Incorporating Price-Responsive Demand in Energy Scheduling Based on Consumer Payment Minimization

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Abstract—In the restructured power industry, demand responsiveness is a key factor whose importance will be boosted due to the impetus provided by the development of smart grids. Within the context of pool-based electricity markets, this paper addresses the incorporation of price-responsive demand in multiperiod energy scheduling driven by consumer payment minimization. Although consumer payment minimization has drawn considerable attention mainly in an operational setting and also recently under a planning framework, available models and solution approaches typically neglect demand-side participation. The proposed scheduling model considers a marginal pricing scheme as well as the effects of both network constraints and intertemporal constraints associated with generation operation. Modeling demand-side participation leads to bilinear payment terms that significantly increase the mathematical complexity of the optimization process. The resulting problem is formulated as a mixed-integer nonlinear bilevel program for which no exact solution technique is currently available. This paper presents a novel methodology by which the original bilevel and bilinear problem is converted into an equivalent single-level mixed-integer linear program suitable for efficient off-the-shelf software. This transformation is based on the application of duality theory of linear programming, integer algebra, and Karush–Kuhn–Tucker optimality conditions. The proposed approach has been successfully applied to the IEEE 118-bus system.

Index Terms—Bilevel programming, bilinear terms, consumer payment minimization, demand response, energy scheduling, marginal pricing, mixed-integer linear equivalent.

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NOMENCLATURE

Indices

b	Demand bid block.
i	Generating unit.
j	Consumer.
l	Transmission line.
n	Bus.
$n(i)$	Bus where generating unit i is located.
$n(j)$	Bus where consumer j is located.
o	Generation offer block.
t	Time period.

Sets

$\mathcal{B}_j$	Indices of the demand bid blocks of consumer j .
$\mathcal{F}_{it}$	Feasibility set for the scheduling variables associated with unit i in period t .
$\mathcal{I}$	Indices of generating units.
$\mathcal{I}_n$	Indices of the generating units located at bus n .
$\mathcal{J}$	Indices of consumers.
$\mathcal{J}_n$	Indices of the consumers located at bus n .
$\mathcal{L}_t^+$	Indices of transmission lines for the constraints imposing the upper bounds on line power flows in period t .
$\mathcal{L}_t^-$	Indices of transmission lines for the constraints imposing the lower bounds on line power flows in period t .
$\mathcal{N}$	Indices of buses.
$\mathcal{O}_i$	Indices of the generation offer blocks of unit i .
$\mathcal{T}$	Indices of time periods.

Constants

A_{ln}	Sensitivity factor for line l and bus n .
C_{bjt}^d	Price of the b th demand block bid by consumer j in period t .
C_{oit}^g	Price of the o th generation block offered by unit i in period t .
$n_{\mathcal{B}_j}$	Cardinality of $\mathcal{B}_j$.
n_I	Number of generating units.
n_J	Number of consumers.
$n_{\mathcal{L}_t^+}$	Cardinality of $\mathcal{L}_t^+$.
$n_{\mathcal{L}_t^-}$	Cardinality of $\mathcal{L}_t^-$.
n_N	Number of buses.
$n_{\mathcal{O}_i}$	Cardinality of $\mathcal{O}_i$.
n_T	Number of time periods.

$\bar{P}_{bjt}^d$	Upper bound for p_{bjt}^d .
$\underline{P}_{jt}^d$	Lower bound for p_{jt}^d .
$\bar{P}_{jt}^d$	Upper bound for p_{jt}^d .
$\bar{P}_l^f$	Power flow capacity of line l .
P_{i0}^g	Initial power output of unit i .
$\underline{P}_{it}^g$	Lower bound for p_{it}^g .
$\bar{P}_{it}^g$	Upper bound for p_{it}^g .
$\bar{P}_{oit}^g$	Upper bound for p_{oit}^g .
R_i^{dn}	Ramp-down rate of unit i .
R_i^{sd}	Shut-down ramp rate of unit i .
R_i^{su}	Start-up ramp rate of unit i .
R_i^{up}	Ramp-up rate of unit i .
V_{i0}	Initial on/off status of unit i .
$\underline{\gamma}_{it}^{\text{lo}}$	Lower bound for γ_{it}^{lo} .
$\bar{\gamma}_{it}^{\text{up}}$	Upper bound for γ_{it}^{up} .
$\bar{\epsilon}_{it}$	Upper bound for ϵ_{it} .
$\bar{\xi}_{it}$	Upper bound for ξ_{it} .
$\underline{\sigma}_{it}$	Lower bound for σ_{it} .

Variables

a_{it}	Auxiliary variable equal to the product $v_{it}\gamma_{it}^{\text{lo}}$.
b_{it}	Auxiliary variable equal to the product $v_{it}\gamma_{it}^{\text{up}}$.
c_{it}	Auxiliary variable equal to the product $v_{it}\xi_{it}$.
d_{it}	Auxiliary variable equal to the product $v_{it-1}\xi_{it}$.
e_{it}	Auxiliary variable equal to the product $v_{it}\epsilon_{it}$.
k_{it}	Auxiliary variable equal to the product $v_{it+1}\epsilon_{it}$.
m_{it}	Auxiliary variable equal to the product $v_{it}\sigma_{it}$.
p_{bjt}^d	Consumption level awarded to block b bid by consumer j in period t .
p_{jt}^d	Power consumption of consumer j in period t .
p_{it}^g	Power output of unit i in period t .
p_{oit}^g	Generation level awarded to block o offered by unit i in period t .
r_{it}	Auxiliary variable equal to the product $v_{it-1}\sigma_{it}$.
s_{it}	Payment for the start-up of unit i in period t .
v_{it}	On/off status of unit i in period t that is equal to 1 if unit i is scheduled on in period t , being 0 otherwise.
α_{it}	Dual variable associated with the definition of p_{it}^g as the sum of the generation levels awarded to the offer blocks.
β_{oit}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{oit}^g .
β_{oit}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{oit}^g .
γ_{it}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{it}^g .
γ_{it}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{it}^g .
δ_{jt}	Dual variable associated with the definition of p_{jt}^d as the sum of the consumption levels awarded to the demand bid blocks.
ϵ_{it}	Dual variable associated with the shut-down ramp rate constraint of unit i in period t .

λ_t	Dual variable associated with the power balance equation in period t .
μ_{bjt}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{bjt}^d .
μ_{bjt}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{bjt}^d .
ξ_{it}	Dual variable associated with the constraint modeling the ramp-up and start-up ramp rates of unit i in period t .
π_{nt}	Locational marginal price (LMP) at bus n in period t .
ρ_{jt}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{jt}^d .
ρ_{jt}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{jt}^d .
σ_{it}	Dual variable associated with the ramp-down rate constraint of unit i in period t .
ϕ_{lt}^{lo}	Dual variable associated with the constraint imposing the lower bound for the power flow of line l in period t .
ϕ_{lt}^{up}	Dual variable associated with the constraint imposing the upper bound for the power flow of line l in period t .

I. INTRODUCTION

SINCE THE pioneering work of Jacobs [1], consumer payment minimization has triggered extensive research in the fields of energy scheduling [2]–[15] and investment planning [16]. However, most works have neglected the effect of demand response, which is crucial for guaranteeing the efficiency and competitiveness of the restructured power industry [17]–[24]. The advent of smart grids provides a suitable framework to boost the long-desired participation of demand at both wholesale market level [17]–[19], [21] and distribution level [20], [22]–[24]. Therefore, the new context where smart grids come into play calls for changes in the operational and planning models that have typically disregarded demand-side participation such as those wherein the consumer payment is minimized [2]–[4], [7]–[9], [11]–[16].

One of those problems is wholesale energy scheduling based on consumer payment minimization in pool-based electricity markets [2]–[4], [7]–[9], [11]–[15]. In the presence of price-responsive demand, the solution to this problem provides the system operator with the set of awarded generation offers and demand bids that yields the minimum payment to consumers. Such market-clearing procedure is implemented while meeting the operational constraints associated with electricity generation and consumption and keeping the privacy of the information submitted by market participants. Energy scheduling driven by consumer payment minimization has been proposed as a remedy to a practical issue in pool-based electricity markets, namely the lack of incentives for suppliers to offer their actual production costs, which are private corporate information. Thus, consumer payment minimization can be viewed as an instrument to protect consumers against dishonest behavior and market power abuse by generation companies.

Rather than discussing the suitability of consumer payment minimization as an adequate optimization goal for wholesale energy scheduling, this paper addresses the challenges and issues associated with the consideration of price-responsive demand, i.e., demand-side bidding, in such operational problem. From a mathematical viewpoint, this practical aspect gives rise to a major complicating feature. Since the consumer payment is defined in terms of products of energy prices and consumption levels, energy prices become decision variables of the optimization process, thereby requiring an explicit mathematical characterization in the problem formulation. Moreover, incorporating demand responsiveness implies extending the set of decision variables to include consumption levels. Thus, the aforementioned payment terms involve products of two continuous variables. It is worth emphasizing that no exact general methodology is currently available to deal with the resulting optimization problem involving such bilinear and highly nonconvex terms.

To the best of the authors' knowledge, demand-side participation has only been accounted for in the framework of consumer payment minimization in [5], [6], and [10]. Demand bids were theoretically considered in the two-layer structure presented in [5] to characterize LMPs but no solution methodology was provided. In [6] and [10], demand-side bidding was also taken into account in the consumer payment minimization problem. However, rather than standard marginal pricing [25], [26], both models adopted alternative pricing schemes which may be of reduced practical applicability. Hence, new models and methodologies are yet to be explored.

The thrust of this paper is the proposal of a novel approach for multiperiod energy scheduling based on consumer payment minimization wherein demand-side bidding is explicitly considered under the widely used marginal pricing scheme. As done in [8] and [15] for the case with inelastic demand, a bilevel programming framework [27] is used to properly account for marginal pricing. The schedule of generating units is determined in the upper-level problem, whereas generation and consumption levels result from the lower-level problem. In addition, dual information from the lower level is used to compute LMPs in the upper level. The resulting problem is formulated as a mixed-integer nonlinear bilevel program with bilinear terms. The presence of bilinear payment terms constitutes the main distinctive modeling aspect with respect to the models described in [8] and [15]. As above mentioned, such salient feature renders the problem unsuitable for existing solution techniques.

The key idea of the proposed solution approach is to convert the original mixed-integer nonlinear bilevel problem into an equivalent single-level mixed-integer linear program. To that end, a two-step procedure is implemented. First, similar to [15], practical modeling aspects allow applying an effective primal-dual transformation [28] and integer algebra results [29] to recast the bilevel scheduling model as a single-level equivalent. It is worth mentioning that, unlike in [15], the resulting single-level optimization is a mixed-integer program with bilinear payment terms for which a global solution method is also unavailable. As a distinctive methodological feature of the proposed approach, a novel,

exact, and computationally inexpensive linearization scheme based on duality theory of linear programming and Karush–Kuhn–Tucker (KKT) optimality conditions is subsequently applied to yield an equivalent single-level mixed-integer linear program. Mixed-integer linear programming guarantees finite convergence to optimality [30]. Moreover, efficient off-the-shelf branch-and-cut software is available [31].

This paper thus features three major novelties with respect to previous works on wholesale energy scheduling driven by consumer payment minimization [2]–[15].

- 1) Unlike [2]–[4], [7]–[9], and [11]–[15], demand-side bidding is explicitly accounted for.
- 2) In contrast to [6] and [10], the model includes locational marginal pricing and intertemporal operational constraints.
- 3) A sound and effective solution approach is proposed.

More specifically, the main contributions of this paper are threefold.

- 1) From a modeling perspective, a novel formulation for energy scheduling driven by consumer payment minimization under marginal pricing is presented wherein price-responsive demand is explicitly incorporated. The resulting model, which includes transmission network constraints, intertemporal operational constraints, and marginal pricing, is formulated as a mixed-integer nonlinear bilevel program with bilinear terms.
- 2) From a methodological perspective, this paper provides a novel and exact linearization of the bilinear terms associated with the consumer payment for energy. Such linearization, in conjunction with a previously reported primal-dual transformation and well-known integer algebra results, allows formulating the original bilevel problem with bilinear terms as a single-level mixed-integer linear equivalent, with no additional binary variables, for which efficient solvers exist.
- 3) Numerical experience is reported from solving a case study significantly larger than those analyzed in previous works addressing demand-side participation in the consumer payment minimization problem.

The ultimate purpose of this paper is to provide the system operator and market agents with a tool that allows a comprehensive assessment of energy scheduling driven by consumer payment minimization so that informed decisions can be made on its eventual implementation in real-world pool-based electricity markets. Moreover, the proposed approach may be useful for the system operator and the regulator in two respects, namely: 1) to monitor the behavior of market agents when the consumer payment is minimized under marginal pricing; and 2) to examine the short- and long-term implications of this trading scheme. Such analyses, which are beyond the scope of this paper, may require dynamic game theoretical models or equilibrium-based models wherein the proposed tool would play a key role. Relevant related works are those by Vázquez *et al.* [4] and Zhao *et al.* [9], where simplified models and approaches disregarding demand-side participation were used.

The remainder of this paper is outlined as follows. Section II presents the formulation for the scheduling model with

price-responsive demand. In Section III, the proposed solution approach is described. Numerical results are discussed in Section IV. Finally, the conclusion is drawn in Section V.

II. PROBLEM FORMULATION

This section presents the mathematical formulation for the network-constrained multiperiod energy scheduling problem driven by consumer payment minimization with price-responsive demand under marginal pricing. Within the context of current industry practice regarding demand-side bidding in pool-based electricity markets [32]–[34], the focus is placed on demand price responsiveness in the form of bids including pairs quantity-price and bounds on consumption levels. This model is simpler to describe and analyze, yet bringing out the issue of demand-side participation in the consumer payment minimization problem. Based on [8] and [15], the proposed auction model can be formulated as the following bilevel program:

$$\text{Minimize}_{s_{it}, v_{it}, \pi_{nt}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \pi_{n(j)t} p_{jt}^d + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} s_{it} \quad (1)$$

subject to

$$\pi_{nt} = \lambda_t - \sum_{l \in \mathcal{L}_t^+} A_{ln} \phi_{lt}^{\text{up}} - \sum_{l \in \mathcal{L}_t^-} A_{ln} \phi_{lt}^{\text{lo}}; \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \quad (2)$$

$$s_{it}, v_{it} \in \mathcal{F}_{it}; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (3)$$

$$v_{it} \in \{0, 1\}; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (4)$$

where p_{jt}^d , λ_t , ϕ_{lt}^{up} , and ϕ_{lt}^{lo} are obtained from the following multiperiod dc optimal power flow:

$$\text{Maximize}_{p_{bjt}^d, p_{jt}^d, p_{oit}^g, p_{oit}^s} \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \quad (5)$$

subject to

$$\sum_{j \in \mathcal{J}} p_{jt}^d = \sum_{i \in \mathcal{I}} p_{it}^g : (\lambda_t); \quad \forall t \in \mathcal{T} \quad (6)$$

$$\sum_{n \in \mathcal{N}} A_{ln} \left(\sum_{i \in \mathcal{I}_n} p_{it}^g - \sum_{j \in \mathcal{J}_n} p_{jt}^d \right) \leq \bar{P}_l^f : (\phi_{lt}^{\text{up}}) \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}_t^+ \quad (7)$$

$$\sum_{n \in \mathcal{N}} A_{ln} \left(\sum_{i \in \mathcal{I}_n} p_{it}^g - \sum_{j \in \mathcal{J}_n} p_{jt}^d \right) \geq -\bar{P}_l^f : (\phi_{lt}^{\text{lo}}) \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}_t^- \quad (8)$$

$$p_{it}^g v_{it} \leq p_{it}^g \leq \bar{P}_{it}^g v_{it} : (\gamma_{it}^{\text{lo}}, \gamma_{it}^{\text{up}}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (9)$$

$$p_{jt}^d \leq p_{jt}^d \leq \bar{P}_{jt}^d : (\rho_{jt}^{\text{lo}}, \rho_{jt}^{\text{up}}); \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (10)$$

$$p_{it}^g = \sum_{o \in \mathcal{O}_i} p_{oit}^g : (\alpha_{it}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (11)$$

$$0 \leq p_{oit}^g \leq \bar{P}_{oit}^g : (\beta_{oit}^{\text{lo}}, \beta_{oit}^{\text{up}}); \quad \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \quad \forall t \in \mathcal{T} \quad (12)$$

$$p_{jt}^d = \sum_{b \in \mathcal{B}_j} p_{bjt}^d : (\delta_{jt}); \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (13)$$

$$0 \leq p_{bjt}^d \leq \bar{P}_{bjt}^d : (\mu_{bjt}^{\text{lo}}, \mu_{bjt}^{\text{up}}); \quad \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \quad \forall t \in \mathcal{T} \quad (14)$$

$$p_{it}^g \leq p_{it-1}^g + R_i^{\text{up}} v_{it-1} + R_i^{\text{su}} (v_{it} - v_{it-1}) + \bar{P}_{it}^g (1 - v_{it}) : (\xi_{it}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (15)$$

$$p_{it}^g \geq p_{it-1}^g - R_i^{\text{dn}} v_{it} - R_i^{\text{sd}} (v_{it-1} - v_{it}) - \bar{P}_{it}^g (1 - v_{it-1}) : (\sigma_{it}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (16)$$

$$p_{it}^g \leq R_i^{\text{sd}} (v_{it} - v_{it+1}) + \bar{P}_{it}^g v_{it+1} : (\epsilon_{it}) \forall i \in \mathcal{I}, \quad \forall t = 1 \dots n_T - 1. \quad (17)$$

For the sake of consistency, hourly time periods are considered; $\sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d = \bar{P}_{jt}^d, \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$; and $\sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g = \bar{P}_{it}^g, \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$.

The adoption of a marginal pricing scheme motivates the bilevel programming structure of (1)–(17). Under marginal pricing [25], locational energy prices for a given feasible schedule, represented by variables v_{it} , are characterized in terms of the Lagrange multipliers or dual variables associated with a multiperiod optimal power flow wherein the declared social welfare is maximized once scheduling variables v_{it} have been fixed [35].

Moreover, based on [5], [6], and [10] and practical auction designs [32]–[34], demand-side participation is limited to price-responsive bids with consumption bounds. Notwithstanding, the mathematical structure of the proposed market-clearing procedure would remain essentially unaltered should other forms of demand response be modeled including on/off consumption scheduling, limited energy consumption over the scheduling horizon, consumption ramping limits, load shifting, operational consumption cycles, time-coupled price elasticity, among others [18]–[24].

The proposed bilevel model (1)–(17) consists of two optimization levels, namely the upper level (1)–(4) and the lower level (5)–(17). Upper-level variables are s_{it} and v_{it} , associated with start-up offers and on/off statuses of generating units, respectively, as well as LMPs π_{nt} . Lower-level variables are power outputs p_{it}^g , power consumptions p_{jt}^d , and awarded levels of generation offer and demand bid blocks p_{oit}^g and p_{bjt}^d , respectively. Note that Lagrange multipliers or dual variables of the lower level are shown in parentheses in (6)–(17).

The upper-level problem minimizes the consumer payment (1), which comprises two terms. The first term represents the energy payment, which, as a major complicating feature, includes bilinear terms involving the products of two continuous variables, namely LMPs and consumption levels. The second term is related to the payment for start-ups. As done in [5], [7]–[9], and [11]–[15], start-up offers are fully compensated in (1).

According to [26], LMPs are defined in (2), where the first term in the right-hand side represents the energy component and the other two terms are the congestion components. Energy and congestion components correspond to the Lagrange multipliers or dual variables associated with the power balance equations and line capacity constraints, respectively, in the lower-level problem. As described in [36], sets $\mathcal{L}_t^+$ and $\mathcal{L}_t^-$ only include the indices of those lines that are prone to experience congestion as determined by

a computationally inexpensive screening performed prior to the optimization process. Expressions (3) characterize generation constraints exclusively involving scheduling variables, i.e., minimum up and down times as well as start-ups. A detailed description of these linear constraints can be found in [37]. Expressions (4) set the integrality of the scheduling variables v_{it} .

In contrast, the lower-level problem maximizes the declared social welfare (5) through a multiperiod optimal power flow particularized in terms of the scheduling variables v_{it} . As is customary in day-ahead energy scheduling [8], [35], [36], [38]–[41], a dc load flow model is used to characterize the behavior of the transmission network, recognizing that the use of such a simplified model leads to results that may be optimistic and that a complete study of the scheduling problem should also consider the effect of reactive power. This generalization would, however, render the problem essentially intractable. This modeling limitation notwithstanding, the solution of the energy scheduling problem based on the dc load flow is acceptable for the purposes of the day-ahead operation of power systems [8], [35], [36], [38]–[41].

Using a model relying on linear sensitivity factors [39], the effect of the transmission network is considered in (6)–(8). Constraints (6) represent the power balance equations. As shown in [36], expressions (7) and (8) guarantee the fulfillment of line capacity limits across the transmission network. Note that the cardinalities of $\mathcal{L}_t^+$ and $\mathcal{L}_t^-$ are less than the number of transmission lines. Hence, the dimension of the model is reduced, thereby leading to computational savings. The interested reader is referred to [36] for the proof of the equivalence between this model and that considering power flow limits for the whole set of transmission lines. Production and consumption limits are set in (9) and (10), respectively. The block structures of generation offers and demand bids are characterized through constraints (11)–(12) and (13)–(14), respectively. Finally, ramping rates are modeled by (15)–(17) as described in [37].

III. SOLUTION APPROACH

The proposed solution approach consists in reformulating the original nonlinear bilevel program (1)–(17) as an equivalent single-level mixed-integer linear programming problem. To that end, duality theory of linear programming, integer algebra, and KKT optimality conditions are used as described next.

A. Mixed-Integer Linear Equivalent for the Lower-Level Problem

Based on the findings of [28], the lower-level problem can be equivalently replaced by its primal feasibility constraints (6)–(17), its dual feasibility constraints, and the equality corresponding to the strong duality theorem wherein the primal and dual objective functions of the lower-level problem are equated. It should be noted that duality theory of linear programming applies here since the lower-level optimization problem (5)–(17) is linear in the continuous lower-level primal variables p_{ijt}^d , p_{jt}^d , p_{it}^g , p_{oit}^g , and since binary variables v_{it} , determined by the upper-level problem, can be considered as parameters in the lower-level problem.

Lower-level dual feasibility constraints are given by

$$-\lambda_t + \sum_{l \in \mathcal{L}_t^+} A_{ln(i)} \phi_{lt}^{\text{up}} + \sum_{l \in \mathcal{L}_t^-} A_{ln(i)} \phi_{lt}^{\text{lo}} + \gamma_{it}^{\text{lo}} + \gamma_{it}^{\text{up}} + \xi_{it} - \xi_{it+1} + \sigma_{it} - \sigma_{it+1} + \epsilon_{it} + \alpha_{it} = 0; \quad \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (18)$$

$$-\lambda_{n_T} + \sum_{l \in \mathcal{L}_{n_T}^+} A_{ln(i)} \phi_{ln_T}^{\text{up}} + \sum_{l \in \mathcal{L}_{n_T}^-} A_{ln(i)} \phi_{ln_T}^{\text{lo}} + \gamma_{in_T}^{\text{lo}} + \gamma_{in_T}^{\text{up}} + \xi_{in_T} + \sigma_{in_T} + \alpha_{in_T} = 0; \quad \forall i \in \mathcal{I} \quad (19)$$

$$-\alpha_{it} + \beta_{oit}^{\text{lo}} + \beta_{oit}^{\text{up}} = -C_{oit}^g; \quad \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (20)$$

$$\lambda_t + \delta_{jt} - \sum_{l \in \mathcal{L}_t^+} A_{ln(j)} \phi_{lt}^{\text{up}} - \sum_{l \in \mathcal{L}_t^-} A_{ln(j)} \phi_{lt}^{\text{lo}} + \rho_{jt}^{\text{lo}} + \rho_{jt}^{\text{up}} = 0; \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (21)$$

$$-\delta_{jt} + \mu_{bjt}^{\text{lo}} + \mu_{bjt}^{\text{up}} = C_{bjt}^d; \quad \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (22)$$

$$\gamma_{it}^{\text{lo}}, \sigma_{it} \leq 0, \gamma_{it}^{\text{up}}, \xi_{it} \geq 0; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (23)$$

$$\epsilon_{it} \geq 0; \quad \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (24)$$

$$\rho_{jt}^{\text{lo}} \leq 0, \rho_{jt}^{\text{up}} \geq 0; \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (25)$$

$$\phi_{lt}^{\text{up}} \geq 0; \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}_t^+ \quad (26)$$

$$\phi_{lt}^{\text{lo}} \leq 0; \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}_t^- \quad (27)$$

$$\beta_{oit}^{\text{lo}} \leq 0, \beta_{oit}^{\text{up}} \geq 0; \quad \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (28)$$

$$\mu_{bjt}^{\text{lo}} \leq 0, \mu_{bjt}^{\text{up}} \geq 0; \quad \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (29)$$

while the equality associated with the strong duality theorem is formulated as

$$\begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \\ &= \sum_{i \in \mathcal{I}} \sum_{l \in \mathcal{L}_t^+} \bar{P}_l^f \phi_{lt}^{\text{up}} - \sum_{i \in \mathcal{I}} \sum_{l \in \mathcal{L}_t^-} \bar{P}_l^f \phi_{lt}^{\text{lo}} \\ &+ \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\bar{P}_{it}^g v_{it} \gamma_{it}^{\text{lo}} + \bar{P}_{it}^g v_{it} \gamma_{it}^{\text{up}} + \sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g \beta_{oit}^{\text{up}} \right) \\ &+ \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \left(\bar{P}_{jt}^d \rho_{jt}^{\text{lo}} + \bar{P}_{jt}^d \rho_{jt}^{\text{up}} + \sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d \mu_{bjt}^{\text{up}} \right) \\ &+ \sum_{i \in \mathcal{I}} \left[P_{i0}^g \xi_{i1} + R_i^{\text{up}} V_{i0} \xi_{i1} + R_i^{\text{su}} (v_{i1} \xi_{i1} - V_{i0} \xi_{i1}) \right. \\ &\quad \left. + \bar{P}_{i1}^g (\xi_{i1} - v_{i1} \xi_{i1}) \right] \\ &+ \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{\text{up}} v_{it-1} \xi_{it} + R_i^{\text{su}} (v_{it} \xi_{it} - v_{it-1} \xi_{it}) \right. \\ &\quad \left. + \bar{P}_{it}^g (\xi_{it} - v_{it} \xi_{it}) \right] \\ &+ \sum_{i \in \mathcal{I}} \left[P_{i0}^g \sigma_{i1} - R_i^{\text{dn}} v_{i1} \sigma_{i1} - R_i^{\text{sd}} (V_{i0} \sigma_{i1} - v_{i1} \sigma_{i1}) \right. \\ &\quad \left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right] \\ &- \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{\text{dn}} v_{it} \sigma_{it} + R_i^{\text{sd}} (v_{it-1} \sigma_{it} - v_{it} \sigma_{it}) \right. \\ &\quad \left. + \bar{P}_{it}^g (\sigma_{it} - v_{it-1} \sigma_{it}) \right] \\ &+ \sum_{i \in \mathcal{I}} \sum_{t=1}^{n_T-1} \left[R_i^{\text{sd}} (v_{it} \epsilon_{it} - v_{it+1} \epsilon_{it}) + \bar{P}_{it}^g v_{it+1} \epsilon_{it} \right]. \quad (30) \end{aligned}$$

Expression (30) is nonlinear due to the presence of products of binary variables v_{it} and continuous Lagrange multipliers or dual variables associated with the lower-level problem. Based on [29], such nonlinearities can be equivalently recast as linear expressions. Thus, the nonlinear strong duality equality (30) is replaced by

$$\begin{aligned}
& \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \\
& = \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}_t^+} \bar{P}_l^f \phi_{lt}^{\text{up}} - \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}_t^-} \bar{P}_l^f \phi_{lt}^{\text{lo}} \\
& + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\bar{P}_{it}^g a_{it} + \bar{P}_{it}^g b_{it} + \sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g \beta_{oit}^{\text{up}} \right) \\
& + \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \left(\bar{P}_{jt}^d \rho_{jt}^{\text{lo}} + \bar{P}_{jt}^d \rho_{jt}^{\text{up}} + \sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d \mu_{bjt}^{\text{up}} \right) \\
& + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \xi_{i1} + R_i^{\text{up}} V_{i0} \xi_{i1} + R_i^{\text{su}} (c_{i1} - V_{i0} \xi_{i1}) \right. \\
& \quad \left. + \bar{P}_{i1}^g (\xi_{i1} - c_{i1}) \right] \\
& + \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{\text{up}} d_{it} + R_i^{\text{su}} (c_{it} - d_{it}) + \bar{P}_{it}^g (\xi_{it} - c_{it}) \right] \\
& + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \sigma_{i1} - R_i^{\text{dn}} m_{i1} - R_i^{\text{sd}} (V_{i0} \sigma_{i1} - m_{i1}) \right. \\
& \quad \left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right] \\
& - \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{\text{dn}} m_{it} + R_i^{\text{sd}} (r_{it} - m_{it}) + \bar{P}_{it}^g (\sigma_{it} - r_{it}) \right] \\
& + \sum_{i \in \mathcal{I}} \sum_{t=1}^{n_T-1} \left[R_i^{\text{sd}} (e_{it} - k_{it}) + \bar{P}_{it}^g k_{it} \right] \\
& \gamma_{it}^{\text{lo}} v_{it} \leq a_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (32) \\
& \gamma_{it}^{\text{lo}} (1 - v_{it}) \leq \gamma_{it}^{\text{lo}} - a_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (33) \\
& 0 \leq b_{it} \leq \bar{\gamma}_{it}^{\text{up}} v_{it}; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (34) \\
& 0 \leq \gamma_{it}^{\text{up}} - b_{it} \leq \bar{\gamma}_{it}^{\text{up}} (1 - v_{it}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (35) \\
& 0 \leq c_{it} \leq \bar{\xi}_{it} v_{it}; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (36) \\
& 0 \leq \xi_{it} - c_{it} \leq \bar{\xi}_{it} (1 - v_{it}); \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (37) \\
& 0 \leq d_{it} \leq \bar{\xi}_{it} v_{it-1}; \quad \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (38) \\
& 0 \leq \xi_{it} - d_{it} \leq \bar{\xi}_{it} (1 - v_{it-1}); \quad \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (39) \\
& 0 \leq e_{it} \leq \bar{\epsilon}_{it} v_{it}; \quad \forall i \in \mathcal{I}, \quad \forall t = 1 \dots n_T - 1 \quad (40) \\
& 0 \leq \epsilon_{it} - e_{it} \leq \bar{\epsilon}_{it} (1 - v_{it}); \quad \forall i \in \mathcal{I}, \quad \forall t = 1 \dots n_T - 1 \quad (41) \\
& 0 \leq k_{it} \leq \bar{\epsilon}_{it} v_{it+1}; \quad \forall i \in \mathcal{I}, \quad \forall t = 1 \dots n_T - 1 \quad (42) \\
& 0 \leq \epsilon_{it} - k_{it} \leq \bar{\epsilon}_{it} (1 - v_{it+1}); \quad \forall i \in \mathcal{I}, \quad \forall t = 1 \dots n_T - 1 \quad (43) \\
& \underline{\sigma}_{it} v_{it} \leq m_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (44) \\
& \underline{\sigma}_{it} (1 - v_{it}) \leq \sigma_{it} - m_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (45)
\end{aligned}$$

$$\underline{\sigma}_{it} v_{it-1} \leq r_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (46)$$

$$\underline{\sigma}_{it} (1 - v_{it-1}) \leq \sigma_{it} - r_{it} \leq 0; \quad \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (47)$$

where (31) is the linearized strong duality equality, whereas (32)–(33), (34)–(35), (36)–(37), (38)–(39), (40)–(41), (42)–(43), (44)–(45), and (46)–(47) correspond to the linearization of $v_{it} \gamma_{it}^{\text{lo}}$, $v_{it} \gamma_{it}^{\text{up}}$, $v_{it} \xi_{it}$, $v_{it-1} \xi_{it}$, $v_{it} \epsilon_{it}$, $v_{it+1} \epsilon_{it}$, $v_{it} \sigma_{it}$, and $v_{it-1} \sigma_{it}$, respectively.

Therefore, the mixed-integer linear equivalent for the lower-level problem includes (6)–(29) and (31)–(47).

B. Linear Equivalent for the Energy Payment

As above mentioned, and as a major distinctive modeling aspect with respect to previous works disregarding demand responsiveness [8], [15], the total payment for energy in (1) comprises bilinear terms involving the products of LMPs and consumption levels, which are both continuous decision variables. Here, we propose a novel and exact linearization of such bilinear terms based on duality theory of linear programming and KKT optimality conditions. From both modeling and computational perspectives, the proposed linearization is characterized by two relevant salient features over other methodologies addressing bilinear terms such as the binary expansion approach [42] and Schur's decomposition [43]: 1) rather than approximations, the resulting linear expressions are equivalent to the original bilinear products and 2) no additional binary variables are required.

The proposed transformation uses equalities (2) and (13), lower-level dual feasibility constraints (21) and (22), as well as the following complementary slackness conditions, which are associated with lower-level constraints (10) and (14):

$$\rho_{jt}^{\text{lo}} (p_{jt}^d - \bar{p}_{jt}^d) = 0; \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (48)$$

$$\rho_{jt}^{\text{up}} (p_{jt}^d - \bar{p}_{jt}^d) = 0; \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (49)$$

$$\mu_{bjt}^{\text{lo}} (p_{bjt}^d - 0) = 0; \quad \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (50)$$

$$\mu_{bjt}^{\text{up}} (p_{bjt}^d - \bar{p}_{bjt}^d) = 0; \quad \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (51)$$

where (48) and (49) are related to (10) while (50) and (51) correspond to (14).

From (48)–(51), the following equalities can be obtained:

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \rho_{jt}^{\text{lo}} p_{jt}^d = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \bar{p}_{jt}^d \rho_{jt}^{\text{lo}} \quad (52)$$

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \rho_{jt}^{\text{up}} p_{jt}^d = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \bar{p}_{jt}^d \rho_{jt}^{\text{up}} \quad (53)$$

$$\sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} \mu_{bjt}^{\text{lo}} p_{bjt}^d = 0 \quad (54)$$

$$\sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} \mu_{bjt}^{\text{up}} p_{bjt}^d = \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} \bar{p}_{bjt}^d \mu_{bjt}^{\text{up}} \quad (55)$$

Using (2) and (21), the energy payment term in (1) can be recast as follows:

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \pi_{n(j)t} p_{jt}^d = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} (-\rho_{jt}^{\text{lo}} - \rho_{jt}^{\text{up}} - \delta_{jt}) p_{jt}^d \quad (56)$$

TABLE I
DIMENSIONS OF THE ORIGINAL BILEVEL MODEL AND D-PM

Model	# of constraints	# of binary variables	# of real variables
Original (1)–(17)	$n_T(n_N + 1) + n_I +$ $\sum_{t \in \mathcal{T}} (n_{L_t^+} + n_{L_t^-}) +$ $n_T(9n_I + 3n_J) +$ $2n_T \sum_{i \in \mathcal{I}} n_{O_i} +$ $2n_T \sum_{j \in \mathcal{J}} n_{B_j}$	$n_I n_T$	$n_T n_N +$ $n_T(2n_I + n_J) +$ $n_T \sum_{i \in \mathcal{I}} n_{O_i} +$ $n_T \sum_{j \in \mathcal{J}} n_{B_j}$
D-PM (59)–(60)	$n_T(n_N + 1) - 11n_I +$ $\sum_{t \in \mathcal{T}} (n_{L_t^+} + n_{L_t^-}) +$ $n_T(34n_I + 4n_J) +$ $3n_T \sum_{i \in \mathcal{I}} n_{O_i} +$ $3n_T \sum_{j \in \mathcal{J}} n_{B_j} + 1$	$n_I n_T$	$n_T(n_N + 1) - 5n_I +$ $\sum_{t \in \mathcal{T}} (n_{L_t^+} + n_{L_t^-}) +$ $n_T(16n_I + 4n_J) +$ $3n_T \sum_{i \in \mathcal{I}} n_{O_i} +$ $3n_T \sum_{j \in \mathcal{J}} n_{B_j}$

TABLE II
PRICES OF DEMAND BID BLOCKS (\$/MWh)

Consumer	Block		
	1	2	3
1	50	20	18
2	50	16	14
3	50	12	10
4	50	8	6
5	50	7	5
6	50	28	26
7	50	25	20
8	50	23	13
9	50	22	15
10	50	13	8
11–20	50	20	18
21–30	50	16	14
31–40	50	12	10
41–50	50	8	6
51–60	50	7	5
61–70	50	28	26
71–80	50	25	20
81–86	50	23	13
87–91	50	22	15

In the same vein, using (13) and (22) in the last term of (56) yields

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \pi_{n(j)t} p_{jt}^d = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} (-\rho_{jt}^{\text{lo}} - \rho_{jt}^{\text{up}}) p_{jt}^d - \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} (\mu_{bjt}^{\text{lo}} + \mu_{bjt}^{\text{up}} - C_{bjt}^d) p_{bjt}^d. \quad (57)$$

Finally, substituting (52)–(55) in (57), the total energy payment is cast as a linear equivalent as follows:

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \pi_{n(j)t} p_{jt}^d = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} (-P_{jt}^d \rho_{jt}^{\text{lo}} - \bar{P}_{jt}^d \rho_{jt}^{\text{up}}) - \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} (\bar{P}_{bjt}^d \mu_{bjt}^{\text{up}} - C_{bjt}^d p_{bjt}^d). \quad (58)$$

This linearization overcomes the difficulties of available approaches handling bilinear terms [42], [43] and allows the application of mixed-integer linear programming to the energy scheduling problem based on consumer payment minimization with price-responsive demand under marginal pricing. Therefore, this step constitutes the main methodological contribution of this paper.

C. Single-Level Mixed-Integer Linear Equivalent

The single-level equivalent of problem (1)–(17) is formulated as the following mixed-integer linear program, which is hereinafter referred to as D-PM:

$$\begin{aligned} & \text{Minimize} \\ & a_{it}, b_{it}, c_{it}, d_{it}, e_{it}, \\ & k_{it}, m_{it}, p_{bjt}^d, p_{jt}^d, p_{jt}^g, \\ & p_{oit}^g, r_{it}, s_{it}, v_{it}, \alpha_{it}, \\ & \rho_{oit}^{\text{lo}}, \rho_{oit}^{\text{up}}, \gamma_{it}^{\text{lo}}, \gamma_{it}^{\text{up}}, \delta_{jt}, \\ & \epsilon_{it}, \lambda_{it}, \mu_{bjt}^{\text{lo}}, \mu_{bjt}^{\text{up}}, \xi_{it}, \\ & \pi_{nt}, \rho_{jt}^{\text{lo}}, \rho_{jt}^{\text{up}}, \sigma_{it}, \phi_{it}^{\text{lo}}, \phi_{it}^{\text{up}} \\ & \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} (-P_{jt}^d \rho_{jt}^{\text{lo}} - \bar{P}_{jt}^d \rho_{jt}^{\text{up}}) \\ & - \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} (\bar{P}_{bjt}^d \mu_{bjt}^{\text{up}} - C_{bjt}^d p_{bjt}^d) \\ & + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} s_{it} \end{aligned} \quad (59)$$

subject to

constraints (2)–(4), (6)–(29), and (31)–(47). (60)

Table I provides the size of both the original bilevel problem (1)–(17) and the single-level mixed-integer linear equivalent D-PM (59), (60) in terms of the numbers of constraints, binary variables, and real variables. It is worth emphasizing that the number of minimum up and down time constraints depends on the initial statuses of generators. Therefore, the number of constraints listed in Table I for each model is an upper bound on the corresponding actual number of constraints. Note also that although the same number of binary variables is required by both models, the transformation process described in Sections III-A and III-B yields a larger number of constraints and real variables for D-PM.

IV. NUMERICAL RESULTS

This section presents and discusses results from a case study based on the IEEE 118-bus system [44], [45]. This case study comprises 186 transmission lines, 54 generating units, and 91 consumers over a 24-h time span. The application of the findings reported in [36] to this system leads to the consideration of 412 line capacity constraints, i.e., only 4.6% of the whole set of line capacity constraints is required to equivalently model the effect of the transmission network. Generation data can be found in [45]. Three-block generation energy offers are obtained from the linearization of the quadratic production costs. Start-up offer prices are set equal to the start-up cost coefficients. It should be noted that generation data and offers remain unchanged over the time span. For illustration purposes, maximum demand bids are based on: 1) the nodal peak demands available in [44], which have been increased by 60%; and 2) the hourly scaling factors reported in [46] for Wednesday of week 35. Moreover, it is assumed that consumers submit three-block demand bids priced as listed in Table II, with $\bar{P}_{1jt}^d$, $\bar{P}_{2jt}^d$, and $\bar{P}_{3jt}^d$ equal to 90%, 5%, and 5% of the corresponding maximum demand bid level, respectively,

TABLE III
PROBLEM DIMENSIONS FOR THE 118-BUS SYSTEM

Model	# of constraints	# of binary variables	# of real variables
Original (1)–(17)	42272	1296	18048
D-PM (59)–(60)	86649	1296	63790

TABLE IV
PAYMENT VERSUS DECLARED SOCIAL WELFARE (\$)

	D-PM	SWM
Payment	1903442.2	1985996.6
Declared social welfare	3736788.0	3786008.5

and with $\bar{p}_{jt}^d = \bar{P}_{1jt}^d, \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$. Similar to producers, consumers do not modify the prices of their respective demand bid blocks over the time span. Table III presents the dimensions of both the original bilevel problem (1)–(17) and D-PM (59), (60) for this case study.

The results of D-PM are compared with those achieved by a conventional scheduling procedure based on the maximization of the declared social welfare [40]. This latter mechanism is hereinafter referred to as SWM. Both D-PM and SWM have been implemented on a Dell PowerEdge R910X64 with four Intel Xeon E7520 processors at 1.866 GHz and 32 GB of RAM using CPLEX 12.1 [31] under GAMS 23.3 [47].

Using a stopping criterion for D-PM based on a 1% optimality gap, the proposed approach required 1231.3 s to attain the solution summarized in Table IV. According to current industry practice [41], [48], achieving a solution within a 1% optimality gap in around 20 min reveals an acceptable computational performance since the proposed algorithm is suitable to provide a solution that is very close to the optimum within a practical amount of time for a reasonably-sized problem. Moreover, the computational gain associated with the use of the network model described in [36] is also evidenced since CPLEX was unable to find a solution within a 1% optimality gap in less than 50 000 s when the complete set of transmission capacity constraints was considered in the formulation of D-PM. Table IV also lists the results associated with the optimal solution to SWM, which was achieved in 4 s. Note that the total payment by consumers drops from \$1985996.6 when a conventional market-clearing procedure is used down to \$1903442.2 when the alternative auction design is implemented. It is worth emphasizing that such substantial 4.2% reduction in the consumer payment is attained by moderately decreasing the declared social welfare by 1.3%.

As shown in Fig. 1, D-PM and SWM yield almost identical consumption profiles, with slightly larger hourly awarded consumption levels under D-PM in most periods. This result reveals that, for this case study, a considerable consumer payment reduction can be achieved without significantly altering the consumption pattern. In addition, Fig. 1 depicts the hourly load-weighted average LMPs for D-PM and SWM. The load-weighted average LMP for time period t , denoted

Fig. 1. Awarded consumption levels and load-weighted average LMPs.

TABLE V
IMPACT OF THE LEVEL OF DEMAND RESPONSIVENESS
ON CONSUMER PAYMENT

$\frac{\bar{P}_{1jt}^d}{\bar{P}_{jt}^d}$ (%)	Payment (\$)
100	Infeasible
95	2019471.6
90	1903442.2
85	1808506.7
80	1694624.1
75	1601892.7

as π_t^{av} , is computed as the sum over all buses of the products of the corresponding LMP and nodal consumption level divided by the total awarded consumption level in that period, i.e., $\pi_t^{\text{av}} = \left(\sum_{n \in \mathcal{N}} \pi_{nt} \left(\sum_{j \in \mathcal{J}_n} p_{jt}^d \right) / \sum_{j \in \mathcal{J}} p_{jt}^d \right)$. It is worth noting that both curves follow the shape of the corresponding consumption profile, as expected under a marginal pricing scheme. Furthermore, it can be observed that load-weighted average LMPs yielded by D-PM are less than those resulting from SWM for all periods.

Finally, in order to assess the effect of the level of demand-side participation, we have run D-PM for different values of $\bar{P}_{bjt}^d$, expressed as percentages of the corresponding maximum demand bid level, also with $\bar{p}_{jt}^d = \bar{P}_{1jt}^d, \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$. For the sake of simplicity, the percent values for $\bar{P}_{2jt}^d$ and $\bar{P}_{3jt}^d$ are identical. Thus, lower percent values for $\bar{P}_{1jt}^d$ indicate larger levels of demand responsiveness. As illustrated in Table V, demand-side participation is beneficial for the purposes of consumer payment minimization. For this particular case study, the case with inelastic demand, i.e., with a percent value for $\bar{P}_{1jt}^d$ equal to 100%, is infeasible due to network limitations.

V. CONCLUSION

Within the context of pool-based electricity markets, this paper has presented a novel and computationally efficient approach to incorporate price-responsive demand in multi-period energy scheduling based on consumer payment minimization under marginal pricing. The salient feature of the proposed method lies in converting the resulting mixed-integer

nonlinear bilevel program with bilinear terms into an equivalent mixed-integer linear programming problem.

The proposed approach has been successfully applied to a case study based on the IEEE 118-bus system with commercially available software. The major findings revealed by the numerical results are as follows.

- 1) High-quality solutions are attained in acceptable computing times.
- 2) The consumer payment is substantially reduced at the expense of moderately decreasing the declared social welfare.
- 3) The consumption pattern is not significantly altered with respect to that resulting from social welfare maximization.
- 4) Larger degrees of demand price responsiveness yield larger reductions in the consumer payment.
- 5) The use of the reduced and equivalent network model is beneficial for computational purposes.

Although a simple albeit practical characterization of demand-side participation has been presented in this paper, it should be noted that the main steps used in the proposed solution approach are readily applicable to incorporate other aspects of electricity consumption such as on/off consumption scheduling, limited energy consumption over the scheduling horizon, consumption ramping limits, load shifting, operational consumption cycles, and time-coupled price elasticity, among others. Such extension would require additional operational constraints as well as some extra notation to properly index variables and parameters. We recognize that the extended model needs further numerical studies.

Further research will be devoted to analyzing the effects of ac network constraints, additional demand-side operational constraints, and uncertainty sources such as wind power generation. Another interesting avenue of research is the analysis of tighter formulations and the application of decomposition techniques for the purposes of improved computational performance.

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