with the mechanical designed proposed, mainly owing to the existence of the elastic deformations in the arm. The deformations in the arm alter the location of the payload and center of mass of the link with regard to the case of the rigid link, and ultimately affect the torque exerted on the motor axis. However, since the flexible links studied in this thesis have only small deflections, the location of the corresponding centers of mass do not vary significantly, and the gravity compensation mechanism for the rigid arm produces a near perfect gravity compensation. To make the construction of the prototype of the mechanism possible, some non-ideal design decisions were assumed. Despite the effort made to arrange the spring outside the line A-B in Fig. 7.1, as indicated in Fig 7.2a, the mechanical design of the gravity compensation mechanism did not achieve the equivalent "zero free length condition" mentioned in the previous section. The reason for this was that, in order to avoid the collision between the wire rope guides (part 4) and the attachment connector (part 5), a small distance was maintained between these two parts. An additional non-ideal construction characteristic was the fact that the rollers in the wire rope guides (piece 4) have non-negligible diameters and, therefore, slightly deform the path of the wire rope. This

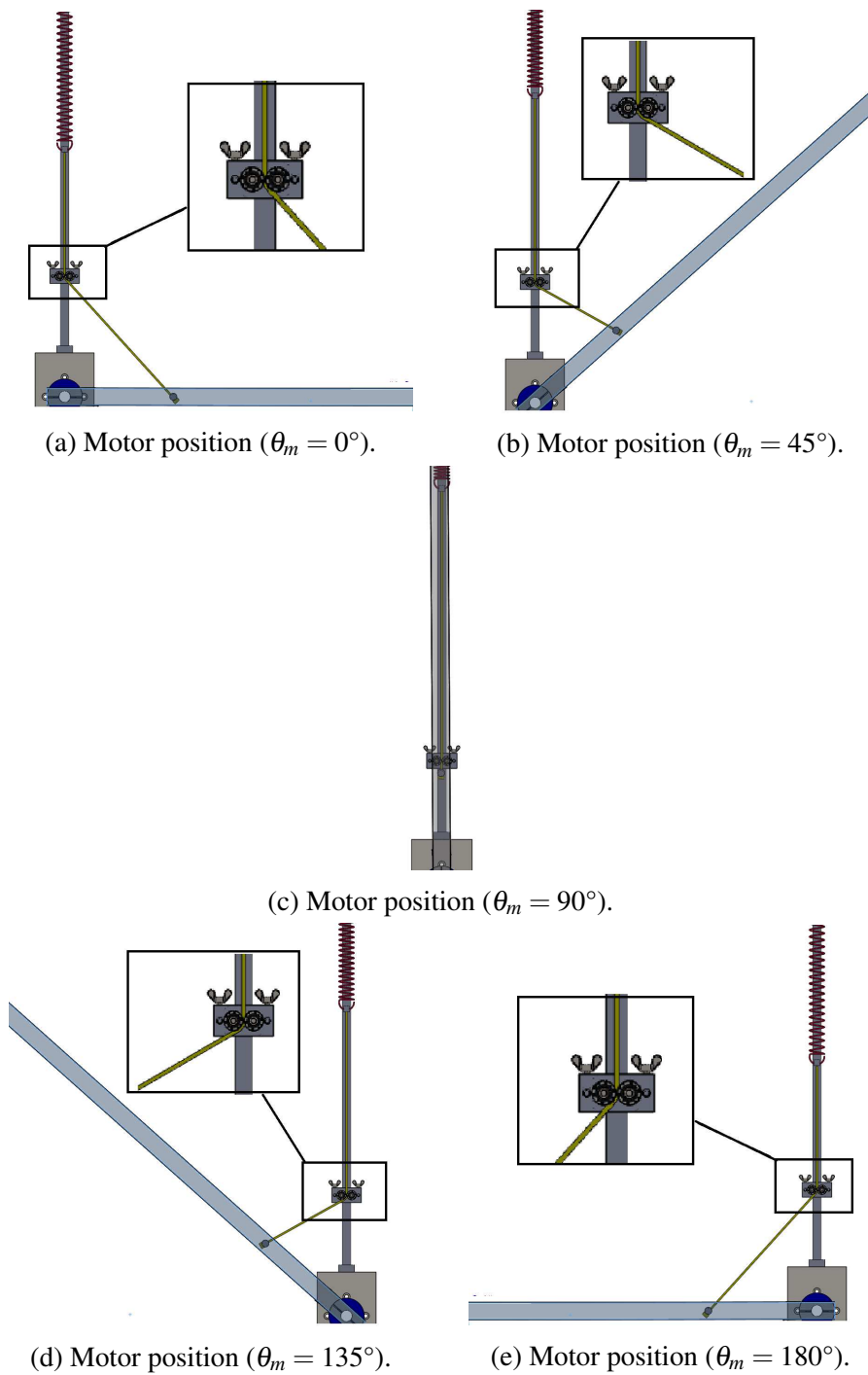


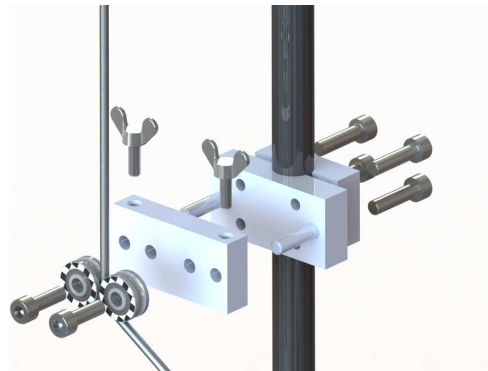
Fig. 7.5 Experimental setup: CAD representation of the 1-DOF flexible link arm with the passive gravity compensation system (cross section view).



Fig. 7.6 Experimental setup: photograph of the spring upper support.

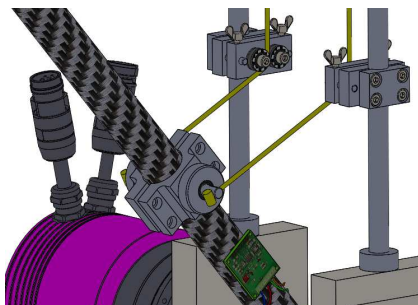


(a) Photograph.



(b) CAD exploded view.

Fig. 7.7 Experimental setup: wire rope guide.



(a) Photograph.



(b) CAD exploded view.

Fig. 7.8 Experimental setup: connector between the wire rope and the flexible link.

affects the accuracy of the compensation mechanism, especially in the case of configurations in which the link is near to the vertical orientation.

### 7.3 Dynamic model

The dynamic model is obtained in this section by following a similar procedure to that shown in Section 3.1.2. Fig 7.9 shows an equivalent diagram representing the single link flexible arm with the gravity compensation system. In order to simplify the analysis, in this figure the spring has been directly considered in the path between the fixed support and the point at which it is attached to the link. It is important to mention that in the real prototype, the spring has been taken to an equivalent, more convenient location similar to the one presented in the design alternative shown Fig 7.2a. In addition to the change of location, in the real prototype the spring was splitted into two. More details about these construction considerations can be found in the previous section.

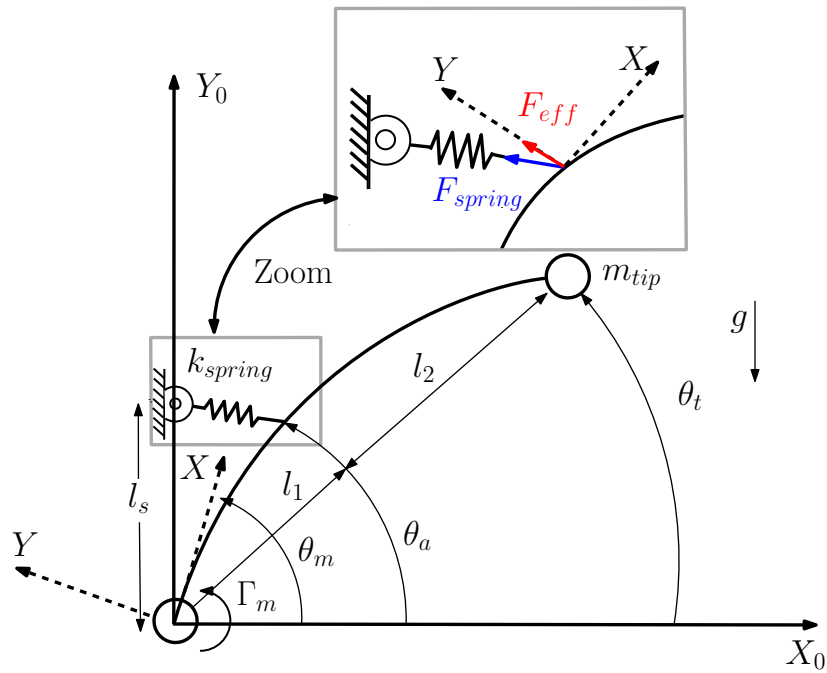


Fig. 7.9 Dynamic modeling: single link flexible robot with passive gravity compensation mechanism described by single lumped mass model.

The existence of an external force applied to an intermediate point of the flexible link imposes the necessity of considering two flexible segments although only one lumped mass is considered. The lengths of these segments are  $l_1$  and  $l_2$ , s that  $l_1 + l_2 = l$ . The deflections along these segments can be expressed as

$$y_1(x) = u_{1,0} + u_{1,1} \cdot (x) + u_{1,2} \cdot (x)^2 + u_{1,3} \cdot (x)^3 \quad 0 \leq x \leq l_1, \quad (7.3)$$

$$y_2(x) = u_{2,0} + u_{2,1} \cdot (x - l_1) + u_{2,2} \cdot (x - l_1)^2 + u_{2,3} \cdot (x - l_1)^3 \quad l_1 \leq x \leq l_1 + l_2, \quad (7.4)$$

The polynomial coefficients  $u_{i,j}$  are not necessarily constant in time and define the dynamical behavior of the flexible link. Upon defining  $\theta_a$  and  $\theta_t$  as the angular position of the end of the first and second segments, respectively, the following relationships can be established

$$y_1(l_1) = u_{1,0} + u_{1,1} \cdot l_1 + u_{1,2} \cdot l_1^2 + u_{1,3} \cdot l_1^3 = l_1(\theta_a - \theta_m), \quad (7.5)$$

$$y_2(l_2) = u_{2,0} + u_{2,1} \cdot l_2 + u_{2,2} \cdot l_2^2 + u_{2,3} \cdot l_2^3 = l_2(\theta_t - \theta_m). \quad (7.6)$$

The values of the polynomial coefficients  $u_{i,j}$  are obtained from the boundary conditions of the problems and from the dynamic equations resulting from applying the Newton's second law. The boundary conditions between the motor shaft and the first segment of the beam are

$$y_1(0) = 0 \implies u_{1,0} = 0, \quad (7.7)$$

$$\frac{dy_1(0)}{dx} = 0 \implies u_{1,1} = 0. \quad (7.8)$$

The deflection, along with its first and its second derivatives, are continuous in the boundary between the first and the second segment, which implies that

$$y_1(l_1) = y_2(0) \implies u_{1,0} + u_{1,1} l_1 + u_{1,2} l_1^2 + u_{1,3} l_1^3 = u_{2,0} \quad (7.9)$$

$$\frac{dy_1(l_1)}{dx} = \frac{dy_2(0)}{dx} \implies u_{1,1} + 2 u_{1,2} l_1 + 3 u_{1,3} l_1^2 = u_{2,1} \quad (7.10)$$

$$\frac{d^2 y_1(l_1)}{dx^2} = \frac{d^2 y_2(0)}{dx^2} \implies u_{1,2} + 3 u_{1,3} l_1 = u_{2,2} \quad (7.11)$$

The torque along the link ( $\Gamma(x)$ ) is related to the second derivative of the deflections. Since no external torque is applied to the link at the tip, the following condition is met

$$\Gamma(l_2) = EI_z \frac{d^2 y_2(l_2)}{dx^2} = 0 \implies 2u_{2,2} + 6u_{2,3}l_2 = 0. \quad (7.12)$$

The torque exerted by the motor on the base of the link, denominated as the coupling torque ( $\Gamma_{coup}$ ), is defined as

$$\Gamma_{coup} = -\Gamma(0) = -EI_z \frac{d^2 y_1(0)}{dx^2} = -2 EI_z u_{1,2}. \quad (7.13)$$

In a similar manner, the torque at the point of attachment ( $\Gamma_a$ ) is defined as

$$\Gamma_a = -\Gamma(l_1) = -EI_z \frac{d^2 y_2(0)}{dx^2} = -2 EI_z u_{2,2}. \quad (7.14)$$

Upon applying the Newton's second law, we obtain the following torque relations

$$\Gamma(0) = -m_{tip} (l_1 + l_2)^2 \ddot{\theta}_t - m_{tip} (l_1 + l_2) g \cos(\theta_t) + F_{eff} l_1, \quad (7.15)$$

$$\Gamma(l_1) = -m_{tip} (l_1 + l_2) l_2 \ddot{\theta}_t - m_{tip} l_2 g \cos(\theta_t), \quad (7.16)$$

where the variable  $F_{eff}$  defines the component of the spring force,  $F_{spring}$ , which affects the dynamics of the flexible arm. As in the previous dynamics models (see Sections 3.1.4 and 3.1.6), the effect of the distributed mass can be approximated by including some additional fictitious masses in the equations resulting from Newton's second law. Since the distributed mass affects the terms owing to force of gravity and the terms related to the acceleration of the system differently, we propose the addition of four fictitious masses ( $m_{link-a1}$ ,  $m_{link-a2}$ ,  $m_{link-g1}$ ,  $m_{link-g2}$ )

$$\Gamma(0) = -m_{a1} (l_1 + l_2)^2 \ddot{\theta}_t - m_{g1} (l_1 + l_2) g \cos(\theta_t) + F_{eff} l_1, \quad (7.17)$$

$$\Gamma(l_1) = -m_{a2} (l_1 + l_2) l_2 \ddot{\theta}_t - m_{g2} l_2 g \cos(\theta_t), \quad (7.18)$$

where  $m_{a1} = m_{tip} + m_{link-a1}$ ,  $m_{g1} = m_{tip} + m_{link-g1}$ ,  $m_{a2} = m_{tip} + m_{link-a2}$  and  $m_{g2} = m_{tip} + m_{link-g2}$ . Let us now define  $\Gamma_{pt-attach}$  as the torque at the connection point between the flexible link and the spring. This torque and the coupling torque ( $\Gamma_{coup}$ ) can be measured by means of the strain gauges in the following manner

$$\Gamma_{coup} \equiv -\Gamma(0) = m_{a1} (l_1 + l_2)^2 \ddot{\theta}_t + m_{g1} (l_1 + l_2) g \cos(\theta_t) - F_{eff} l_1 \quad (7.19)$$

$$\Gamma_{pt-attach} \equiv -\Gamma(l_1) = m_{a2} (l_1 + l_2) l_2 \ddot{\theta}_t + m_{g2} l_2 g \cos(\theta_t) \quad (7.20)$$

These torques are related to the voltage measurements at the output of the strain gauge amplifiers in the following form

$$\Gamma_{coup} = k_{gauge-base} v_{gauge-base} \quad (7.21)$$

$$\Gamma_{pt-attach} = k_{gauge-attach} v_{gauge-attach} \quad (7.22)$$

The spring force vector is defined as

$$\vec{F}_{spring} = k_{spring} (-l_1 \cos(\theta_a), l_s - l_1 \sin(\theta_a)). \quad (7.23)$$

The effective force  $F_{eff}$  acting on the flexible link dynamics is the projection  $\vec{F}_{spring}$  over a unitary vector in the direction of  $\mathbf{Y}$ ,  $\hat{v}_\perp = (-\sin(\theta_m), \cos(\theta_m))$ . The effective force can be written as

$$F_{eff} = k_{spring} (l_1 \sin(\theta_m - \theta_a) + l_s \cos(\theta_m)). \quad (7.24)$$

Since the small deflection hypothesis is assumed, the term  $\sin(\theta_m - \theta_a) \approx (\theta_m - \theta_a)$ , and the following approximation can be made

$$F_{eff} \approx k_{spring} (l_1 (\theta_m - \theta_a) + l_s \cos(\theta_m)). \quad (7.25)$$

The following steps summarize the methodology used to obtain the dynamic model of the system and the relationship between the sensor measurements and the variables relevant for the control algorithm. For the sake of simplifying the notation of the resulting model, the distance  $l_1$  will be defined as  $l_1 \equiv \lambda l$ .

**Step 1:** Solve  $u_{1,2}$  and  $u_{2,2}$ , and eliminate  $u_{1,0}$ ,  $u_{1,1}$ ,  $u_{1,3}$ ,  $u_{2,0}$ ,  $u_{2,1}$  and  $u_{2,3}$  from the equations of the deflection, (equations (7.5) and (7.6)), and the boundary conditions (equations (7.7)-(7.11) and (7.12)). Replace the values found for  $u_{1,2}$  and  $u_{2,2}$  into equations (7.13) and (7.14), and substitute the result into the expressions for  $\Gamma_{coup}$  and  $\Gamma_{pt-attach}$  (equations (7.19) and (7.20)). The resulting expressions are

$$\Gamma_{coup} = \frac{6 E I_z}{l \lambda (\lambda^2 - 5 \lambda + 4)} \left( \theta_m (2 - 2 \lambda) + \theta_a (\lambda - 2) + \lambda \theta_t \right), \quad (7.26)$$

$$\Gamma_{pt-attach} = \frac{6 E I_z}{l (\lambda^2 - 5 \lambda + 4)} \left( \theta_m (\lambda - 1) + \theta_a (3 - \lambda) - 2 \theta_t \right). \quad (7.27)$$

**Step 2:** Replace the expressions from Step 1 into the equations that resulted from applying Newton's second law (equations 7.19 and 7.20). Solve these equations for the variables  $\theta_a$  and  $\ddot{\theta}_t$ .

**Step 3:** Consider the expression for the  $\theta_a$  of the previous step, and substitute it into equation (7.25) to obtain the effective force  $F_{eff}$  in terms of the state variable  $\theta_t$  and the motor position  $\theta_m$ .

**Step 4:** Rearrange the expression for  $\ddot{\theta}_t$  in Step 2 to obtain the equation describing the dynamics of the system in terms of the input of the system  $\theta_m$  and the effective force applied by the spring to the link  $F_{eff}$ . This equation is presented below.

$$\gamma_1 \ddot{\theta}_t + \gamma_2 \cos(\theta_t) + l \left( \frac{\lambda^3 - 3\lambda^2}{2} \right) F_{eff} + \frac{3 E I_z}{l} (\theta_t - \theta_m) = 0, \quad (7.28)$$

where

$$\gamma_1 = l^2 (m_{tip} + m_{link-a1} \left( \frac{3\lambda - \lambda^2}{2} \right) + m_{link-a2} \left( \frac{2 - 3\lambda + \lambda^2}{2} \right)), \quad (7.29)$$

$$\gamma_2 = l g (m_{tip} + m_{link-g1} \left( \frac{3\lambda - \lambda^2}{2} \right) + m_{link-g2} \left( \frac{2 - 3\lambda + \lambda^2}{2} \right)). \quad (7.30)$$

**Step 5:** In order to make this dynamic model useful for a control algorithm design, it is necessary to estimate the values of the variables  $\theta_a$ ,  $\theta_t$  and  $F_{eff}$  from the measurements available  $\Gamma_{coup}$ ,  $\Gamma_{pt-attach}$  and  $\theta_m$ . The estimations  $\hat{\theta}_a$  and  $\hat{\theta}_t$  are obtained from equations (7.26) and (7.27).

$$\hat{\theta}_a = \theta_m - \frac{l \lambda}{6 E I_z} (\Gamma_{pt-attach} + 2 \Gamma_{coup}), \quad (7.31)$$

$$\hat{\theta}_t = \theta_m + \frac{l}{6 E I_z} \left( (\lambda - 2) \Gamma_{pt-attach} + (-3\lambda + \lambda^2) \Gamma_{coup} \right). \quad (7.32)$$

The estimation  $\hat{F}_{eff}$  can be obtained in two ways. First, an estimation that does not depend on the spring model is possible by eliminating the variable  $\ddot{\theta}_t$  from equations (7.19) and (7.20). This results in

$$\hat{F}_{eff} = \rho_1 \cos(\hat{\theta}_t) - \frac{1}{l_1} \Gamma_{coup} + \rho_2 \Gamma_{pt-attach}, \quad (7.33)$$

where  $\rho_1$  and  $\rho_2$  are

$$\rho_1 = \frac{m_{tip} g}{\lambda (m_{tip} + m_{link-a2})} (m_{link-a2} - m_{link-a1} - m_{link-g2} + m_{link-g1}) + \frac{g}{\lambda (m_{tip} + m_{link-a2})} (m_{link-a2} m_{link-g1} - m_{link-a1} m_{link-g2}), \quad (7.34)$$

$$\rho_2 = \frac{(m_{tip} + m_{link-a1})}{l (1 - \lambda) (\lambda) (m_{tip} + m_{link-a2})}. \quad (7.35)$$

The second way in which to estimate the effective force is obtained from the model of the spring (equation (7.25)) as follows;

$$\hat{F}_{eff} = k_{spring} (\lambda l (\theta_m - \hat{\theta}_a) + l_s \cos(\theta_m)). \quad (7.36)$$

## 7.4 Identification of the dynamic model

In this section, we focus on the identification of the composite flexible link with the gravity compensation mechanism. It is assumed that the actuator has already been identified and controlled in position. Refer to Sections 3.2.1 and 4.2, respectively, for a description of the procedure followed to identify the actuator and design the position controller. The identification procedure in which we are interested in this stage was divided into four stages. Each stage considers experiments in which the flexible link holds the payload masses  $m_{tip-c21}$ ,  $m_{tip-c22}$  and  $m_{tip-c23}$ . The resulting parameters in each stage are normally used in the subsequent stage.

**First stage:** In this stage, the strain gauge constants ( $k_{gauge-base}$  and  $k_{gauge-attach}$ ) and the equivalent gravitational link masses ( $m_{link-g1}$  and  $m_{link-g2}$ ) were identified in the system with no gravity compensation mechanism. The experiments considered the system in static conditions for different poses (motor positions between 0 and 180 degrees). The measurements considered were the tip position ( $\theta_t$ ), obtained by the motion capture cameras, and the strain gauge measurements ( $V_{g1}$  and  $V_{g2}$ ). The parameters were identified by means of linear regression of the following expressions

$$\Gamma_{coup} = k_{gauge-base} v_{gauge-base} = (m_{link-g1} + m_{tip}) l \cos(\theta_t), \quad (7.37)$$

$$\Gamma_{pt-attach} = k_{gauge-attach} v_{gauge-attach} = (m_{link-g2} + m_{tip}) l (1 - \lambda) \cos(\theta_t). \quad (7.38)$$

These equations are the result of combining equations (7.19-7.21) and ignoring the terms corresponding to the tip acceleration ( $\ddot{\theta}_{tip}$ ) and the effective force ( $F_{eff}$ ).

**Second stage:** In this stage, we identified the flexural rigidity ( $EI_z$ ) and the equivalent inertial link masses ( $m_{link-a1}$  and  $m_{link-a2}$ ) from the system with the gravity compensation mechanism installed. The data for the identification was obtained from the Frequency Response Function (FRF) between the motor position ( $\theta_m$ ) and the coupling torque ( $\Gamma_{coup}$ ) around an intermediate operating point ( $\theta_{m0} = 45$  degrees). The frequency response was obtained by commanding the motor to track a chirp signal of an amplitude of 1 degree with frequency variations ranging from 0 to 60 Hz. In order to prevent the static friction from interfering with the tracking of the very small motor trajectories, the excitation signal was added to a sinusoidal signal with an amplitude of 5 degrees and a frequency of 0.1 Hz. The identification procedure consisted of using non-linear regression to obtain the parameters that permit the model to match the first vibrational frequency in each case. The expression for the vibrational frequencies was obtained by linearizing the model of the flexible link (without the motor) around the aforementioned operating point. The frequency adjustment matched the experimental frequencies with a relative error of below 1%. As a final comment on this stage, it is important to mention that similar results were obtained by repeating the steps described in this stage for any other operating point and without the gravity compensation mechanism, since, at least in the case of our experimental setup, the vibrational frequencies do not notably change. It is possible to hypothesize that these conditions are less likely to occur in the case of spring with a very significant inertia, in which a more complex model of the spring would be required.

**Third stage:** In this stage, we identified the equivalent spring constant ( $K_{spring}$ ) and fine-tuned the value for the distance  $l_s$  (see Fig. 7.9). The experiments considered the system in static conditions for different poses, as in the first stage. The value of the distance  $l_s$  is approximately known, since it can be directly measured from the platform. However, it was fine-tuned to compensate for the non-ideal construction characteristics of the compensation mechanism. On the real platform, the attachment to the flexible link and the guiding of the wire rope is achieved by means of mechanical pieces of non-negligible dimensions, which contrasts with the ideal point of attachments and the ideal pulleys represented in Fig. 7.2a. In this respect, in order to avoid the situation of the dimensions of the two rollers of the mechanical part "wire rope guide" significantly affecting the identification of the  $K_{spring}$ , we based the identification on configurations of the flexible link in which the motor position was

far from the vertical position of the arm, i.e., between 140 and 180 degrees. The parameters were computed by means of linear regression of the following expression

$$\Gamma_{coup} = (m_{link-g1} + m_{tip}) l \cos(\theta_t) - (k_{spring}) (\lambda^2 l^2 \sin(\theta_m - \theta_a) - (k_{spring} l_s) \lambda l l_s \cos(\theta_m)), \quad (7.39)$$

which was obtained by substituting the  $F_{eff}$  from equation (7.24) in equation (7.19), and ignoring the term of the acceleration of the tip position. The angular positions  $\theta_a$  and  $\theta_t$  were estimated from equations (7.31) and (7.32).

Fig. 7.10 presents a comparison among the torque measurements obtained from the strain gauges and estimated from the spring model. Fig. 7.11 depicts the relative error between the signal measurements and the estimation. In these figures, the results corresponding to the parameters identified up to this stage are indicated as "Based on the spring model (initial)". The figures present the comparison for the whole quadrant ( $\theta_m$  between 90 to 180 degrees), even when the identification in this stage was carried out considering a reduced part of the work space (between 140 and 180 degrees). With regard to the coupling torque measurements, it is worth mentioning that the compensation mechanism very significantly reduces the gravity torque exerted at the motor axis (the coupling torque). In the case of payload mass  $m_{tip-c22}$ , for which the gravity compensation mechanism was explicitly configured, the coupling torque was maintained below . This is a very small torque if we keep in mind that the composite link is 2 meters long and holds the payload  $m_{tip-c22} = 2.13$  Kg. . In relation to the comparison between the coupling torque measurements and the estimations, there are some differences, especially in the case of those that tend toward verticality. These differences are apparently significant. However, it should be noted that the magnitudes under consideration are quite small. The fact that the the biggest errors take place in configurations near to the vertical orientation reflects the effects that the dimensions of the two rollers of the "wire rope guide" part have on the gravity compensation, and on the precision of the estimation when they are taken into account in the model.

Figs 7.12 and 7.13 show the fitting between the attachment torque measured from the strain gauges and that estimated from the spring model. In this case, the fitting of the signal is highly consistent with a maximum relative error, which is about 4 %.

There is an alternative way in which to study the effect of the non-ideal construction characteristics of the gravity compensation mechanism that consists of reformulating equation (7.36) in terms of an auxiliary variable  $Def f$ , hereafter denominated as the effective distance. Let us define  $Def f = \lambda l (\theta_m - \hat{\theta}_a) + l_s \cos(\theta_m)$ , such that its substitution into equation (7.36) yields the following expression:  $F_{eff} = k_{spring} Def f$ . Taking into consid-

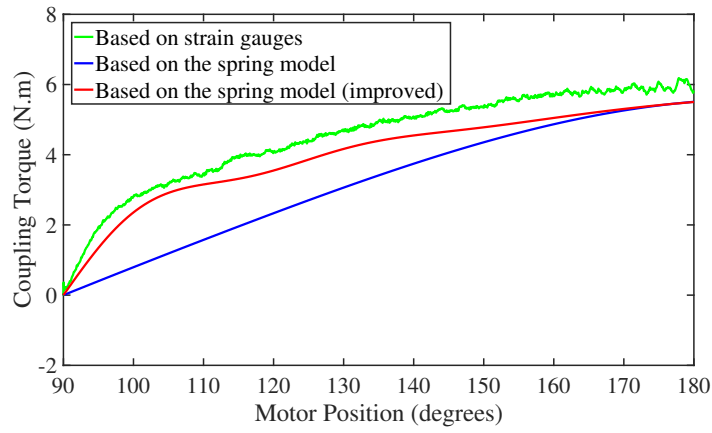
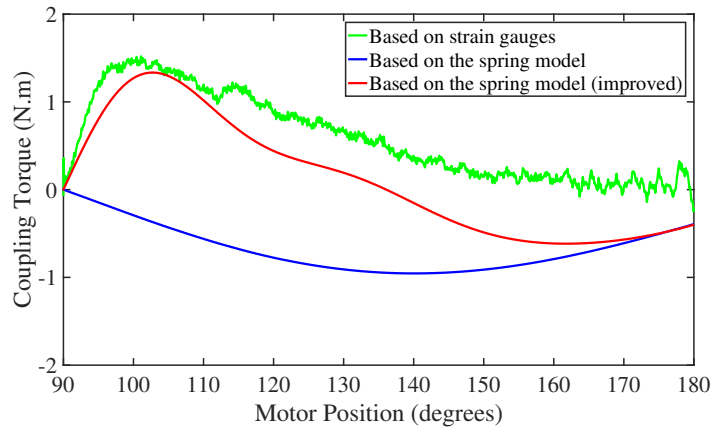
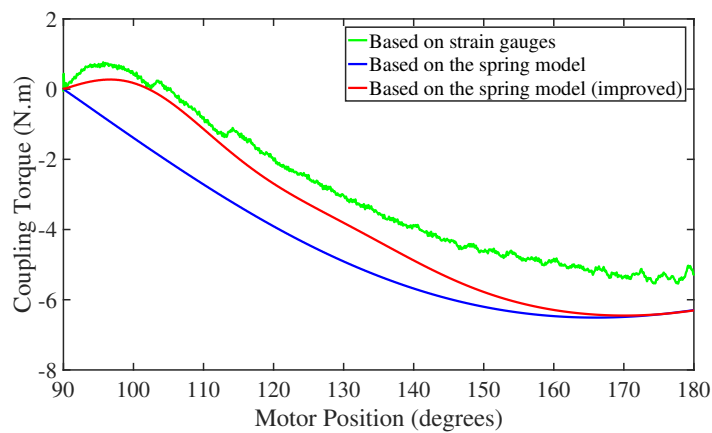
(a) Payload mass  $m_{tip-c21}$ .(b) Payload mass  $m_{tip-c22}$ .(c) Payload mass  $m_{tip-c23}$ .

Fig. 7.10 Identification - composite link: fitting of the coupling torque for the static situation.

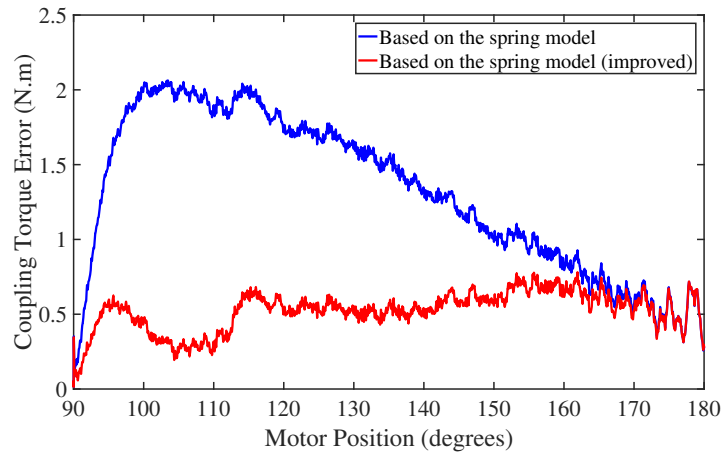
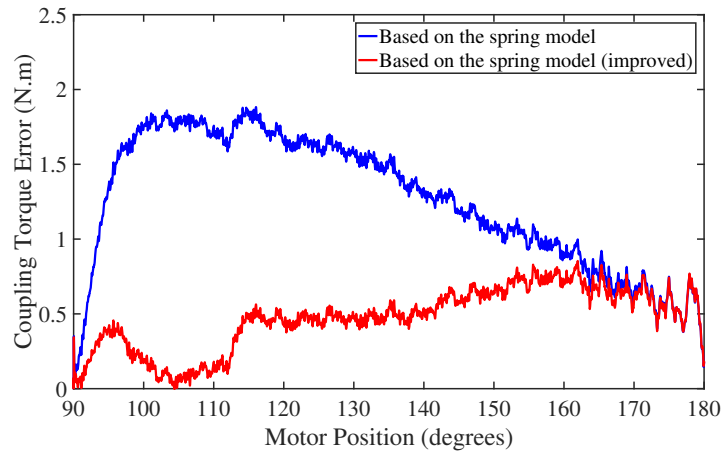
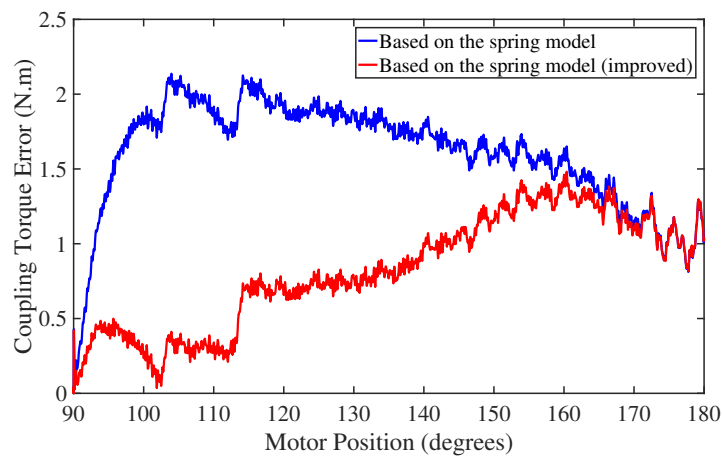
(a) Payload mass  $m_{tip-c21}$ .(b) Payload mass  $m_{tip-c22}$ .(c) Payload mass  $m_{tip-c23}$ .

Fig. 7.11 Identification - composite link: relative error of the fitting of the coupling torque for the static situation.

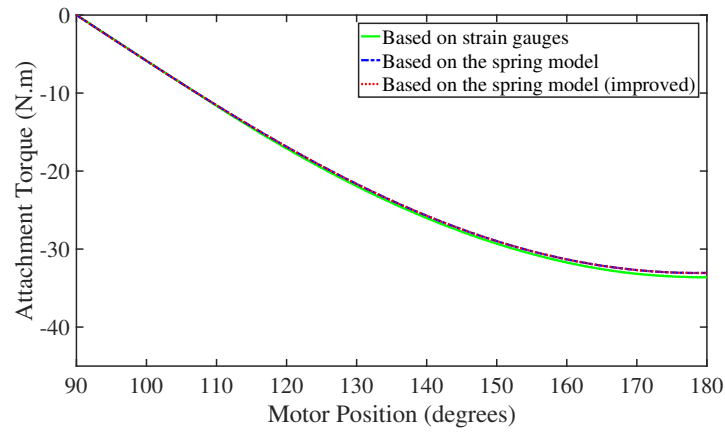
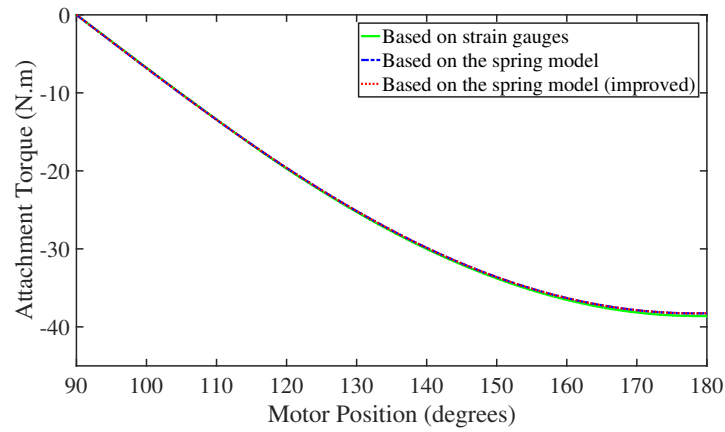
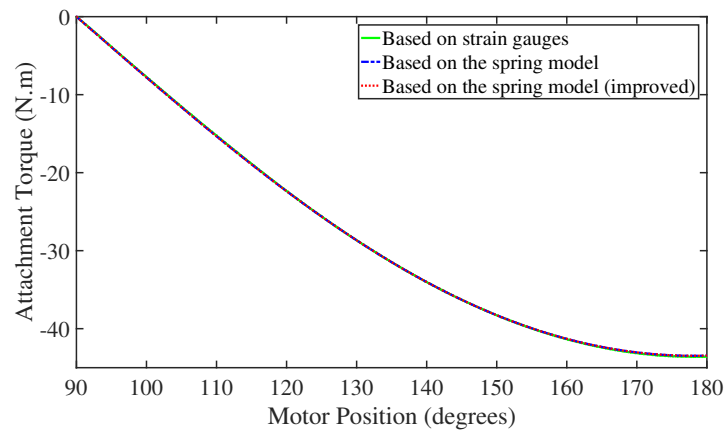
(a) Payload mass  $m_{ip-c21}$ .(b) Payload mass  $m_{ip-c22}$ .(c) Payload mass  $m_{ip-c23}$ .

Fig. 7.12 Identification - composite link: fitting of the torque at the attachment point for the static situation.

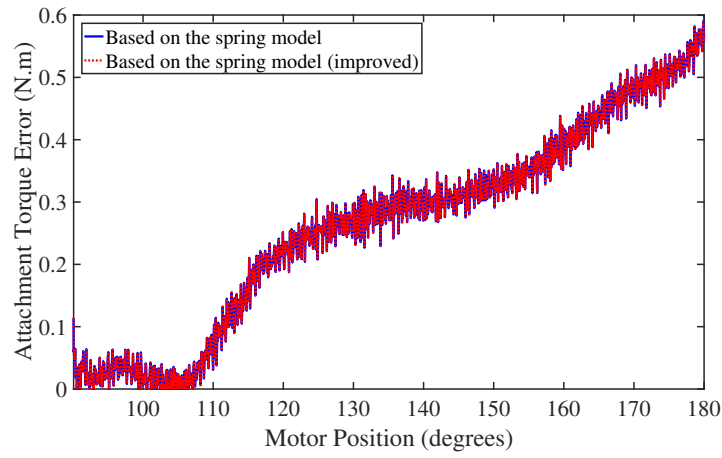
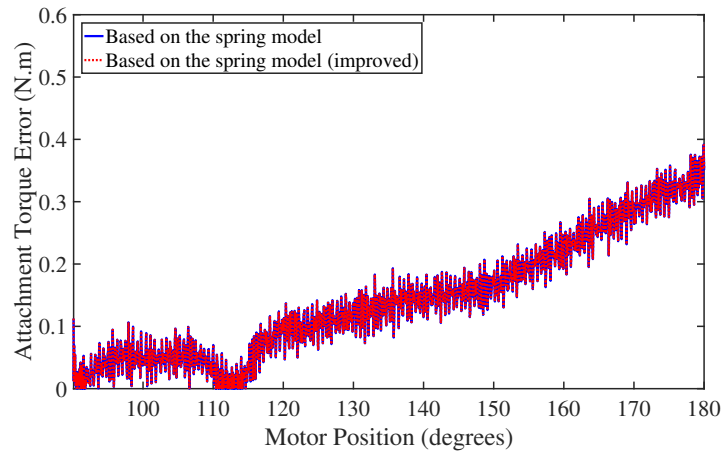
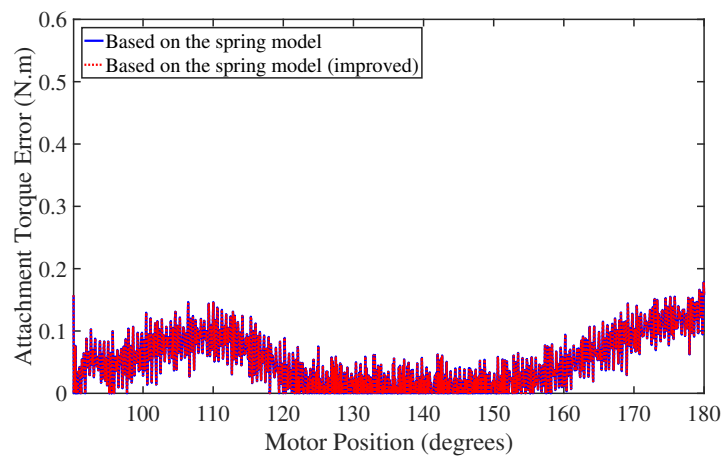
(a) Payload mass  $m_{tip-c21}$ .(b) Payload mass  $m_{tip-c22}$ .(c) Payload mass  $m_{tip-c23}$ .

Fig. 7.13 Identification - composite link: relative error of the fitting of the attachment torque for the static situation.

eration that the effective force can be estimated from the sensor measurements from equation (7.33), the idea consists of plotting the effective force versus the effective distance and observing how much the resulting plot differs from the perfectly straight line crossing the origin that could be expected from the spring model (equation (7.36)). Fig 7.14 presents this relationship: it shows that the relationship has no significant variation with respect to the payload mass considered ( $m_{tip-c21}$ ,  $m_{tip-c22}$  or  $m_{tip-c23}$ ); it can also be observed that although the spring model identified in this stage, denominated as the "Spring model (initial)", closely predicts the effective force in terms of the effective distance, there are some visible errors in the situations in which the effective distances are small, i.e., the arm is near the vertical orientation. In the following stage, we present an alternative spring model with which to obtain a better description of the effective force.

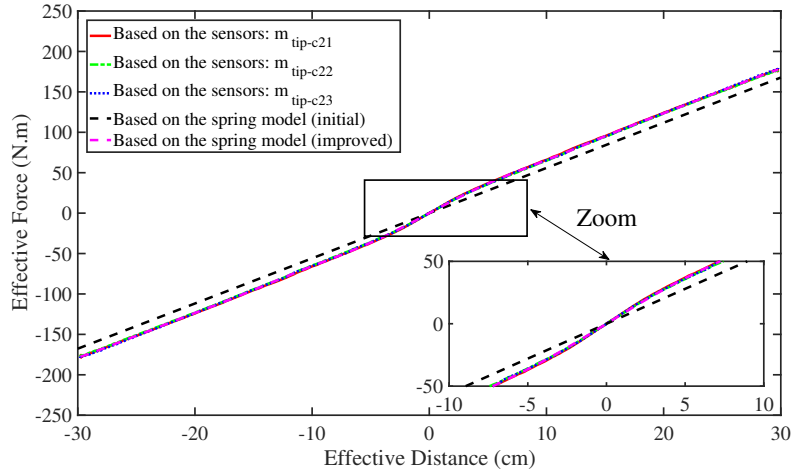


Fig. 7.14 Identification - composite link: effective force vs effective distance.

**Fourth stage:** In this stage, we pursued a fine adjustment of the model so as to approximately emulate the effect of the non-ideal construction characteristics of the mechanical part denominated as the "wire rope guide". Likewise, the ideas presented in this stage are valid as regards compensating for other non-ideal properties in the compensation mechanism, such as the non-linearities in the springs. The proposal in this stage was to fit a curve for the effective force ( $F_{eff}$ ) in terms of the effective distance ( $D_{eff}$ ), and to consider the resulting curve in order to replace the original equation (7.36), which stated that  $F_{eff} = k_{spring} (\lambda l (\theta_m - \hat{\theta}_a) + l_s \cos(\theta_m)) = k_{spring} D_{eff}$ . The result of the fitting is indicated in Fig 7.14 under the name of "improved". The improvements made to the fitting of the coupling torque can be seen in Figs 7.10 and Fig 7.11. The attachment torque does not

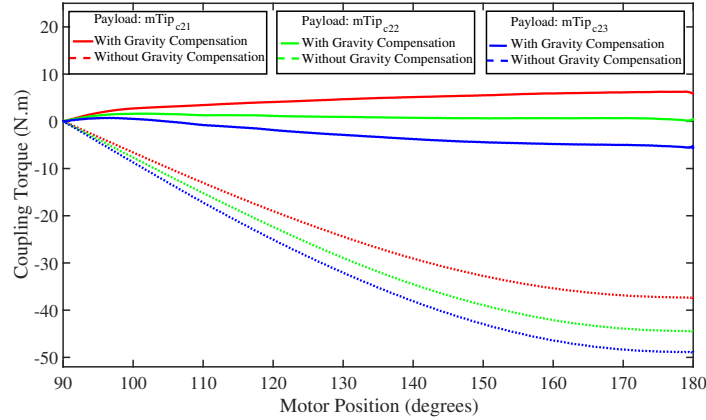


Fig. 7.15 Identification - composite link: comparison of the coupling torque of the flexible arm with and without gravity compensation.

change with regard to the initial model of the spring, since it does not directly depend on the effective force (see Figs 7.12 and 7.11).

The parameters identified are presented in Appendix A in Table A.2. In order to evaluate the effectiveness of the gravity compensation mechanism, it is necessary to compare the original system with the system with the gravity compensation mechanism. More specifically, it is necessary to evaluate the coupling torque which is the torque exerted by the motor in order to move the flexible link arm. Fig 7.15 shows the comparison for the payload masses  $m_{tip-c21}$ ,  $m_{tip-c22}$  and  $m_{tip-c23}$ . In the case of payload mass  $m_{tip-c22}$ , for which the gravity compensation mechanism has been adjusted, the torque reduction is 44.06 N.m for the situation of maximum gravitational torque, which represents a reduction of 98.4% with regard to the system without gravity compensation and a reduction of 40 % in relation to the continuous stall torque of the motor (108 N.m).

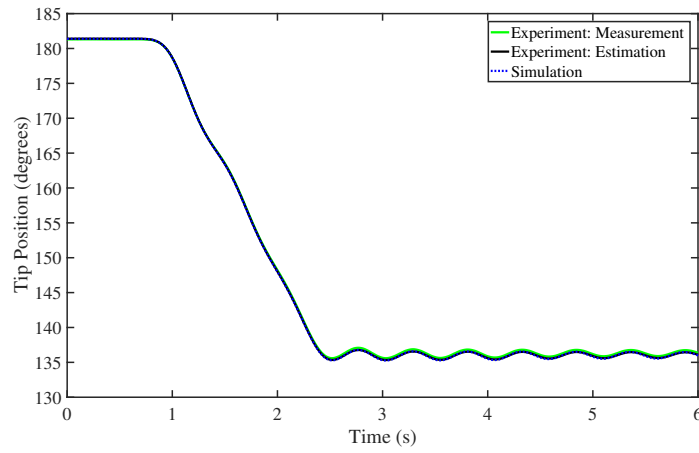
## 7.5 Experimental validation of the dynamic model

The dynamic model was validated by comparing the time response of the system when the motor tracked two different trajectories and no controller was applied to cancel the vibrations in the flexible link. The first trajectory corresponded to a fourth order trajectory that goes from 180 to 135 degrees in 2 seconds. The second trajectory consisted of a more complex maneuver composed of several fourth order trajectories of various amplitudes, concatenated and covering a major area of the workspace of the manipulator. The payload masses considered were  $m_{tip-c21}$ ,  $m_{tip-c21}$  and  $m_{tip-c23}$ , presented in Table A.4 of Appendix A. The signals compared were the tip position, the coupling torque and the attachment torque.

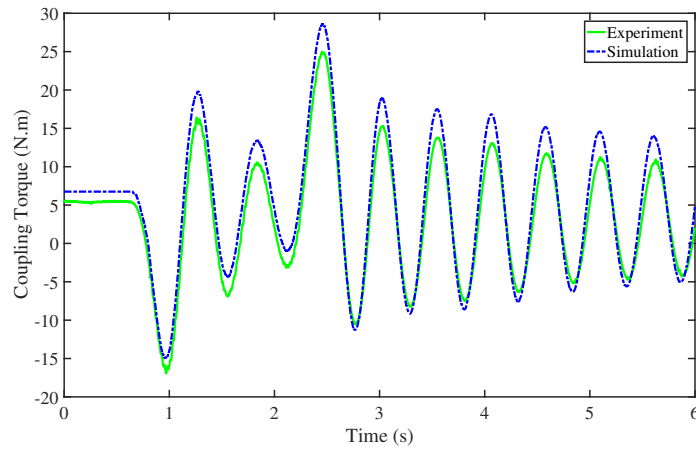
With regard to the precision of the measurement system, it is important to mention that the precision of the camera tracking system was about 1 mm for the current experimental setup, which is equivalent to a precision error of 0.028 degrees in the measurement of the tip position.

Figs. 7.16-7.18 depict the comparison between the experiments and the simulation of the model in the case of the simple fourth order trajectory. The fit of both torques was good for all the payload masses. With regard to the tip position, Fig. 7.19 shows that the maximum error between the measurement and the simulation was about 6.5 mm, which occurred in the case of payload mass  $m_{tip-c22}$ . In the same case, which represents the worst case scenario, the fitness of the model was of 99.9 % for the tip position variable, calculated as the Normalized Mean Square Error (NMSE).

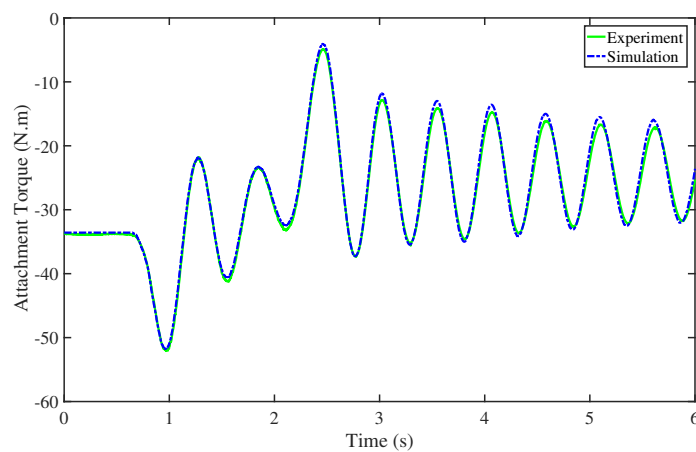
Figs. 7.20-7.22 show comparisons between the experiments and the simulations in the case of a complex maneuver. The results show that the dynamic model closely predicts the behavior of the real system in all its variables. In relation to the tip position, Fig. 7.19 shows that the maximum error between the measurement and the simulation was under 6 mm, which occurred in the case of payload mass  $m_{tip-c23}$ . The fitness of the model was, in this case, 99.9 % for the tip position signal.



(a) Angular position comparison.

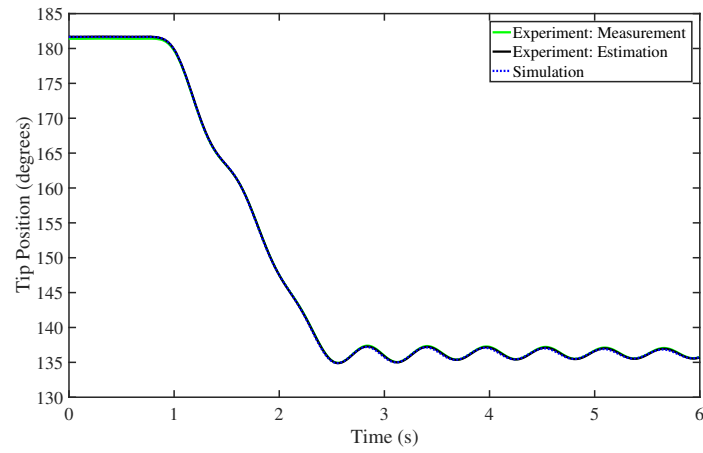


(b) Coupling torque.

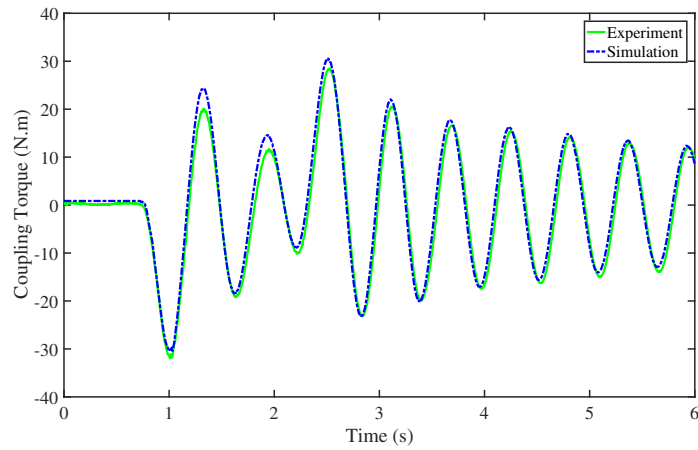


(c) Attachment torque.

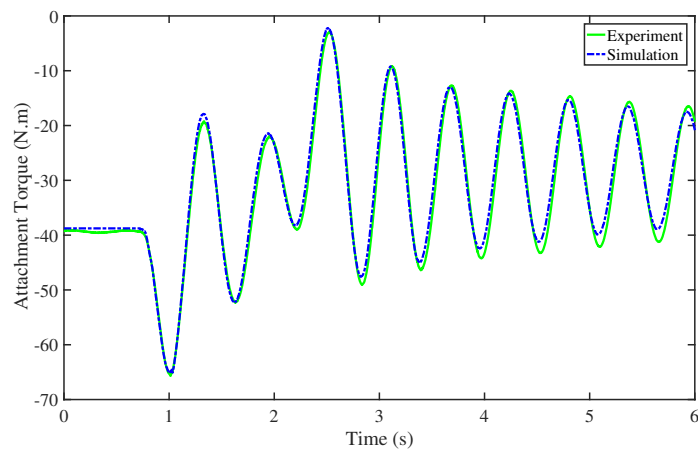
Fig. 7.16 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c21}$  when a simple trajectory is commanded.



(a) Angular position comparison.

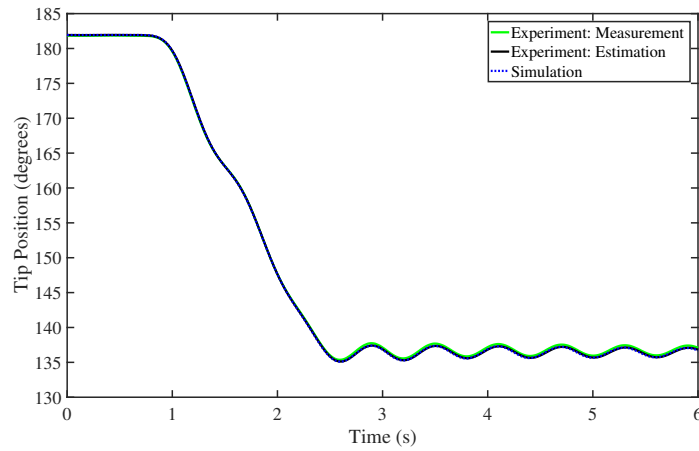


(b) Coupling torque.

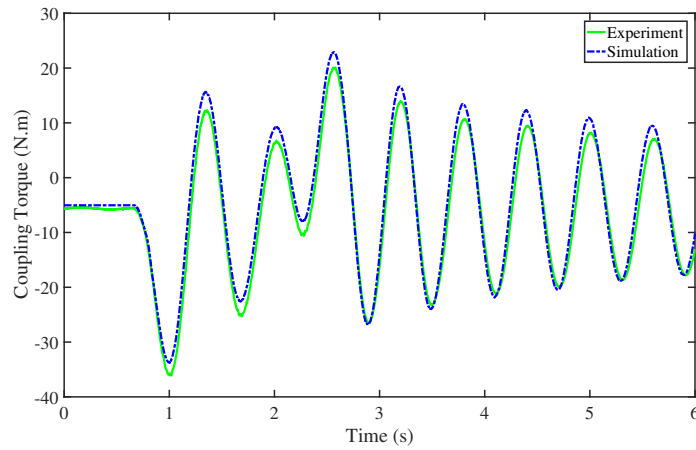


(c) Attachment torque.

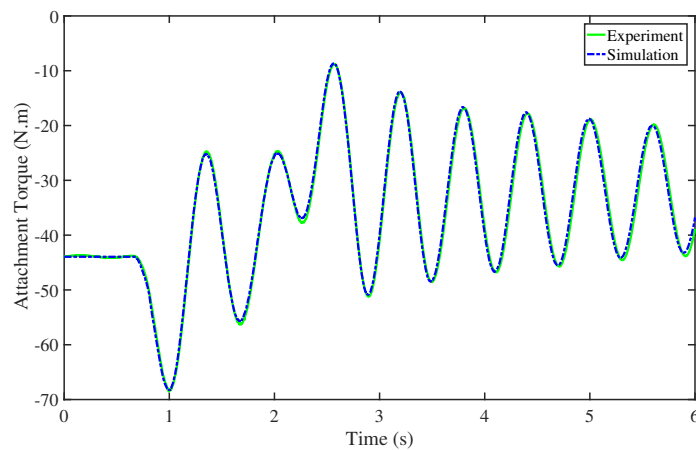
Fig. 7.17 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c22}$  when a simple trajectory is commanded.



(a) Angular position comparison.



(b) Coupling torque.



(c) Attachment torque.

Fig. 7.18 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c23}$  when a simple trajectory is commanded.

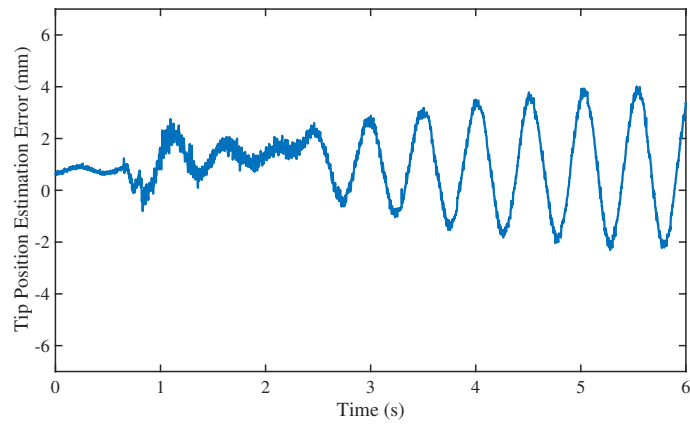
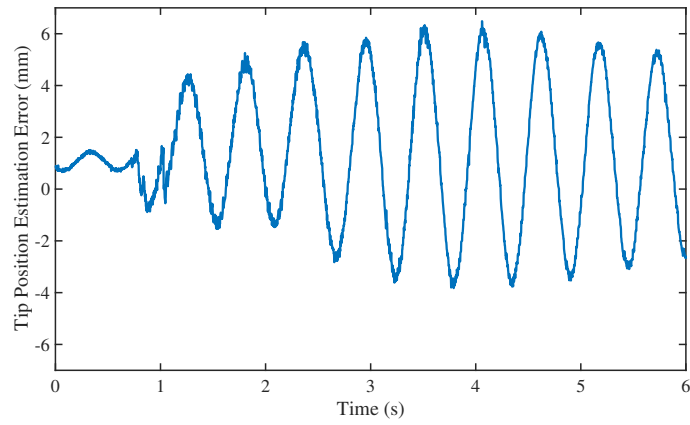
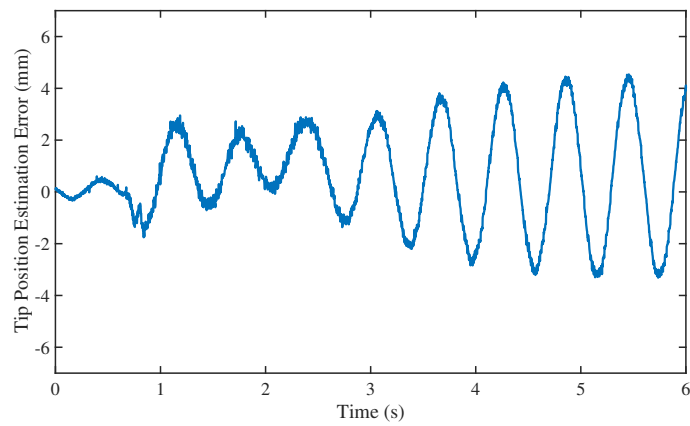
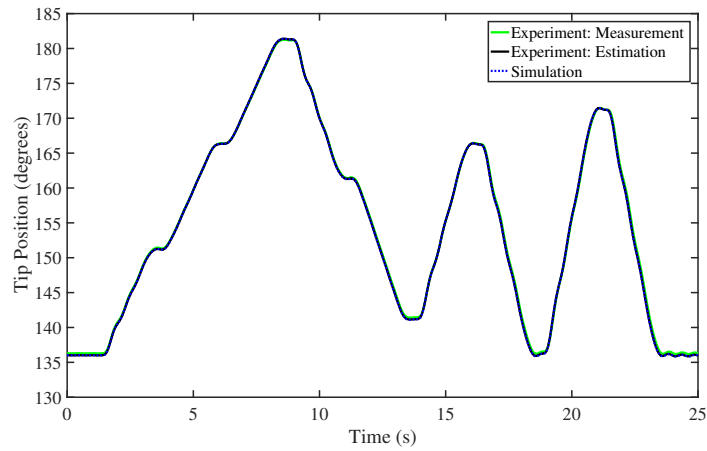
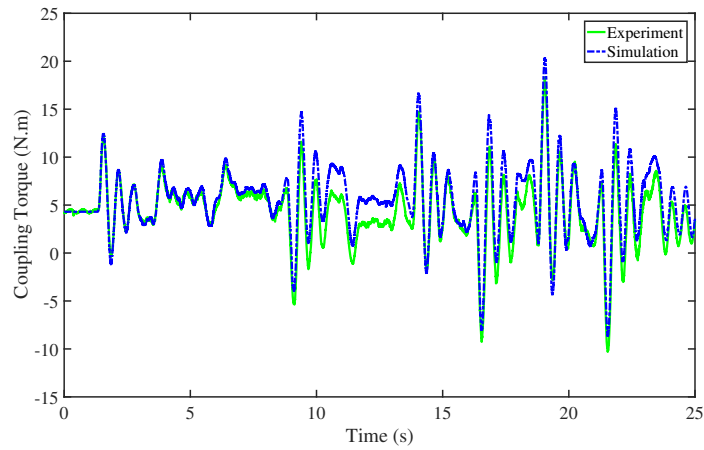
(a) Payload mass:  $m_{tip-c21}$ .(b) Payload mass:  $m_{tip-c22}$ .(c) Payload mass:  $m_{tip-c23}$ .

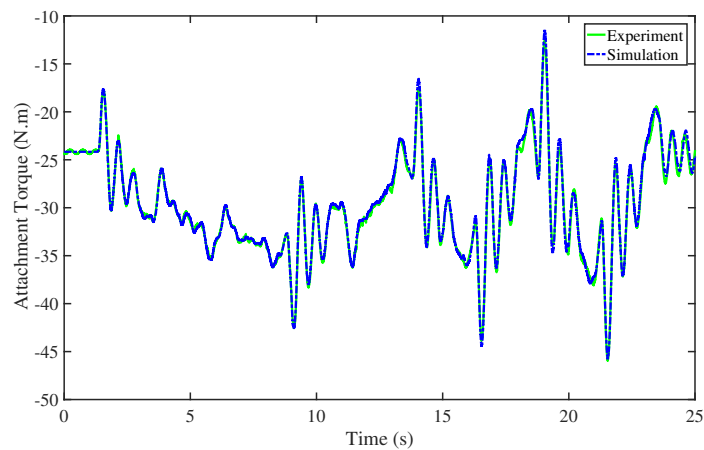
Fig. 7.19 Validation - composite link: tip position error in the tracking of a simple motor trajectory.



(a) Angular position comparison.

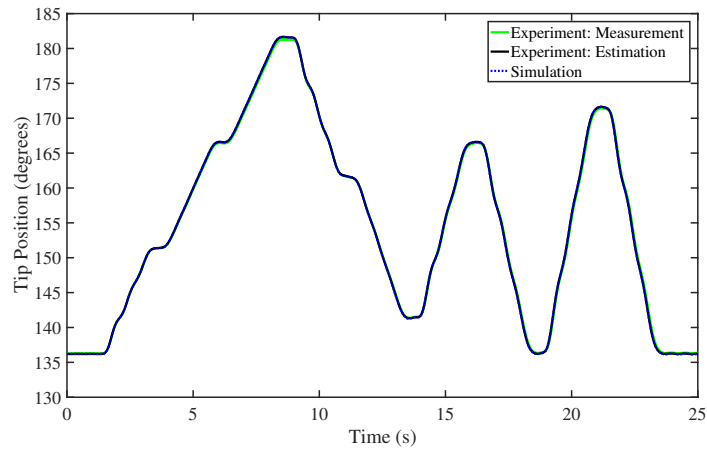


(b) Coupling torque.

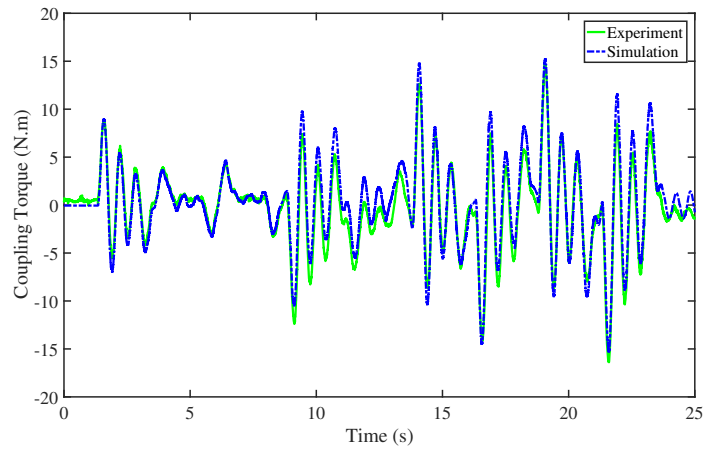


(c) Attachment torque.

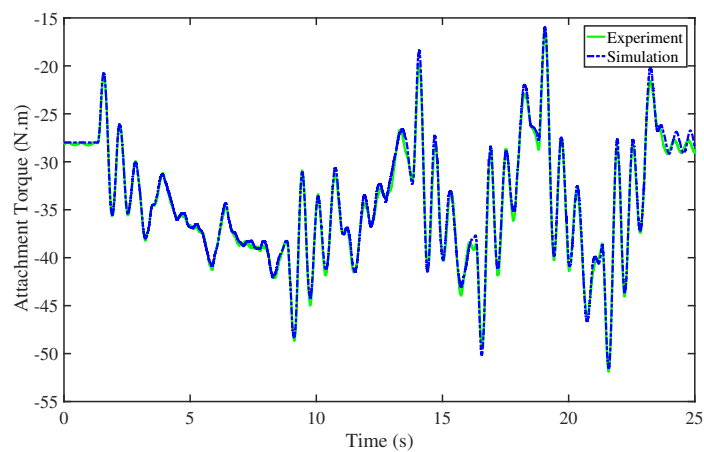
Fig. 7.20 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c21}$  when a complex trajectory is commanded.



(a) Angular position comparison.

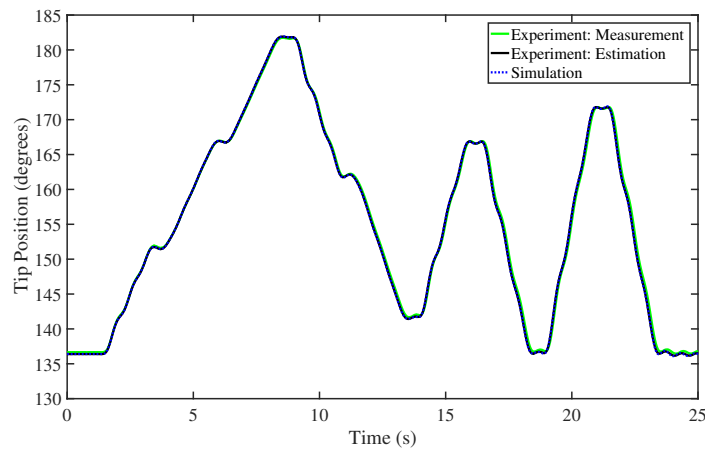


(b) Coupling torque.

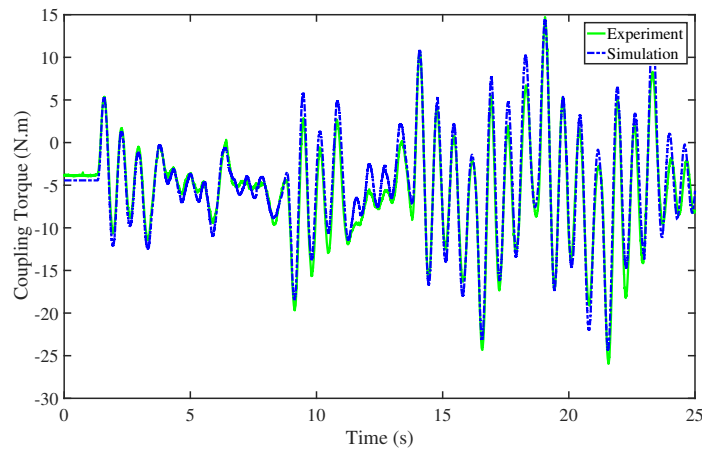


(c) Attachment torque.

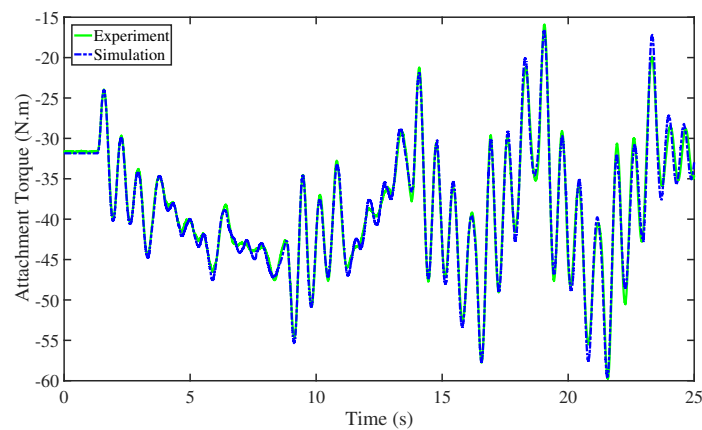
Fig. 7.21 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c22}$  when a complex trajectory is commanded.



(a) Angular position comparison.



(b) Coupling torque.



(c) Attachment torque.

Fig. 7.22 Validation - composite link: comparison of the time response for the payload mass  $m_{tip-c23}$  when a complex trajectory is commanded.

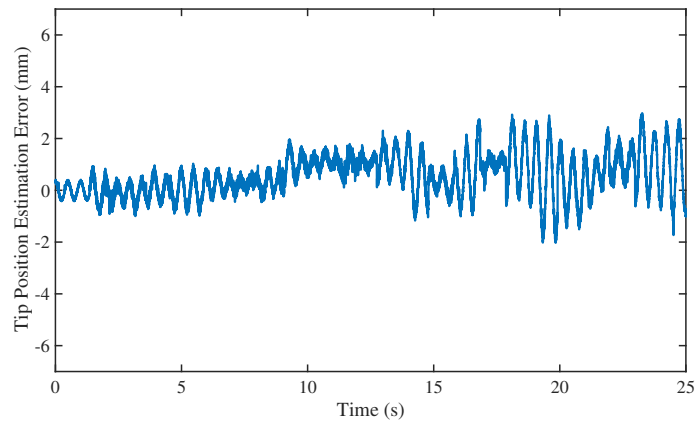
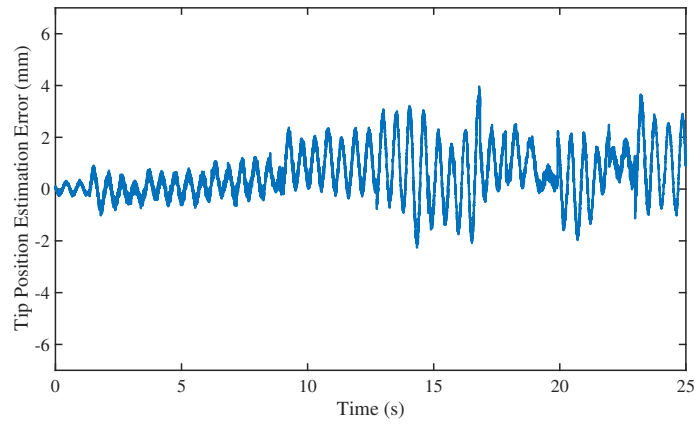
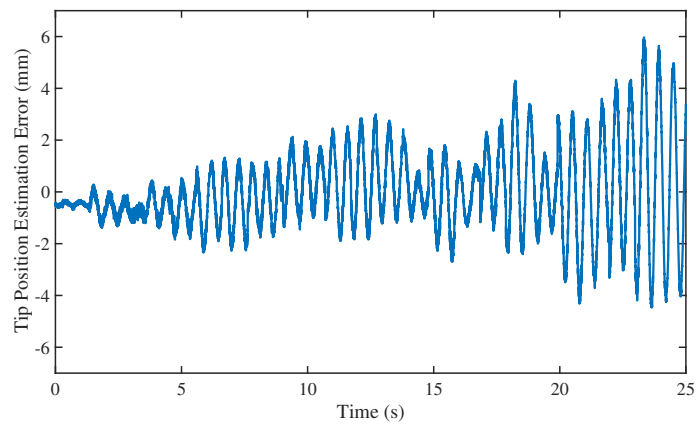
(a) Payload mass:  $m_{tip-c21}$ .(b) Payload mass:  $m_{tip-c22}$ .(c) Payload mass:  $m_{tip-c23}$ .

Fig. 7.23 Validation - composite link: tip position error in motor tracking of a complex maneuver.

## 7.6 Controller design

In accordance with the controller designs presented in the previous two chapters, an input-state feedback linearization controller implemented in a double loop control scheme is considered as the overall approach. In this strategy, the inner loop addresses the motor position control in the presence of static/Coulomb friction, whereas the outer loop deals with the tip position trajectory tracking control and guarantees the vibration suppression of the flexible link arm with the gravity compensation mechanism. This section is concerned with the outer loop controller design. It is assumed that the inner loop implements the two-degree-of-freedom controller explained in Chapter 4.

Let us begin by considering the inner loop dynamics, given by equation (4.23), and the dynamics of the gravity compensated flexible arm described by equation (7.28). Let us also define the state vector  $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ , such that  $x_1 = \theta_t$ ,  $x_2 = \dot{\theta}_t$ ,  $x_3 = \theta_m$ ,  $x_4 = \dot{\theta}_m$ . The space state representation of the system is then given by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, F_{eff}) + \mathbf{g}(\mathbf{x}) u \\ &= \begin{bmatrix} x_2 \\ \frac{3 E I_z}{\eta l} (x_3 - x_1) - \frac{\eta}{\eta} \cos(x_1) - \frac{l}{\eta} \frac{\lambda^3 - 3 \lambda^2}{2} F_{eff} \\ x_4 \\ -\frac{1}{\alpha^2} x_3 - \frac{2}{\alpha} x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\alpha^2} \end{bmatrix} u \end{aligned} \quad (7.40)$$

The input-state feedback linearization technique requires that the plant dynamics satisfies the controllability and involutivity conditions [42]. Before studying these conditions, let us consider that the  $F_{eff}$  is an external disturbance that can be measured and effectively canceled, and that it can, therefore, be neglected in the dynamics. The vector field  $\mathbf{f}(\mathbf{x}, F_{eff})$  can be written as  $\mathbf{f}(\mathbf{x}, F_{eff})$ . Thus, the controllability and involutivity conditions in terms of the Lie bracket of  $\mathbf{f}$  and  $\mathbf{g}$  ( $ad_{\mathbf{f}}\mathbf{g}$ ) and its repeated Lie brackets ( $ad_{\mathbf{f}}^i\mathbf{g}$ ) as  $[\mathbf{g}, ad_{\mathbf{f}}\mathbf{g}, ad_{\mathbf{f}}^2\mathbf{g}, ad_{\mathbf{f}}^3\mathbf{g}]$  are as result defined as:

- Controllability condition. This condition is satisfied if, and only if, there is a closed connected region  $\Omega$  in  $\mathbf{R}^n$  such that every vector in the controllability matrix is linearly independent. The controllability matrix is defined as  $[\mathbf{g}, ad_{\mathbf{f}}\mathbf{g}, ad_{\mathbf{f}}^2\mathbf{g}, ad_{\mathbf{f}}^3\mathbf{g}]$ .
- Involutivity condition. This condition is met if the set  $[\mathbf{g}, ad_{\mathbf{f}}\mathbf{g}, ad_{\mathbf{f}}^2\mathbf{g}]$  is involutive in the aforementioned region  $\Omega$ .

The substitution of  $f(\mathbf{x})$  and  $g(\mathbf{x})$  into the controllability matrix leads to

$$\begin{bmatrix} 0 & 0 & 0 & \frac{-3 E I_z}{\gamma_1 \alpha^2 l} \\ 0 & 0 & \frac{3 E I_z}{\alpha^2 \gamma_1 l} & \frac{6 E I_z}{\gamma_1 \alpha^3 l} \\ 0 & \frac{-1}{\alpha^2} & \frac{-2}{\alpha^3} & \frac{-3}{\alpha^4} \\ \frac{1}{\alpha^2} & \frac{2}{\alpha^3} & \frac{3}{\alpha^4} & \frac{4}{\alpha^5} \end{bmatrix} \quad (7.41)$$

Since the previous matrix has full rank for  $EI > 0$  and  $0 < m, l, \alpha, \gamma_1 < \infty$ , the controllability condition is satisfied. The involutivity condition is trivially met by considering that the vector fields  $[g, ad_f g, ad_f^2 g]$  are constant. In this case, the input-state linearization of the system is global as long as both conditions, controllability and involutivity, are satisfied everywhere and, therefore, for any region  $\Omega$ .

The following step is to find the state and the input transformations that permit the exact linearization of the system. The procedure employed to find the state transformation  $\mathbf{z}(\mathbf{x}) = [z_1, z_2, z_3, z_4]^T$  consists of finding an expression for transformed state  $z_1$  in terms of the system parameters and states  $\mathbf{x}$ , such that the following constraints are met:

$$\nabla_{z_1} ad_{\mathbf{f}}^k \mathbf{g} = 0 \quad k = 0, 1, 2, \quad (7.42)$$

$$\nabla_{z_1} ad_{\mathbf{f}}^3 \mathbf{g} \neq 0. \quad (7.43)$$

These conditions are met when  $z_1 = x_1 = \theta_l$ , which is the variable to be controlled. Once the transformation for state  $z_1$  is obtained, the whole system transformation can be calculated in terms of the Lie derivatives of  $z_1$  with respect to  $\mathbf{f}$  ( $L_{\mathbf{f}} z_1$ ) and its repeated derivatives ( $L_{\mathbf{f}}^i z_1$ ) as

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ L_{\mathbf{f}} z_1 \\ L_{\mathbf{f}}^2 z_1 \\ L_{\mathbf{f}}^3 z_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \frac{3 E I_z}{\gamma_1 l} (x_3 - x_1) - \frac{\gamma_2}{\gamma_1} \cos(x_1) - \frac{l}{\gamma_1} \frac{\lambda^3 - 3 \lambda^2}{2} F_{eff} \\ \frac{3 E I_z}{\gamma_1 l} (x_4 - x_2) + \frac{\gamma_2}{\gamma_1} \sin(x_1) x_2 - \frac{l}{\gamma_1} \frac{\lambda^3 - 3 \lambda^2}{2} \dot{F}_{eff} \end{bmatrix} \quad (7.44)$$

Applying this transformation to the original dynamics equations yields

$$\dot{\mathbf{z}} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ z_4 \\ \frac{3 E I_z}{\gamma_1 l} \left( \frac{1}{\alpha^2} z_1 + \frac{2}{\alpha} z_2 + z_3 \right) - \frac{1}{\alpha^2} z_3 - \frac{2}{\alpha} z_4 \\ \frac{\gamma_2}{\gamma_1} \left[ \left( z_2^2 - \frac{1}{\alpha^2} \right) \cos(z_1) + \left( \frac{2}{\alpha} z_2 + z_3 \right) \sin(z_1) \right] \\ \left( \frac{3 \lambda^2 - \lambda^3}{\gamma_1} l \right) \left[ \frac{1}{2 \alpha^2} F_{eff} + \frac{1}{\alpha} \dot{F}_{eff} + \frac{1}{2} \ddot{F}_{eff} \right] \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{3 E I_z}{\alpha^2 \gamma_1 l} \end{bmatrix} u \quad (7.45)$$

The linearizing control law to be applied has the following form:  $u = \psi_1(\mathbf{z}) + \beta(\mathbf{z})v$ , where  $v$  is the new input of the linearized system, and  $\psi_1(\mathbf{z})$  and  $\psi_2(\mathbf{z})$  are

$$\psi_1(\mathbf{z}) = -\frac{L_f^4 z_1}{L_g L_f^3 z_1}, \quad (7.46)$$

$$\psi_2(\mathbf{z}) = \frac{1}{L_g L_f^3 z_1}. \quad (7.47)$$

The control law can be defined as

$$\begin{aligned} u = & z_1 + 2 \alpha z_2 + \left( \alpha^2 - \frac{\gamma_1 l}{3 E I_z} \right) z_3 - \frac{2 \alpha \gamma_1 l}{3 E I_z} z_4 + \frac{\gamma_2 l}{3 E I_z} (\alpha^2 z_2^2 - 1) \cos(z_1) \\ & + \frac{\gamma_2 l}{3 E I_z} (2 \alpha z_2 + \alpha^2 z_3) \sin(z_1) - \frac{l^2}{2 E I_z} \left( \lambda^2 + \frac{\lambda^3}{2} \right) F_{eff} + -\frac{\alpha l^2}{E I_z} \left( \lambda^2 + \frac{\lambda^3}{3} \right) \dot{F}_{eff} \\ & + \frac{\alpha^2 l^2}{2 E I_z} \left( \lambda^2 - \frac{\lambda^3}{3} \right) \ddot{F}_{eff}. \end{aligned} \quad (7.48)$$

As a result of substituting equation (7.48) into equation (7.45), the following linear system is obtained

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v \quad (7.49)$$

The following linear control law is proposed to make the tip position  $z_1 = x_1 = \theta_t$  track a predefined trajectory  $z_1^*$

$$v = z_1^{*(4)} - k_3(z_1^{(3)} - z_1^{*(3)}) - k_2(\ddot{z}_1 - \ddot{z}_1^*) - k_1(\dot{z}_1 - \dot{z}_1^*) - k_0(z_1 - z_1^*) . \quad (7.50)$$

The tracking error dynamics can be expressed, in terms of the error  $e_1 = z_1 - z_1^*$ , as

$$e_1^{(4)} + k_3 e_1^{(3)} + k_2 e_1^{(2)} + k_1 \dot{e}_1 + k_0 e_1 = 0 . \quad (7.51)$$

where the parameters  $k_i$  determine the stability and the performance of the system. For the sake of simplicity, let us place the poles of the closed loop system,  $p_o$ , in the same position along the negative abscissa axis of the s-plane. The constants  $k_i$  can consequently be determined by equating the coefficients of the actual and the desired characteristic polynomial

$$s^4 + k_3 s^3 + k_2 s^2 + k_1 s + k_0 = (s - p_o)^4 . \quad (7.52)$$

In order to complete the description of the controller, it is necessary to express the non-linear control law in terms of the sensor measurements. The measurements available are the motor position ( $\theta_m$ ), the coupling torque ( $\Gamma_{coup}$ ) and the torque at the attachment point of the spring ( $\Gamma_{pt-attach}$ ). These measurements and equation (7.32) allow the state vector  $\mathbf{x}$  to be fully estimated as

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \begin{bmatrix} \hat{\theta}_t \\ \dot{\hat{\theta}}_t \\ \hat{\theta}_m \\ \dot{\hat{\theta}}_m \end{bmatrix} = \begin{bmatrix} \theta_m + \frac{l}{6 E I_z} \left( (\lambda - 2) \Gamma_{pt-attach} + (-3\lambda + \lambda^2) \Gamma_{coup} \right) \\ \dot{\theta}_m + \frac{l}{6 E I_z} \left( (\lambda - 2) \dot{\Gamma}_{pt-attach} + (-3\lambda + \lambda^2) \dot{\Gamma}_{coup} \right) \\ \theta_m \\ \dot{\theta}_m \end{bmatrix} \quad (7.53)$$

where states  $\hat{x}_2$  and  $\hat{x}_4$  are obtained by means of numerical differentiation of states  $\hat{x}_1$  and  $\hat{x}_3$ , respectively. Once states  $\mathbf{x}$  are calculated, state vector  $\mathbf{z}$  is obtained from equation (7.44). Finally, in order to compute the non-linear control law, the  $F_{eff}$ , need to be estimated from the measurements. As was stated in Section 7.3, there are two ways of estimating the  $F_{eff}$ : (1) by using the estimation independent from the spring model (equation (7.33)); (2) by using the estimation based on the spring model (equation (7.36)). Although an estimation that is independent of the spring model should be preferred, since it considers fewer parameters, this estimation is strongly affected by the noise present in the strain gauge measurements. This effect is not very significant in the computation of  $F_{eff}$ , but it seriously affects the esti-

mation of the first and second derivatives of the signal which must be computed by means of numerical differentiation.  $F_{eff}$  is, therefore, computed from equation (7.36).

To recapitulate, Fig. 7.24 presents a scheme of the controller presented in this section, including all the equations which are necessary for the implementation and simulation of the overall system.

## 7.7 Experimental validation of the control algorithm

In this section, we present the experimental validation of the controller for the end-point positioning of the flexible link. The intention of this section is, besides proving the validity of the controller design, to highlight the benefits of the gravity compensation mechanism in terms of the motor torque and the coupling torque. In order to fulfill these aims, we compare the performance of the controller presented in Chapter 5 for the system without gravity compensation, with the performance of the controller designed in the previous section for the system with the gravity compensation mechanism. In both cases, the validation considered as a tip position reference a fourth order trajectory that goes from 180 to 135 degrees in 2 s, created as in [48]. The poles of the inner and outer loop were set at  $p_i = -20$  and  $p_o = -18$ , respectively. The payload considered corresponded to  $m_{tip-c13} = 1.46$  Kg. Fig 7.25 presents the comparison of the tip position, motor torque, coupling torque and attachment torque for both controllers.

With regard to the tip position (Fig. 7.25a) there were no major differences in the performance of both controllers, despite the fact that the controllers were designed by considering different dynamic models. The controller in Chapter 5 considers a dynamic model with

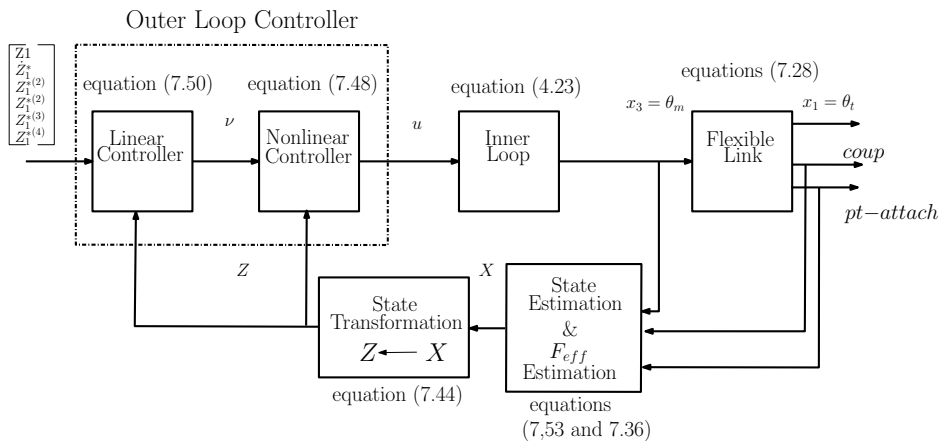
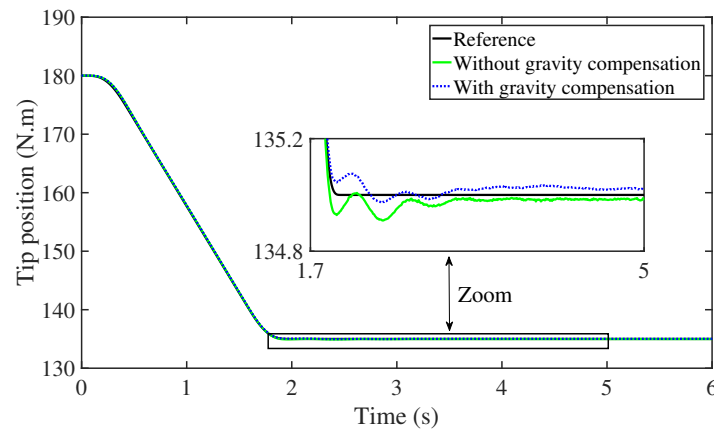


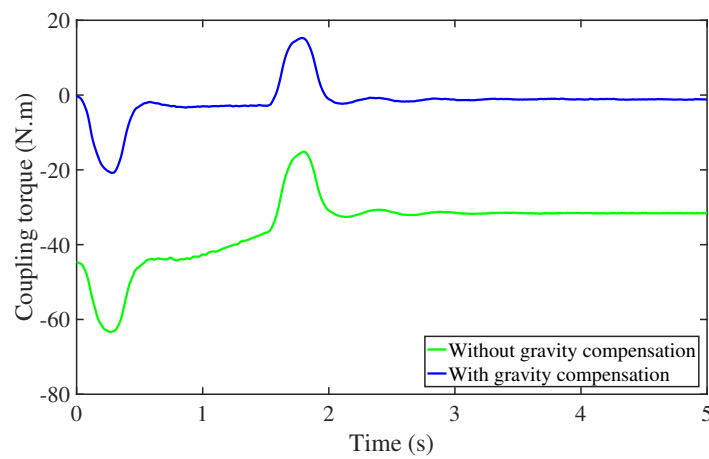
Fig. 7.24 Tip position controller design for the system with the gravity compensation mechanism: schematic representation.

one lumped mass and one massless flexible element, whereas the controller designed in the present chapter considers one lumped mass but two massless flexible elements, in addition to the interacting forces corresponding to the gravity compensation mechanism. In both cases, it can be observed that the vibration of the arm was successfully canceled. Based on the estimation of the tip position based on the strain gauges, the steady state error were 1.1 mm and 1.5 mm, in the first and the second case, respectively. The external camera tracking system, whose precision is limited to 1 mm for the current setup, measures steady state error of 1 mm in both cases.

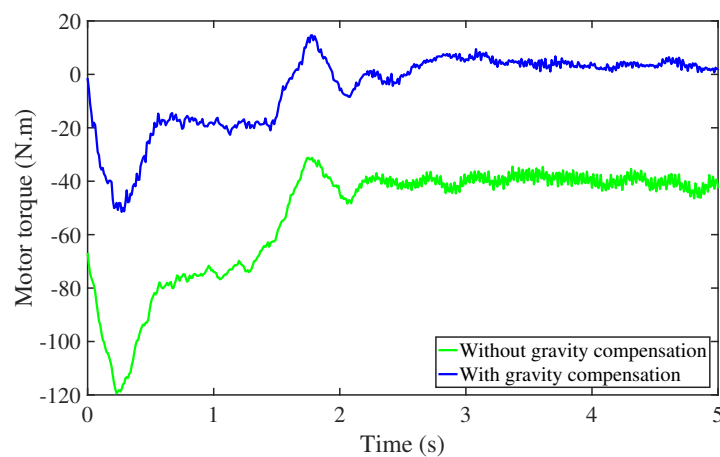
The analysis of the torque should be carried out in two parts: that corresponding to the overall motor torque, which is directly estimated from the control signal of the motor; and that of the coupling torque, which is measured with the strain gauges located on the base of the flexible link. At this point, it is important to recall that the motor torque is the sum of four terms, and is stated in equation (3.1). These terms are: (1) the torque used to accelerate the rotor inertia; (2) the viscous friction torque ; (3) the static/Coulomb friction torque; and finally, (4) the coupling torque. In relation to the motor torque, Fig. 7.25b shows clear improvements to the torque demands of the system when performing the same task. The maximum absolute value of the motor torque was reduced from 119.8 N.m, in the system without the gravity compensation mechanism, to 48.7 N.m, in the case with the gravity compensation mechanism. This improvement represents a reduction of 59.3 %. The problem as regards analyzing the improvement to the gravity compensation mechanism in terms of the motor torque is that the static frictions does not make possible to elucidate the actual torque reduction achieved by the mechanism. In static situations, for instance, in which the torque consumption for the gravity compensated mechanism should be approximately zero, the motor controller sets a current reference value that is beneath the fringe of the static friction, that does not produce any movement. It is evident that there is a certain error in the motor positioning owing to the limited precision of the sensor. A better way in which to assess the actual improvements of the gravity compensation mechanism is to consider the coupling torque, which measures the actual torque that the motor exerts on the base of the flexible link. Fig. 7.25c shows, at the beginning and the end of the trajectory, that the torque demands required to maintain the system in a static position are near zero in both cases. During the execution of the trajectory, the maximum absolute value of the coupling torque was reduced from 64.3 N.m to 22.8 N.m, which represents a reduction of 64.5 %.



(a) Tip position.



(b) Coupling Torque.



(c) Motor torque.

Fig. 7.25 Validation - composite link: comparison of the time response of the tip position controller with and without the gravity compensation mechanism.

# Chapter 8

## Conclusions, contributions and suggested future research

### 8.1 Summary

Over four decades have passed since the publication of the Ph.D thesis by Prof. Wayne Book entitled "Modeling, design and control of flexible manipulator arms"[84], which is considered to be the starting point for the prolific research field of flexible link robotics. Despite all the time that has elapsed, the contributions of greatest significance in real applications have mostly been constrained to the aerospace industry, e.g., the Shuttle Remote Manipulator System (see Section 1.2). This contrasts with terrestrial applications, in which the field has not yet found a clear niche that counteracts the difficulties involved. The framework in which this thesis was developed comprises two research projects, the first of which is entitled "Design and Control of a very Lightweight Flexible Robot Made of Composite Material. Application to a Robotic based inspection system" (DPI2009-09956). The second project is an extension of this and is entitled "Development and Comparison of New Control Techniques for Very Large Lightweight Flexible Arms Mounted on Mobile Platforms Under Environmental Disturbances" (DPI2012-37062-CO2-01). Mobile terrestrial applications are one of the most significant areas that can benefit from flexible link robotics, mainly owing to their properties of lower weight, reduced torque demands and, therefore, low energy consumption.

This thesis deals with the design, modeling and control of a single-link flexible arm with a unique spring-based gravity compensation mechanism. This arm constitutes the first degree of freedom of the flexible link robotic manipulator to be considered on board the mobile platform of the aforementioned projects. For the purposes of this thesis, the flexible

arm is considered to be mounted on a very robust and static base. The present document presents a detailed explanation of the mechanical design of the platform, innovative dynamic models and control schemes for these cases in which the gravity compensation mechanism is or is not installed. The following sections present the main conclusions and contributions of the thesis, the publications derived from it, and suggest directions for future research.

## 8.2 Conclusions and contributions

**Experimental setup:** The research work started with the development of an experimental platform on which to validate the theoretical developments. The experimental system consists of a single-link flexible link arm, whose movements are on a vertical plane and are, therefore, significantly affected by the force of gravity. The system is actuated by a brushless DC motors, and considers as sensors one encoder and two sets of strain gauges to estimate the end effector position of the robot. We have, additionally, designed and built a spring based gravity compensation mechanism for the single link flexible arm that makes it possible to reduce 98.4% of the torque in static with regard to the system without gravity compensation, which represents a reduction of 40 % in relation to the continuous stall torque of the motor (108 N.m). To date, we have found no spring based gravity compensation mechanisms for flexible link robots, but only those based on counterweights. Unlike the mechanisms based on counterweights, the mechanism proposed does not add significant inertia to system, which implies that the improvements obtained for static situations do not increase the dynamic torque demands.

**Dynamic modeling:** We have developed several models that describe the dynamics of the system very accurately. Specifically, we have developed two dynamic models for the flexible link arm without a gravity compensation mechanism: the first dynamic model describes only the first significant mode frequency of the system, whereas the second dynamic model describes the first two significant vibrational frequencies of the system. The first dynamic model was validated with a duraluminum link of 1 m and a composite link of 2 m, with a maximum error of 0.8 mm and 3.9 mm when the motor tracked an abrupt trajectory. With regard to the system with the gravity compensation mechanism, we have developed a dynamic model that describes only the first vibrational mode but that considers the interacting forces between the gravity compensation mechanism and the flexible link arm. The system was validated only for the 2 meter-long composite link with a maximum error of 6.2 mm when the motor tracked an abrupt trajectory.

**Control algorithms:** Three control algorithms were proposed:

- **Flexible link tip position controller:** this served as the basis for the following two control algorithms. This algorithm considered the minimum phase properties of the single lumped mass dynamic model in order to implement an input-state feedback control law in a double loop cascade control. Unlike the approaches typically applied in the bibliography, in which only a single loop is considered, the double loop cascade approach rejects the static/Coulomb friction disturbances in the motor without requiring an explicit model for the friction, while simultaneously permitting a fine tracking of the end-effector trajectories. The results of this approach when applied to the control of the 1 meter duraluminum link showed tracking errors of below 0.5 mm in the tracking of a abrupt trajectory. The performance of the algorithm was clearly superior to that of other alternatives found in literature that do not considered double loop cascade control.
- **Flexible link tip position adaptive controller:** this corresponds to an adaptive controller with which to improve the performance of the system under conditions of payload mass uncertainty. The algorithm complements the first control algorithm with an algebraic identification algorithm that carries out the online identification of the payload mass while the flexible manipulator is moving, and subsequently updates the control law to improve the tracking of the trajectory. The Algebraic Identification Algorithm achieves payload mass estimates of 5 % error with respect to the real payload mass in less than 0.45 s and 0.97 s for the 1 meter long duraluminum link and the 2 meter long composite link, which respectively represent 1/8 and 1/2 of the time required by the Least Mean Squares Algorithm. These results correspond to the most demanding situation in which the flexible link starts from the vertical position where no initial gravity torque exists.
- **Flexible link tip position controller for the system with the gravity compensation mechanism:** this an adaptation of the first algorithm to a more complex case in which the dynamic model includes the spring based gravity compensation mechanism. The maximum error of the trajectory tracking of an abrupt trajectory was of 1.5 mm for the 2 meter long composite link.

## 8.3 List of publications

**Journal papers:**

- J. C. Cambera and V. Feliu-Batlle, “Input-state feedback linearization control of a single-link flexible robot arm moving under gravity and joint friction,” *Robotics and Autonomous Systems*, vol. 88, pp. 24–36, Elsevier, 2017
- J. C. Cambera, A. San-Millan, and V. Feliu-Batlle, “Payload mass identification of a single-link flexible arm moving under gravity: an algebraic identification approach,” *Shock and Vibration*, vol. 2015, Hindawi Publishing Corporation, 2015

### Conference papers:

- J. C. Cambera, D. Feliu-Talegon, S. Gharab, and V. Feliu-Batlle, “Flexible link arm with a passive gravity compensation mechanism: Design and modelling,” in *XXXIX Jornadas de Automática*, (Extremadura, Spain), CEA/IFAC, 2018
- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Feedback linearizing controller for a single link flexible arm with a passive gravity compensation mechanism,” in *Intelligent Robots and System (IROS), IEEE International Conference on*, (Madrid, Spain), IEEE, 2018
- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Modeling and identification of a single link flexible arm with a passive gravity compensation mechanism,” in *Robotics and Automation (ICRA), 2018 IEEE International Conference on*, (Brisbane, Australia), IEEE, 2018
- A. S.-M. Rodriguez, J. C. Cambera, and V. Feliu-Batlle, “Online algebraic identification of the payload changes in a single-link flexible manipulator moving under gravity,” in *International Federation of Automatic Control (IFAC) World Congress 2014*, vol. 47, (Cape Town, South Africa), pp. 8397–8402, Elsevier, 2014
- J. C. Cambera, J. A. Chocoteco, and V. Feliu-Batlle, “Feedback linearizing controller for a flexible single-link arm under gravity and joint friction,” in *ROBOT2013: First Iberian Robotics Conference*, (Madrid, Spain), pp. 169–184, Springer, 2014

### Other publications not related to the thesis:

- S. Gharab, V. Feliu-Batlle, and J. C. Cambera, “Fractional-order identification based on time domain methodology for hydraulic canal system,” in *XXXIX Jornadas de Automática*, (Extremadura, Spain), CEA/IFAC, 2018

- D. Feliu-Talegon, J. C. Cambera, A. San-Millan, and V. Feliu-Batlle, “A new truck prototype for transporting flexible link manipulators,” in *XXXIX Jornadas de Automática*, (Extremadura, Spain), CEA/IFAC, 2018

## 8.4 Prizes and grants

During the development of this Thesis the following prizes and grants were obtained:

- Prize in the Jornadas de Automática in Extremadura in España: Prize to the second best paper related to Modelling, Simulation and Optimization (Accésit al mejor trabajo en Modelado, Simulación y Optimización presentado a las Jornadas (patrocinado por Empresarios Agrupados) (September 2018).
- UCLM Research Stay Grant: Funded by Castilla-La Mancha University to do a 6-months research stay in Dortmund Technical University, Germany (April 2014-September 2014).
- FPI Research Grant: Funded by the Spanish Ministry of Economy and Competitiveness for Master and Doctoral training (October 2010 - September 2014).

## 8.5 Open topics for future research

Suggestions for future research are outlined below:

- The design of more compact gravity compensation mechanisms for a single-link flexible arm. It would be ideal to consider a gravity compensation mechanism that does not require the two supporting rods considered on both sides of the flexible link arm. One suitable solution would be to consider torsion springs rather than extension springs.
- The design of position control algorithms for a single link flexible arm with several significant vibrational modes and a gravity compensation mechanism. These algorithms would be especially relevant as regards improving the robustness of the system with respect to the *spill-over* in the case of payload mass uncertainty.
- An extension of the the current gravity compensation mechanism in order to adapt to different payload masses. The current gravity compensation mechanism provides close to perfect gravity compensation for the payload mass it was adjusted for ( $m_{tipc22}$ ),

but loses effectiveness when the actual payload differs from it. It would be interesting to consider a gravity compensation mechanism that can adapt to different payload masses, although this mechanism would probably need to include some sort of active element.

- The design and modeling of a two-link flexible arm with a gravity compensation mechanism for. It would be especially relevant for these models to include the effect of external forces applied to the end-effector, this making it useful for manipulation task.
- The design of force control algorithms for a flexible arm with several significant vibrational modes and a gravity compensation mechanism. These algorithms would allow the robot to interact with the environment.

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# **Appendix A**

## **Technical characteristics**

This appendix summarizes the main technical characteristics of the components considered in the experimental setup, explained in Chapter 2.

Table A.1 Properties of the flexible link 1 (duraluminum)

| Description                                                        | Value                    |
|--------------------------------------------------------------------|--------------------------|
| Type of section                                                    | Tubular                  |
| Material                                                           | Duraluminum              |
| Mass                                                               | 0.61 Kg                  |
| External diameter                                                  | 25.4 mm                  |
| Internal diameter                                                  | 18 mm                    |
| Nominal length                                                     | 1 m                      |
| Equivalent length ( $l$ )                                          | 0.96 m                   |
| Moment of inertia of the section ( $I_z$ )                         | $1.53 \cdot 10^{-8} m^4$ |
| Payload mass 1 ( $m_{tip-d1}$ )                                    | 1.13 Kg                  |
| Payload mass 2 ( $m_{tip-d2}$ )                                    | 2.16 Kg                  |
| Payload mass 3 ( $m_{tip-d3}$ )                                    | 3.13 Kg                  |
| Identified Parameters for the Modified Single Lumped Dynamic Model |                          |
| Flexural rigidity ( $E I_z$ )                                      | $803.67 N m^2$           |
| Link mass gravitational contribution ( $m_{link-g}$ )              | 0.305 Kg                 |
| Link mass inertial contribution ( $m_{link-a}$ )                   | 0.151 Kg                 |
| Strain gauge gain constant ( $K_{gauge}$ )                         | $39.55 N m/V$            |
| Additional Parameters                                              |                          |
| 1st vibrational frequency for payload mass 1 ( $\omega_{d1-1st}$ ) | 7.32 Hz                  |
| 1st vibrational frequency for payload mass 2 ( $\omega_{d2-1st}$ ) | 5.43 Hz                  |
| 1st vibrational frequency for payload mass 3 ( $\omega_{d3-1st}$ ) | 4.53 Hz                  |

Table A.2 Properties of the flexible link 2 (composites) (Part I)

| Description                                                        | Value                            |
|--------------------------------------------------------------------|----------------------------------|
| Type of section                                                    | Tubular                          |
| Material                                                           | Carbon composite                 |
| Mass                                                               | 0.27 Kg                          |
| External diameter                                                  | 30 mm                            |
| Internal diameter                                                  | 28 mm                            |
| Nominal length                                                     | 2 m                              |
| Equivalent length ( $l$ )                                          | 2.02 m                           |
| Moment of inertia of the section ( $I_z$ )                         | $9.59 \cdot 10^{-9} \text{ m}^4$ |
| Payload mass 11 ( $m_{tip-c11}$ )                                  | 0.73 Kg                          |
| Payload mass 12 ( $m_{tip-c12}$ )                                  | 1.20 Kg                          |
| Payload mass 13 ( $m_{tip-c13}$ )                                  | 1.46 Kg                          |
| Payload mass 21 ( $m_{tip-c21}$ )                                  | 1.83 Kg                          |
| Payload mass 22 ( $m_{tip-c22}$ )                                  | 2.13 Kg                          |
| Payload mass 23 ( $m_{tip-c23}$ )                                  | 2.43 Kg                          |
| Identified Parameters for The Modified Single Lumped Dynamic Model |                                  |
| Flexural rigidity ( $E I_z$ )                                      | $746.57 \text{ N m}^2$           |
| Link mass gravitational contribution ( $m_{link-g}$ )              | 0.138 Kg                         |
| Link mass inertial contribution ( $m_{link-a}$ )                   | 0.035 Kg                         |
| Strain gauge gain constant ( $K_{gauge}$ )                         | 23.14 N m/V                      |
| Identified Parameters for the Modified Two Lumped Dynamic Model    |                                  |
| Flexural rigidity ( $E I_z$ )                                      | $720.05 \text{ N m}^2$           |
| Link mass gravitational contribution ( $m_{link-g}$ )              | 0.303 Kg                         |
| Link mass inertial contribution ( $m_{link-a}$ )                   | 0.126 Kg                         |
| Strain gauge gain constant ( $K_{gauge}$ )                         | $23.27 \text{ N m/V}$            |

Table A.3 Properties of the flexible link 2 (composites) (Part II)

| Identified Parameters for the Modified One Lumped Mass Model<br>(Two Flexible Segments and the Gravity Compensation Mechanism) |                |
|--------------------------------------------------------------------------------------------------------------------------------|----------------|
| Flexural rigidity ( $E I_z$ )                                                                                                  | 841.78 $N m^2$ |
| Link mass gravitational contribution from the base( $m_{link-g1}$ )                                                            | 0.1372 $Kg$    |
| Link mass inertial contribution from the base( $m_{link-a1}$ )                                                                 | 0.1166 $Kg$    |
| Link mass gravitational contribution from the attachment point( $m_{link-g2}$ )                                                | 0.0797 $Kg$    |
| Link mass inertial contribution from the attachment point( $m_{link-a2}$ )                                                     | 0.0322 $Kg$    |
| Strain gauge gain constant ( $K_{gauge-base}$ )                                                                                | 24.40 $N m/V$  |
| Strain gauge gain constant ( $K_{gauge-attach}$ )                                                                              | 20.59 $N m/V$  |
| Spring constant (equivalent to two springs in parallel)( $K_{spring}$ )                                                        | 597.80 $N/m$   |
| Distance between the motor axis and the point<br>of attachment of the spring along the axis $Y_0$ ( $l_s$ )                    | 0.29 $m$       |
| Distance between the motor axis and the point<br>of attachment of the spring along the link ( $l_1$ )                          | 0.25 $m$       |
| Additional parameters                                                                                                          |                |
| 1st vibrational frequency for payload mass $m_{tip-c11}$ ( $\omega_{c11-1st}$ )                                                | 2.97 Hz        |
| 1st vibrational frequency for payload mass $m_{tip-c12}$ ( $\omega_{c12-1st}$ )                                                | 2.34 Hz        |
| 1st vibrational frequency for payload mass $m_{tip-c13}$ ( $\omega_{c13-1st}$ )                                                | 2.15 Hz        |
| 2nd vibrational frequency for payload mass $m_{tip-c11}$ ( $\omega_{c11-2nd}$ )                                                | 44 Hz          |
| 2nd vibrational frequency for payload mass $m_{tip-c12}$ ( $\omega_{c12-2nd}$ )                                                | 44 Hz          |
| 2nd vibrational frequency for payload mass $m_{tip-c13}$ ( $\omega_{c13-2nd}$ )                                                | 44 Hz          |
| 1st vibrational frequency for payload mass $m_{tip-c11}$ ( $\omega_{c21-1st}$ )                                                | 2.01 Hz        |
| 1st vibrational frequency for payload mass $m_{tip-c12}$ ( $\omega_{c22-1st}$ )                                                | 1.87 Hz        |
| 1st vibrational frequency for payload mass $m_{tip-c13}$ ( $\omega_{c23-1st}$ )                                                | 1.75 Hz        |
| 2nd vibrational frequency for payload mass $m_{tip-c11}$ ( $\omega_{c21-2nd}$ )                                                | 39 Hz          |
| 2nd vibrational frequency for payload mass $m_{tip-c12}$ ( $\omega_{c22-2nd}$ )                                                | 39 Hz          |
| 2nd vibrational frequency for payload mass $m_{tip-c13}$ ( $\omega_{c23-2nd}$ )                                                | 39 Hz          |

Table A.4 Technical characteristic of the Harmonic Drive DC Motor with gearbox and break

| Description                                               | Value                           |
|-----------------------------------------------------------|---------------------------------|
| Model                                                     | Harmonic Drive CHA-25A-100-D-SP |
| Gear ratio                                                | 100                             |
| Average backlash                                          | 0°                              |
| Maximum output torque                                     | 157 <i>N m</i>                  |
| Brake holding torque                                      | 157 <i>N m</i>                  |
| Continuous stall torque                                   | 108 <i>N m</i>                  |
| Maximum output speed                                      | 35 <i>rpm</i>                   |
| Torque constant at output ( $K_m$ <i>n</i> )              | 7.4 <i>N m/A</i>                |
| Moment of inertia at output                               | 3.88 <i>Kg m<sup>2</sup></i>    |
| Identified Parameters                                     |                                 |
| Coulomb/static friction constant at output ( $\Gamma_c$ ) | 18.15 <i>N m</i>                |
| Motor viscous friction at output ( $v$ )                  | 1.32 <i>N m s</i>               |
| Moment of inertia at output ( $J$ )                       | 3.78 <i>Kg m<sup>2</sup></i>    |

Table A.5 Technical characteristic of the digital servo drive.

| Description                       | Value                                                                                                                                                    |
|-----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                             | Advanced Motion Controls DPCANTR-040B080                                                                                                                 |
| DC Supply Voltage                 | 20 – 80 <i>VDC</i>                                                                                                                                       |
| Maximum Peak Output Current       | 40 <i>A</i> / (28.3 <i>Arms</i> )                                                                                                                        |
| Maximum Continuous Output Current | 20 <i>A</i>                                                                                                                                              |
| Communication Interfaces          | CANopen                                                                                                                                                  |
| Feedback supported                | ±10 <i>VDC</i> Position, Auxiliary Incremental Encoder, Resolver, Tachometer                                                                             |
| Modes of Operation                | Profile Current, Profile Velocity, Profile Position, Cyclic Synchronous Current Mode, Cyclic Synchronous Velocity Mode, Cyclic Synchronous Position Mode |

Table A.6 Technical characteristic of the resolver.

| Description          | Value                    |
|----------------------|--------------------------|
| Model                | LTN RE-35-1-V02          |
| Pole pairs           | 1                        |
| Transformation ratio | $0.5 \pm 10\%$           |
| Input voltage        | $7 V_{rms}$              |
| Input frequency      | $5 kHz$                  |
| Input current        | typ. $55 mA$             |
| Phase shift          | $+7.5^\circ \pm 3^\circ$ |
| Accuracy             | $\pm 10' spread$         |

Table A.7 Technical characteristic of the external encoder.

| Description             | Value                   |
|-------------------------|-------------------------|
| Model                   | Baumer G0356-175R073000 |
| Type                    | Incremental             |
| Sensing method          | Optical                 |
| Resolution              | $10000 pulses/turn$     |
| Maximum operating speed | $10000 rpm$             |
| Input Voltage           | $4.75$ to $30 V$        |

Table A.8 Technical characteristic of strain gauges for aluminum.

| Description     | Value                  |
|-----------------|------------------------|
| Model           | Kyowa KFRP-5-120-C1-1  |
| Grid length     | $5 mm$                 |
| Grid width      | $1.4 mm$               |
| Gage factor     | $2.10 \pm 1.0 \%$      |
| Gage resistance | $119.8 \pm 0.2 \Omega$ |

Table A.9 Technical characteristic of strain gauges for composite materials.

| Description     | Value                  |
|-----------------|------------------------|
| Model           | Kyowa KFG-5-120-C1-23  |
| Grid length     | $5 mm$                 |
| Grid width      | $1.4 mm$               |
| Gage factor     | $2.1 \pm 1.0 \%$       |
| Gage resistance | $119.8 \pm 0.2 \Omega$ |

Table A.10 Technical characteristic of strain gauges amplifier

| Description       | Value               |
|-------------------|---------------------|
| Model             | Vishay BA660        |
| Accuracy class    | 0.1                 |
| Bridge resistance | 87 to 5000 $\Omega$ |
| Bridge excitation | 5 V                 |
| Bandwidth         | 250 Hz              |
| Output voltage    | 5 V or 10 V         |

Table A.11 Technical characteristic of the magnetic reed switch

| Description        | Value                            |
|--------------------|----------------------------------|
| Model              | Assemtech PTC 130/30 Reed switch |
| Maximum current    | 250 mA                           |
| Maximum AC voltage | 130 V                            |
| Switching distance | 3 mm                             |
| Shape              | Cylindrical                      |
| Total length       | 328 mm                           |

Table A.12 Technical characteristic of the target tracking camera

| Description             | Value                                                   |
|-------------------------|---------------------------------------------------------|
| Model                   | Qualisys ProReflex MCU 240                              |
| Number of cameras       | 3                                                       |
| Maximum sampling time   | 240 Hz                                                  |
| Max number of markers   | 150 at 60 Hz                                            |
| Image sensor resolution | 658 x 500 pixels                                        |
| Resolution              | Depends on the camera placement                         |
| Flash light             | Infra red at 880 nm                                     |
|                         | Standard deviation of 0.5 mm approx. on the calibration |

Table A.13 Technical characteristic of the PXI platform

| Description                   | Value                                                          |
|-------------------------------|----------------------------------------------------------------|
| Chassis model                 | National Instruments PXIe-1082                                 |
| Slot numbers                  | 8                                                              |
| Bandwidth                     | Up to 1 <i>GB/s</i> per slot dedicated,<br>7 <i>GB/s</i> total |
| Embedded controller model     | National Instruments PXIe-8108                                 |
| Processor                     | Intel Core 2 Duo T9400 - 2.53 <i>GHz</i>                       |
| RAM                           | 4 <i>GB</i>                                                    |
| Data acquisition card model   | National Instruments PXIe-6363                                 |
| Analog inputs channels        | 32                                                             |
| Analog input resolution       | 16 <i>bits</i>                                                 |
| Analog input range            | $\pm 10$ <i>V</i>                                              |
| Analog outputs channels       | 4                                                              |
| Analog outputs resolution     | 16 <i>bits</i>                                                 |
| Analog outputs range          | $\pm 10$ <i>V</i>                                              |
| Digital input/output channels | 48                                                             |
| Counters/timers channels      | 4                                                              |

Table A.14 Technical characteristic of the PC

| Description      | Value                                  |
|------------------|----------------------------------------|
| Model            | Toshiba Satellite L500-20z             |
| Processor        | Intel Dual-Core T4400- 2.26 <i>GHz</i> |
| RAM              | 4 <i>GB</i>                            |
| Storage          | 500 <i>GB</i> (5400 <i>rpm</i> ) SATA  |
| Graphics         | Intel Graphics Media Accelerator       |
| Operating System | Windows 7 Home Premium- 64 bits        |

# Appendix B

## Additional formulations

### B.1 Linearization of the modified single loop mass model

Let us to combine the equations of the dynamics (3.26) and (3.27) by eliminating  $\Gamma_{coup}$  to obtain the following single equation

$$\frac{3 E I_z}{l}(\theta_m - \theta_t) = m_a l^2 \frac{d^2 \theta_t}{dt^2} + m_g g l \cos(\theta_t) \quad (B.1)$$

If the system input is defined as  $u = \theta_m$  and  $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]$  is consider to be the state vector, where  $x_1 = \theta_t$  and  $x_2 = \dot{\theta}_t$ , the space state representation of the model can be written as follows

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(\mathbf{x}, u) = \begin{bmatrix} x_2 \\ -\frac{3 E I_z}{m_a l^3} x_1 - \frac{m_g g}{m_a l} \cos(x_1) + \frac{3 E I_z}{m_a l^3} u \end{bmatrix} \quad (B.2)$$

The equilibrium points are defined by  $\mathbf{x}_0 = [0, \theta_{t0}]$  and  $u_0 = \theta_{m0}$ , where  $\theta_{t0}$  and  $u_0$  are related by the following equation

$$\frac{3 E I_z}{l}(\theta_{m0} - \theta_{t0}) = m_g g l \cos(\theta_{t0}) \quad (B.3)$$

The linearized model in terms of the incremental variables  $\mathbf{x}_\delta = \mathbf{x} - \mathbf{x}_0$  and  $u_\delta = u - u_0$  is given by

$$\dot{\mathbf{x}}_\delta = \underbrace{\frac{\partial f(\mathbf{x}, u)}{\partial \mathbf{x}}}_{\text{state matrix}} \mathbf{x}_\delta + \underbrace{\frac{\partial f(\mathbf{x}, u)}{\partial u}}_{\text{input matrix}} u_\delta = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{3 E I_z}{m_a l^3} + \frac{m_g g}{m_a l} \sin(\theta_{t0}) & 0 \end{bmatrix}}_{\text{state matrix}} \mathbf{x}_\delta + \underbrace{\begin{bmatrix} 0 \\ \frac{3 E I_z}{m_a l^3} \end{bmatrix}}_{\text{input matrix}} u_\delta \quad (B.4)$$

The natural frequencies of the system for a certain equilibrium point correspond to the imaginary part of the poles, which can be computed as the eigenvalues of the state matrix of the linearized system. Recalling that  $m_a = m_{tip} + m_{link-a}$  and  $m_g = m_{tip} + m_{link-g}$ , the natural frequencies are given as:

$$\omega(\theta_{t0}) = \sqrt{\frac{3 E I_z}{m_a l^3} - \frac{m_g}{m_a} \frac{g}{l} \sin(\theta_{t0})} \quad (B.5)$$

$$= \sqrt{\frac{3 E I_z}{(m_{tip} + m_{link-a}) l^3} - \frac{(m_{tip} + m_{link-g})}{(m_{tip} + m_{link-a})} \frac{g}{l} \sin(\theta_{t0})} \quad (B.6)$$

## B.2 Euler-Bernoulli cantilever beam formulas

The following formulas cover the case of small deflections of a thin beam that is only subjected to lateral loads and that is fixed at one end and free at the other one. It is assumed that the beam has a mass  $m_{link}$ , a length  $l$ , a constant flexural rigidity along its length  $E I_z$ , and that it supports a payload mass  $m_{tip}$  at its free end with negligible polar moment of inertia.

**Natural frequencies:** the natural frequencies are computed by finding the roots  $\Omega_n$  of the following expression [87]

$$\begin{aligned} \frac{m_{tip} \Omega_n}{m_{link}} (\cos(\Omega_n) \sinh(\Omega_n) - \sin(\Omega_n) \cosh(\Omega_n)) \\ + \cos(\Omega_n) * \cosh(\Omega_n) + 1 = 0 \end{aligned} \quad (B.7)$$

where the correspondence between these roots and the natural frequencies is the following

$$\omega_n = \frac{\Omega_n^2}{2 \pi} \sqrt{\frac{E I_z l}{m_{link}}} \quad (B.8)$$

The previous formulas were developed considering a beam constrained to an horizontal plane, i.e., without any distributed force, as the gravity, acting on it. Interestingly, in our experiments we found that no major vibrational frequencies occur depending on the orientation of the beam at least for the links considered.

**Vertical static deflections:** Considering that the fixed-end of the beam is completely horizontal, which is equivalent to consider the single link flexible robot at  $\theta_m = 0$ , the vertical deflection of the beam at its tip due to the gravity force, determined by the expression [88]

$$\Delta Y = \frac{m_{tip} g l^3}{3 E I_z} + \frac{m_{link} g l^3}{8 E I_z} \quad (\text{B.9})$$

**Torque at the fixed end:** Considering again that the fixed-end of the beam is oriented horizontally, the torque at the fixed end can be computed as [88]

$$\Gamma_{coup-o} = m_{tip} g l + \frac{m_{link} g l}{2} \quad (\text{B.10})$$

