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“Decision making process via binary decision diagram”

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Decision making process via binary decision diagram

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Firms require to optimize any decisions taken for increasing their competitiveness. Each decision will depend of a large number of causes. The relations between the causes and the decisions are effected by the decision making process. An approach to finding the best solution to each logistical or organizational problem is to identify the main causes which are involved for the best decision. A qualitative and a quantitative analysis to facilitate the decision making process is carried out in this paper employing logical decision trees together with binary decision diagrams. This paper presents a novel comparison between different ranking methods applied to several case studies of logical decision trees. The ranking methods considered in this paper are 'Level', 'AND' and the topological heuristic methods Top-Down-Left-Right, Depth-First-Search and Breath-First-Search. Importance measures have been used for ranking the basic events of the logical decision trees according to their contribution to the top event in order to establish the main causes and, therefore, to determine the critical causes. The importance measures methods employed in this paper are: Fussell–Vesely, Birnbaum and criticality. Finally, an analysis of these importance measures is performed that is based on a comparative study of the different methods for setting the importance measures in order to determine the differences between them.

Keywords: decision making; binary decision diagrams; ~~Level; AND; Top-Down-Left-Right; Depth-First-Search; Breath-First-Search; Fussell–Vesely importance measure; Birnbaum importance measure; criticality~~

JEL Classification:

1. Introduction

The decision making process is used to choose a decision over several alternative scenarios (Baker et al., 2001; Harris, 2013). The scenarios are conditioned by constraints found in the resources available in the business (Menguzzato & Renau, 1991).

A main problem can be decomposed into smaller problems, or basic causes (BCs), that can be involved in the creation of that specific problem.

The scheme of the main problem can be represented using logical decision trees (LDTs), where a graphical interpretation can be drawn. This graph will lead to the analysis of the logical relationship between different BCs.

A quantitative study is very complex in cases where there are a large number of BCs. In this paper, the authors propose the transformation of the LDT into binary decision diagrams (BDDs) in order to analyse the problem quantitatively (Artigao, 2009; Pliego, 2012). BDDs can be used to obtain an analytic expression by the Boolean expression for the probability of occurrence of the main problem.

Finally, importance measures are applied to evaluate the BCs in the decision making process.

2. Logical decision tree

Decision trees represent one of the most commonly used tools for depicting an issue in a business so as to

encompass all the alternatives involved. An LDT provides an alternative method for depicting a decision making issue including the interrelations between all the BCs (Lopez & Van Slyke, 1977).

LDTs describe the roots and relationships of a certain problem and its interrelations graphically. The logical operators 'AND' and 'OR' are introduced for a more complex analysis of the main problem (Mallo & Merlo, 1995). These operators will allow a qualitative analysis of the problem and leading to establishing the conversion from LDT to BDD.

It is assumed that a delay in the repair order causes an undesired situation in a business. This will be termed the 'main problem' (see Figure 1).

The terminology used in this paper for the LDT is as follows.

- **Top box.** It will be desirable to minimize its occurrence probability. It will be defined as the 'main problem'.
- **AND logic gates.** These are needed for both descent BCs in order to set the above non-basic cause.
- **OR logic gates.** These are required for only one BC in order to generate the above non-basic cause.
- **Problems inside the circles.** These are the direct causes relating to the main problem. They will be denoted as BCs.

3. Binary decision diagram

BDDs, as a data structure that represents Boolean functions, were introduced by Lee (1959), and further

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Figure 1. Delay in repair orders represented by an LDT.

popularized by Akers (1978), Moret (1982) and Bryant (1986). The BDD is used in order to analyse the DT. It will allow us to obtain an analytical expression depending on the occurrence probability and the logical structure of the tree of every single basic event.

A BDD is a directed acyclic graph (V, N) , with vertex set V and index set N (the position of v in the order of variables). Vertex sets contain two types of branch. On the one hand, a terminal vertex has the following values as attributes: $value(v) \{0,1\}$, where a '1' state corresponds to the occurrence of the main problem, and a '0' state corresponds to the non-occurrence of the main problem. All the paths that have 1 state provide the cut sets of the decision tree. On the other hand, a non-terminal vertex v has as attributes an argument index $index(v) N\{0,1,\dots,n\}$, and two descendants, $low(v)$ and $high(v)$, that are connected by a branch.

Each one has a vertex 0-branch that represents a non-occurrence basic event, or a 1-branch that represents an occurrence basic event. For any non-terminal vertex v , if $low(v)$ is also non-terminal, then $index(v) < index(low(v))$, and if $high(v)$ is non-terminal, then $index(v) < index(high(v))$.

A BDD has a root vertex v that leads to denoting a function f_v defined recursively as follows. Firstly, if v is a terminal vertex and $value(v) = 1$, then $f_v = 1$. In another case, if $value(v) = 0$, then $f_v = 0$. Secondly, if v is a non-terminal vertex with $index(v) = i$, then f_v will be

$$f_v(x_1, \dots, x_n) = x_i f_{low(v)}(x_1, \dots, x_n) + x_i f_{high(v)}(x_1, \dots, x_n). \quad (1)$$

Figure 2 shows a BDD where there are two branches from each variable (b_1, b_2, b_3, b_4). If it is a non-terminal vertex, its children could be either a following variable or a

Figure 2. Binary decision diagram.

1-0 vertex. Terminal vertices are those in which the path is finished, and only 1 or 0 are possible values. Each single variable has two branches. A 0-branch corresponds to cases where the variable's value is 0, and it is graphically represented by a dashed line. On the other hand, 1-branch cases are those where the event is being carried out and they correspond to variable's having values 1, and a solid line depicts them.

Every single path starts with a root vertex and finishes with a terminal vertex. The cut sets (CS) are the paths to the ones and provide significant information due to the fact that the probability of occurrence of the main problem could be achieved from them. In Figure 2, the cut sets are

$$CS_1 = b_1 \cdot b_2 \cdot b_3$$

$$CS_2 = b_1 \cdot \bar{b}_2 \cdot b_3$$

4. LDT to BDD conversion

LDT to BDD conversion provides some advantages in terms of both efficiency and accuracy (Masahiro, Hisanori, & Bobuaki, 1988).

The *Ite* (If-Then-Else) conditional expression is one of the main expressions employed in BDD (Artigao, 2009). An adequate ranking of the BCs is crucial in order to reduce the size, and thus the computational time to solve the BDD. There are many different methods, and any particular one of them will be more appropriate to use according to the problem structure, number of variables, etc. (Lee, 1959).

The proposed ranking for BCs in the conversion shown in Figure 3 is as follows.

- (1) Wrong planning of job position.
- (2) Limited number of tools.
- (3) Low tools reliability.
- (4) Limited connection between departments.
- (5) Manufacturing order not scheduled.

An example of the conversion from the LDT to the BDD is shown in Figure 3.

Once the BDD has been depicted, the cut sets are easily identified. It will be enough to follow those paths which lead from the BC placed in the first position of

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Figure 3. LDT to BDD conversion.

the ranking to terminal vertex with 1 value. Using the method defined, the following cut sets have been obtained:

$$CS_1 = e_1$$

$$CS_2 = \bar{e}_1 \cdot e_2 \cdot e_3$$

$$CS_3 = \bar{e}_1 \cdot e_2 \cdot \bar{e}_3 \cdot e_4 \cdot e_5$$

$$CS_4 = \bar{e}_1 \cdot \bar{e}_2 \cdot e_4 \cdot e_5$$

where e_1 = wrong planning of job position occurrence; e_2 = limited number of tools occurrence; e_3 = low tools reliability occurrence; e_4 = limited connection between departments occurrence; e_5 = manufacturing order not scheduled occurrence; $\bar{e}_i$ = BC defined by 'i' non-occurrence.

Once the cut sets are determined, the next step is to assign probability values to each BC. $P(e_i)$ is defined as the probability of occurrence assigned to the i th BC. An analytic expression for the probability of occurrence of the main problem (PMP) is possible to achieve due to the fact that the different paths are mutually exclusive, and may be expressed as the sum of probabilities of all the BDD paths, as follows:

$$\begin{aligned} PMP &= P(e_1) + (1 - P(e_1)) \cdot P(e_2) \cdot P(e_3) \\ &\quad + (1 - P(e_1)) \cdot P(e_2) \cdot (1 - P(e_3)) \cdot P(e_4) \cdot P(e_5) \\ &\quad + (1 - P(e_1)) \cdot (1 - P(e_2)) \cdot P(e_4) \cdot P(e_5). \end{aligned} \quad (2)$$

5. Importance of the correct BC ordering

BDD size and CPU runtime have a great dependence on variable ordering. Non-optimal variable ordering generates poor efficiency in computational terms due to the increase in the amount of cut sets generated.

BDD variable ordering has been widely treated over the last decades. Nevertheless, today there is no method capable of providing an optimal solution for adapting to every BDD.

Heuristic methods have been used taking into account that the best variable ordering is desired to be achieved. These methods are well known in the BDD environment and may be described as follows.

(1) Topological heuristic methods

- Top-Down-Left-Right*. The tree is read from the top to the bottom and from the left to the right (Bartlett, 2003; Bartlett & Andrews, 2001).
- Depth-First-Search*. The tree is read from top to bottom and starting at its left (Cormen, Leiserson, Rivest, & Stein, 2001).
- Breadth-First-Search*. The tree is read from left to the right and the variable ordering depends on the place they appear in the tree following those steps (Jensen & Veloso, 2000; Rune, Bryant, & Veloso, 2002).

(2) Level method

It makes a difference between the BCs depending on the position in the tree where they are located. It is directly related with the number of gates that there are above the BC. Events that are repeated will have major importance when some BCs are at the same level. In cases where some BCs have the same above logic gates and are located at the same level, those that emerge first will be given major importance (Malik, Wang, Brayton, & Vincentelli, 1988).

(3) AND method

This method takes into account the number of AND logic gates that are in the path to the main problem of a given BC. In such cases, the lower number of AND

Figure 4. Equivalent BDDs from an LDT.

logic gates have the major importance associated have the BC (Xie, Tan, Goh, & Huang, 2000). Fundamentally, it is based on the idea that BCs under an AND logic gate are less important than those below OR logic gates (Bryant, 1986).

In Figure 4 it can be observed that, starting from the same LDT, different BDDs can be obtained by ordering the BCs in different ways.

A set of LDTs has been chosen in order to compare different ranking methods (Pliego, 2012). The best method will correspond to the minimum number of cut sets obtained (see Table 1).

6. Importance measure analysis

Importance measures provide information about the BCs over the LDT. They will be used for weighting each BC according to the state of the main problem. In this paper we have are considered three of the most relevant measures applied to LDTs: Fussell–Vesely, Birnbaum and Criticality. Further information can be found in Umm-e-Habiba and Asghar (2009).

Table 1. Ranking methods analysis according to the cut sets.

	LDT characteristics			Ranking methods				
	No BCs	No AND	No OR	1. TDLR	2. DFS	3. BFS	4. M_Nivel	5. AND
LDT 1	7	3	3	2	2	2	2	2
LDT 2	15	2	11	28	29	39	28	25
LDT 3	15	4	10	30	30	155	30	30
LDT 4	27	3	10	16	47	56	16	16
LDT 5	27	6	7	18	47	61	18	18
LDT 6	22	8	10	216	160	260	120	120
LDT 7	11	5	5	12	24	36	12	12
LDT 8	25	6	16	64	142	176	64	22
LDT 9	20	6	10	99	207	257	99	55
LDT 10	12	5	3	9	7	7	9	9
LDT 11	10	1	7	9	12	21	9	9
LDT 12	20	6	12	44	76	192	44	44
LDT 13	31	10	16	1012	1292	3456	1012	1012

6.1 Fussell–Vesely importance

Fussell–Vesely importance is defined as the quotient between the cut sets that contain a BC itself and the probability of the main problem (Fussell, 1975). Cut sets are required for this measure.

$$I_i^{FV} = \frac{P(CS_1^i \cup CS_2^i \cup CS_3^i \dots CS_n^i)}{PMP}, \quad (3)$$

where I_i^{FV} = Fussell–Vesely importance; $P(CS_1^i \cup CS_2^i \dots \cup CS_n^i)$ = sum of probabilities of all cut sets containing the i th BC; PMP = probability of occurrence of the main problem.

6.2 Birnbaum importance

Birnbaum importance gives the value associated with the direct relation between the probability of the main problem and the corresponding BC (Birnbaum, 1969):

$$I_i^{Birm} = PMP(1_{i,q}) - PMP(0_{i,q}) = \frac{\partial PMP}{\partial q_i}, \quad (4)$$

where I_i^{Birm} = Birnbaum importance value of the i th BC; PMP = probability of occurrence of the main problem; q_i = probability assigned to the i th BC.

6.3 Criticality importance

Criticality importance, unlike Birnbaum, takes into account the probability of occurrence of the BC itself (Lambert, 1975). It rectifies the drawbacks presented in Birnbaum, weighting the values obtained.

$$I_i^{Crit} = \left(\frac{q_i}{PMP} \right) \cdot \left(\frac{\partial PMP}{\partial q_i} \right) = \left(\frac{q_i}{PMP} \right) \cdot I_i^{Birm}, \quad (5)$$

where I_i^{Crit} = criticality importance value of the i th BC; I_i^{Birm} = Birnbaum importance value of the i th BC; PMP = probability of occurrence of the main problem; q_i = probability assigned to the i th BC.

Figure 5. Importance measures analysis.

Table 2. Importance measures outcomes.

Importance measure	BC classification
Fussell–Vesely	$e_4 = e_5 > e_1 > e_2 > e_3$
Birnbaum	$e_1 = e_5 > e_4 > e_2 > e_3$
Criticality	$e_4 = e_5 > e_1 > e_2 > e_3$

Figure 5 shows the results of applying the importance measure methods, using the example given in Figure 1, and assigning probability values to each BC, such as

$$P(e_1) = 0.1; P(e_2) = 0.2; P(e_3) = 0.3; P(e_4) = 0.4; P(e_5) = 0.3.$$

Figure 5 shows that basic causes can be ordered, attending to different importance measures, as follows:

Table 3. Importance measures (IMs) comparison.

Wrong planning of job position occurrence (e_1)		
IM method	Value	Conclusion
Fussell–Vesely	0.3914	This is the third most important BC according to the Fussell–Vesely results, which indicates that there is a 39.14% probability that wrong planning of job position had been one of the causes of the main problem
Birnbaum	0.8272	This is the most important BC according to the Birnbaum results, which means that wrong planning of job position occupies a very important place within the logical structure of the LDT
Criticality	0.3237	This is the second most important BC according to Criticality results. If a comparison between Birnbaum and Criticality is carried out, it is possible to determine that the relevance of this BC does not reside in its probability of occurrence but in its logical position within the LDT structure

Fussell–Vesely provides information about the probability that the main problem is occurring due to a global state in which BC analysed is happening, i.e. it provides the probability that BC has occurred if the main problem BC occurs.

Birnbaum importance provides the different between *PMP* value when BC is occurring and when it is not. Otherwise, it provides information about the *PMP* variation when the probability of the BC analysed has changed. Therefore it can be considered as a structural importance measure because the BC probability is not taken into account and only provides information about the position of the BC within the logical structure of the main problem.

On the other hand, Criticality importance aims to complete the information provided by Birnbaum importance using a correction factor defined as the quotient between the BC probability and the probability of occurrence of the main problem. This correction factor has the same value for every BC in cases where all BCs have the same probability of occurrence.

The analysis made to evaluate the importance of the individual BCs is shown in Table 3.

It is possible to establish an importance ranking according to different selection criteria if every BC is analysed. A weighting procedure can be carried out in order to specify a particular order for the importance of the BC.

7. Conclusions

This paper described an efficient way to carry out a probabilistic assessment in order to conduct a decision making process through the use of probability data concerning BCs.

8.

The LDT representation leads to a qualitative analysis of the problem. A quantitative analysis of the LDT is proposed via an analytical expression about the probability of occurrence of the main problem obtained by BDDs. The conversion from LDT to BDD requires a good variable ordering in order to be efficient. Different ranking methods are explained and a comparison made between them on their evaluation of differently structured LDTs. The methods considered are: Level, AND and the topological heuristic methods Top-Down-Left-Right, Depth-First-Search and Breath-First-Search. This study is new in the literature.

The expression provided by BDDs allows the application of different techniques of importance measures. These techniques provide information about the relevance of the BC in order to optimize the decision making process. A novel comparison between the Fussell–Vesely, Birnbaum and Criticality methods is carried out in order to know the BC. The value of importance measure is due to either its intrinsic probability of occurrence or to its position within the logical structure of the main problem defined by the corresponding LDT.

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