A Stochastic Robust Optimization Approach for the Bidding Strategy of an Electric Vehicle Aggregator

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Abstract

This paper analyzes the bidding strategy problem of an electric vehicle aggregator that participates in the day-ahead energy market. The problem is formulated using a stochastic robust optimization model in which uncertainties in the day-ahead market prices and in the driving requirements of electric vehicles are modeled using scenarios and confidence bounds, respectively. The output of the proposed model is used to build the bidding curves to be submitted by the aggregator to the day-ahead market. We assume that the electric vehicle aggregator behaves as a price-taker in this market. A case study is analyzed to illustrate the main features of the proposed approach, as well as its applicability. We also compare the results with those achieved by considering other strategies. Results show that the proposed approach allows the aggregator to reduce the charging costs in comparison with other charging strategies. Moreover, the solution obtained is robust in the sense that driving requirements of electric vehicle users are met.

Keywords: Aggregator, electric vehicles, robust optimization, stochastic programming, strategic bidding, uncertainty

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1 Notation

2 The main notation used in this paper is stated below for quick reference,
 3 while other symbols are defined as needed throughout the text. A subscript
 4 t / ω in the symbols below denotes their values in the t th time period / ω th
 5 scenario.

6 *Indices:*

7 t Time periods.

8 ω Scenarios.

9 *Parameters:*

10 E^A / E^D Energy contribution / drop of EVs arriving to / departing
 11 from the EV aggregation [kWh].

12 E_{t_0} Initial energy content of the virtual battery representing the
 13 EV aggregation [kWh].

14 $\underline{E}^{\max}$ / $\overline{E}^{\max}$ Lower / upper bound for the maximum energy content of the
 15 virtual battery representing the EV aggregation [kWh].

16 $\underline{E}^{\min}$ / $\overline{E}^{\min}$ Lower / upper bound for the minimum energy content of the
 17 virtual battery representing the EV aggregation [kWh].

18 $\underline{P}^{+, \max}$ / $\overline{P}^{+, \max}$ Lower / upper bound for the maximum charging power of
 19 the virtual battery representing the EV aggregation [kW].

20 $\underline{P}^{-, \max}$ / $\overline{P}^{-, \max}$ Lower / upper bound for the maximum discharging power of
 21 the virtual battery representing the EV aggregation [kW].

22 Δt Time-step duration [h].

23 η^+ / η^- Charging / discharging efficiency of the virtual battery rep-
 24 resenting the EV aggregation [%].

25 *Random Variables:*

26 $E^{\max} / E^{\min}$ Maximum / minimum energy content of the virtual battery
 27 representing the EV aggregation [kWh].

28 $P^{+, \max} / P^{-, \max}$ Maximum charging / discharging power of the virtual battery
 29 representing the EV aggregation [kW].

30 λ Price in the DA market [\$/MWh].

31 *Optimization Variables:*

32 E Energy content (state-of-charge) of the virtual battery rep-
 33 resenting the EV aggregation [kWh].

34 P Power bought from (if positive) / sold to (if negative) the
 35 DA market [kW].

36 P^+ / P^- Charging / discharging power of the virtual battery repre-
 37 senting the EV aggregation [kW].

38 *Acronyms:*

39 ARIMA Autoregressive integrated moving average.

40 DA Day-ahead.

41 EMS Energy management system.

42 EV Electric vehicle.

43 LP Linear programming.

44 RO Robust optimization.

45 VPP Virtual power plant.

46 V2G Vehicle-to-grid.

47 1. Introduction

48 1.1. Aim and Approach

49 The number of electric vehicles (EVs) is expected to increase in the near
50 future due to, mainly, two reasons. First, the maturity of their technology makes
51 EVs available for long-distance trips, which was one of the main drawbacks in
52 the past. Second, due to the reduction of CO₂ emissions and other greenhouse
53 emissions if renewable energy sources are used to supply EVs [1]. This has
54 motivated the use of incentive policies by some countries to encourage consumers
55 to buy EVs instead of conventional vehicles, e.g., in Norway [2].

56 A large penetration of EVs supposes a big challenge. Traditional charging
57 strategies consider that EV owners start charging their EVs as soon as they are
58 parked and until the next trip or until the battery is full. In this sense, note
59 that most of the population arrive at home/work at similar times. Thus, most
60 EVs are expected to charge at the same time. If the number of EVs is very
61 large, this may translate in a peak in the energy consumption at these times.

62 However, EVs are parked and available for charging most of the time. Thus,
63 it would be possible to shift its charging to the most suitable time, e.g., to a time
64 with low demand or with low prices. This would reduce the peak energy and/or
65 would allow EV users to reduce their charging costs. Nevertheless, managing
66 the charging of every individual EV has two problems. On the one hand, the
67 limited flexibility of an individual EV. On the other hand, the high computation
68 effort to deal with the management of a very large number of EVs. To overcome
69 these two issues, it is generally convenient to use the figure of an aggregator,
70 which can be seen as an energy management system (EMS) [3, 4] in charge of
71 an EV fleet. The aggregator manages the charging of these EVs and ensures
72 that the driving requirements of EV owners are met.

73 The figure of an EV aggregator has been proposed and considered in the
74 technical literature using different approaches, e.g., considering unidirectional
75 [5, 6] or bidirectional charging [7, 8]. This bidirectional charging is usually
76 known as vehicle-to-grid (V2G) and allows the EV aggregator to behave as a

77 producer or as a consumer [9] depending on the market prices and the driving
78 requirements.

79 Among the different problems faced by an EV aggregator, in this paper we
80 focus on the problem of determining its bidding strategy in an energy mar-
81 ket. In particular, we determine the bidding strategy of an EV aggregator that
82 participates in a day-ahead (DA) energy market.

83 A key point at the time of determining the bidding strategy of the EV
84 aggregator is the modeling the different sources of uncertainty that condition
85 this bidding strategy. These uncertainties include the market prices and the
86 driving requirements of EV owners. To do so, in this paper we use scenarios to
87 model the uncertainty in the market prices. This allows us to use a stochastic
88 programming model [10]. On the other hand, we use confidence bounds to
89 model the uncertainty in the driving requirements. This allows us to use robust
90 optimization (RO) [11]. Thus, the bidding strategy problem is formulated as a
91 hybrid stochastic robust model.

92 1.2. Literature Review

93 The technical literature about bidding strategy problems is extensive. Many
94 methods have been proposed for different entities, e.g., conventional power pro-
95 ducers [12], demand-response aggregators [13], virtual power plants (VPP) [14],
96 microgrids [15]. [12] proposes a bi-level model that allows representing the in-
97 fluence of the power producer on market prices. [13] compares stochastic and
98 robust models for a demand-response aggregator. [14] proposes a stochastic
99 model for a VPP participating in both energy and reserve markets. [15] pro-
100 poses a hybrid stochastic robust model for a microgrid participating in the DA
101 and real-time energy markets.

102 Besides these methods, different approaches to deal with the bidding problem
103 of an EV aggregator have been proposed, e.g., [16, 17, 18, 19, 20, 21, 22, 23].

104 Some of these methods consider the participation of EVs in both energy
105 and capacity markets. [16] and [17] consider the participation of the EV ag-
106 gregator in the DA and reserve markets. [18] proposes a stochastic approach

107 and considers the participation of the EV aggregator in the DA and regulation
108 markets. [19] analyzes the optimization of charging and frequency regulation
109 capacity bids using a stochastic dynamic programming model. The problem
110 is formulated in this case using a Markov decision problem. [20] proposes a
111 stochastic mathematical programming with equilibrium constraints problem for
112 the participation of EV aggregators in the DA and ancillary services markets.

113 Other methods consider the participation of EVs only in energy markets. [21]
114 and [22] formulate the bidding problem in the DA market using a bi-level model,
115 which explicitly represents the clearing of the DA market. [23] models the risk
116 associated with the bidding decisions in the DA market using the conditional
117 value-at-risk.

118 However, none of the above methods for EV bidding considers a robust ap-
119 proach. In these references, uncertainties are generally modeled using scenarios.
120 Thus, the driving requirements of EVs may not be met if these scenarios are
121 not properly generated.

122 Note that both [21] and this paper deal with the same problem, namely, the
123 bidding strategy of an EV aggregator in the DA market. However, these two
124 works have three main differences: i) [21] considers that the EV aggregator is a
125 price-maker while in this paper it is considered as a price-taker; ii) uncertainties
126 in [21] are modeled using scenarios, while in this paper we model uncertainties
127 using both scenarios and confidence bounds; and iii) [21] proposes a stochastic
128 model, while in this paper we describe a hybrid stochastic robust model.

129 1.3. Contributions

130 The main contribution of this paper is to provide a new approach based on
131 a hybrid stochastic robust model for the bidding strategy of an EV aggregator
132 in the DA market.

133 On the one hand, stochastic programming has been widely used for offering
134 strategy problems [9, 12]. On the other hand, RO has been recently used in
135 offering and bidding strategy problems of conventional units [24], concentrating
136 solar power plants [25], and VPPs [26], since it constitutes a flexible tool that

137 can be easily included in an EMS, e.g., in an EV aggregator in charge of an EV
138 fleet.

139 However, to the best of our knowledge, there is no paper in the technical
140 literature that combine stochastic programming and robust optimization for the
141 bidding problem of an EV aggregation.

142 The advantage of the method proposed in this paper over other methods
143 based solely on stochastic programming, e.g., [14], is that driving requirements
144 are generally difficult to forecast. Thus, a very large number of scenarios would
145 be needed to obtain an accurate representation of the uncertainty in the driving
146 requirements. This may result in a very computationally complex model.

147 On the other hand, the advantage of this method in comparison with other
148 methods based solely on RO, e.g., [24, 26], is that these methods are usually too
149 conservative. Moreover, there are many methods that provide good forecasts
150 of market prices. Thus, the number of scenarios needed to obtain an accurate
151 representation of the uncertainty in the market prices is not that large as in the
152 case of the driving requirements.

153 As in this paper, [13] and [15] analyze a combination of stochastic and ro-
154 bust models for the bidding problem of a demand response aggregator and a
155 microgrid, respectively. However, in [13] both models are analyzed indepen-
156 dently. On the other hand, [15] uses confidence bounds to model uncertainty in
157 real-time market prices, and scenarios to model uncertainty in DA market price
158 scenarios and wind-power production. Note that while prices appear in the ob-
159 jective function of the optimization problem described in [15], the wind-power
160 production appears within the constraints. Thus, this may result in suboptimal
161 results or even in an infeasible solution if the actual wind-power production (or
162 the driving requirements in the case of this paper) is different to one of the
163 scenarios considered.

164 As a summary and given the above context, the contributions of this paper
165 are fourfold:

- 166 1. To propose a novel approach for the bidding strategy in the DA market of

- 167 an EV aggregator that faces uncertain market prices and uncertain driving
168 patterns.
- 169 2. To re-formulate the problem in 1 as a stochastic robust problem.
 - 170 3. To use the outputs of problem in 2 to build the bidding curves of the EV
171 aggregator.
 - 172 4. To provide and analyze the results of a case study.

173 1.4. Paper Organization

174 The remainder of the paper is organized as follows. Section 2 describes the
175 main features of the bidding strategy problem. Section 3 provides the model
176 of an EV aggregator that manages an EV fleet. The characterization of the
177 different sources of uncertainties is described in Section 4. The bidding problem
178 is formulated in Section 5 using a deterministic approach, which is extended
179 in Section 6 using a stochastic robust approach that takes into account the
180 uncertainties in the market prices and in the driving requirements. Section 7
181 provides the results of a case study. Section 8 concludes the paper with some
182 relevant remarks.

183 2. Problem Description

184 We consider a number of EVs within an EV fleet. These EVs are managed
185 by an EV aggregator that works as an EMS. The individual EVs communicate
186 the aggregator their driving requirements, and, then, the aggregator determines
187 the optimal charging time of each EV with the aim of minimizing the overall
188 charging costs but, at the same time, meeting the driving requirements of EV
189 users. We assume that charging points and EVs are equipped with the V2G
190 technology so that EVs can be also used to supply energy to the grid if needed.

191 In order to supply the EVs, the aggregator participates in the DA energy
192 market. To do so, instead of considering each EV individually, the aggrega-
193 tor considers the aggregation of all EVs in the fleet [8]. Then, considering the
194 driving requirements of the EVs in the fleet, the aggregator decides the bidding

195 strategy and sends these bidding decisions to the market operator. With these
 196 bidding decisions, as well as with the offering and bidding decisions of other mar-
 197 ket participants, the market operator clears the DA market and communicates
 198 the aggregator the market outputs.

199 This problem structure is schematically represented in Fig. 1.

Figure 1: Problem structure.

200 We consider a price-taker approach that assumes that the EV aggregator
 201 cannot affect market prices. If the size of the EV fleet managed by the aggrega-
 202 tor is sufficiently large, it may be able to alter market prices. In such a case, it is
 203 necessary to represent the clearing of the market within the optimization prob-
 204 lem so that the resulting problem is a mathematical program with equilibrium
 205 constraints [27] and the models proposed in [21] and [22] may be considered.
 206 However, in this paper we assume that the size of the EV fleet is not that large.

207 The problem is solved for a planning horizon of one day. For the sake
 208 of simplicity, the planning horizon is discretized into 24 hourly time periods.
 209 However, note that the duration of these time steps may be reduced in order to
 210 obtain a detailed modeling of the charging process. This is out of the scope of

211 this paper and will be subject of future research.

212 3. EV Aggregator

213 We consider an aggregation of EVs managed by the so-called EV aggregator.
 214 The EV aggregation is represented by a virtual battery that takes into account
 215 the driving patterns of EV users, the physical characteristics of the individual
 216 EV batteries, as well as the physical characteristics of the charging infrastruc-
 217 ture. To do so, we use the model proposed in [8], which comprises the following
 218 equations:

$$E_t = E_{t-1} + P_t^+ \Delta t \eta^+ + E_t^A - \frac{P_t^- \Delta t}{\eta^-} - E_t^D \quad \forall t \quad (1)$$

$$E_{t_f} \geq E_{t_0} \quad (2)$$

$$P_t = P_t^+ - P_t^- \quad \forall t \quad (3)$$

$$0 \leq P_t^+ \leq P_t^{+, \max} \quad \forall t \quad (4)$$

$$0 \leq P_t^- \leq P_t^{-, \max} \quad \forall t \quad (5)$$

$$E_t^{\min} \leq E_t \leq E_t^{\max} \quad \forall t. \quad (6)$$

219 Constraints (1) represent the energy balance of the virtual battery represent-
 220 ing the EV aggregation. The energy content of the virtual battery at each time
 221 period is equal to the energy content in the previous time period, the charging
 222 energy, and the energy contributions of EVs arriving to the EV aggregation, mi-
 223 nus the discharging energy and the energy drops of EVs departing from the EV
 224 aggregation at that time period. Constraint (2) imposes a lower bound on the
 225 energy stored in the virtual battery at the end of the planning horizon. If this
 226 constraint is not included, the battery would tend to be depleted over the plan-
 227 ning horizon. Constraints (3) define the power that is sold to / bought from the
 228 DA market as the charging minus the discharging powers. Note that we assume
 229 that EVs can both charge and discharge if it is economically profitable, i.e., we
 230 assume that EVs are equipped with V2G technology. Finally, constraints (4),

231 (5), and (6) impose bounds on the charging power, the discharging power, and
232 the energy content of the virtual battery, respectively.

233 We consider two different variables for the charging and discharging of the
234 EV aggregation, respectively. Thus, binary variables to avoid charging and
235 discharging at the same time would be needed. However, we also consider
236 charging and discharging efficiencies. Since these constraints will be used in
237 an optimization problem that aims at minimizing / maximizing the charging /
238 discharging costs / profits, it is not optimal to charge and discharge at the same
239 time. Thus, it is not necessary to include these binary variables. *Note that this
240 assumption is valid provided that charging and discharging efficiencies are not
241 1.* Note also that not including such binary variables allows us to reduce the
242 computation burden of the proposed approach.

243 The power and energy bounds in constraints (4), (5), and (6) are obtained
244 by considering the driving requirements of EV users, as well as the physical
245 characteristics of the EV fleet and the charging points. Additional details about
246 how these bounds are computed are provided in [8].

247 **4. Uncertainty Characterization**

248 The bidding decisions in the DA market must be made one day in advance
249 and for all the hours of the following day. At that time, the EV aggregator does
250 not know neither the DA market prices nor the driving requirements of the EV
251 fleet. Thus, in order to obtain informed bidding decisions, it is necessary to
252 represent the uncertainty in these variables.

253 There are generally two ways of representing uncertainties within this type of
254 problems. One is to use a set of scenarios that model different realizations of the
255 uncertain variables. The other option is to consider that uncertain variables take
256 values within known confidence bounds. In this paper, we use a combination of
257 both alternatives.

258 On the one hand, we use a set of scenarios indexed by ω to model the uncer-
259 tainty in the DA market prices. These scenarios are obtained using historical

260 data of DA market prices and the ARIMA model described in [28].

261 While there are many methods in the technical literature to generate accu-
 262 rate DA market price scenarios, this is not the case for driving requirements.
 263 Thus, instead of scenarios, we use confidence bounds to model the uncertainty
 264 in the driving requirements, namely, in the maximum charging power, in the
 265 minimum discharging power, as well as in the minimum and maximum energy
 266 content of the virtual battery. We assume that these power and energy limits
 267 are not known at the time the bidding decisions in the DA market are made;
 268 however, we know that they take values within known confidence bounds. That
 269 is:

$$\underline{P}_t^{+, \max} \leq P_t^{+, \max} \leq \overline{P}_t^{+, \max} \quad \forall t \quad (7)$$

$$\underline{P}_t^{-, \max} \leq P_t^{-, \max} \leq \overline{P}_t^{-, \max} \quad \forall t \quad (8)$$

$$\underline{E}_t^{\min} \leq E_t^{\min} \leq \overline{E}_t^{\min} \quad \forall t \quad (9)$$

$$\underline{E}_t^{\max} \leq E_t^{\max} \leq \overline{E}_t^{\max} \quad \forall t. \quad (10)$$

270 The above upper and lower power and energy limits may be obtained using
 271 historical data to forecast the future driving requirements. In this paper, we
 272 simply take the data provided in [8] and consider that power and energy lim-
 273 its vary within $\pm 5\%$ of their expected values. An accurate modeling of these
 274 confidence bounds is out of the scope of this paper.

275 It is important to note that the uncertainty in the driving requirements
 276 decreases as we increase the number of EVs managed by the EV aggregator as
 277 shown in [22].

278 The advantage of using confidence bounds to model the uncertainty in the
 279 driving requirements is that the solution would be feasible for every realization
 280 of the uncertain power and energy limits within the considered bounds. If, in-
 281 stead, scenarios are used to model these uncertainties, then, the solution may
 282 be infeasible if the actual realizations of the power and energy limits are dif-
 283 ferent from the considered scenarios. Moreover, it is generally easier to obtain

284 confidence bounds than generating accurate scenarios.

285 The above uncertainty representation allows us to formulate the bidding
286 problem as a stochastic robust model. It is stochastic [10] from the point of
287 view of market prices and robust [11] from the point of view of power and
288 energy bounds.

289 Finally, note that the market price scenarios and the confidence bounds for
290 driving requirements described above are considered as input data of the model
291 formulated in the following section. This model is general and other methods
292 may be considered to obtain more accurate scenarios and confidence bounds.

293 5. Deterministic Formulation

294 The self-scheduling problem of an EV aggregator that participates in the
295 DA market can be formulated using the linear programming (LP) model below:

$\min_{\Theta_t}$

$$\sum_t \lambda_t P_t \Delta t \quad (11)$$

subject to

$$\text{Constraints (1) – (6),} \quad (12)$$

296 where variables in set $\Theta_t = \{P_t, P_t^+, P_t^-, E_t\}, \forall t$, are the optimization variables
297 of problem (11)-(12).

298 The objective function (11) represents the minimization of the costs of pur-
299 chasing energy in the DA market. Note that if the EV aggregator sells power to
300 the market in time period t , then $P_t < 0$ and these costs are negative, i.e., in this
301 case term $\lambda_t P_t \Delta t$ represents the minus profit achieved by the EV aggregator.

302 Constraints of problem (11)-(12) include the constraints of the virtual bat-
303 tery used to represent the EV aggregation as described in Section 3.

304 Problem (11)-(12) allows the EV aggregator to determine the charging and
305 discharging powers at each time period that minimize its charging costs over the
306 planning horizon and meet the driving requirements of the EV fleet. However,

307 it is important to note that at the time this problem is solved by the EV aggre-
 308 gator, it does not know neither the values of the market prices nor the driving
 309 requirements of the EV users. Thus, it is necessary to model this uncertainty as
 310 explained in Section 4. This uncertainty representation allows us to formulate
 311 the stochastic robust model described in the following section.

312 Note that by solving problem (11)-(12) we obtain the aggregated charging
 313 and discharging powers of the EV aggregator. Then, in a second step, the
 314 aggregator should distribute these powers among the individual EVs. This
 315 second step is out of the scope of this paper.

316 6. Stochastic Robust Formulation

317 The problem of determining the bidding strategy in the DA market of an
 318 EV aggregator that faces uncertain market prices and driving requirements can
 319 be formulated using the stochastic robust model below:

$$\min_{\Theta_{\omega,t}, \Theta_{\omega,t}^R} \sum_{\omega} \pi_{\omega} \sum_t \lambda_{\omega,t} P_{\omega,t} \Delta t \quad (13)$$

subject to

$$E_{\omega,t} = E_{\omega,t-1} + P_{\omega,t}^+ \Delta t \eta^+ - \frac{P_{\omega,t}^- \Delta t}{\eta^-} + E_t^A - E_t^D \quad \forall \omega, t \quad (14)$$

$$E_{\omega,t_f} \geq E_{t_0} \quad \forall \omega \quad (15)$$

$$P_{\omega,t} = P_{\omega,t}^+ - P_{\omega,t}^- \quad \forall \omega, t \quad (16)$$

$$P_{\omega,t} \leq P_{\tilde{\omega},t} \text{ if } \lambda_{\omega,t} \geq \lambda_{\tilde{\omega},t} \quad \forall \omega, \tilde{\omega}, t \quad (17)$$

$$P_{\omega,t}^+ + \Gamma_{\omega,t}^{P+} v_{\omega,t}^{P+} + \mu_{\omega,t}^{P+} \leq \frac{1}{2} \left(\overline{P}_t^{+, \max} + \underline{P}_t^{+, \max} \right) \quad \forall \omega, t \quad (18)$$

$$v_{\omega,t}^{P+} + \mu_{\omega,t}^{P+} \geq \frac{1}{2} \left(\overline{P}_t^{+, \max} - \underline{P}_t^{+, \max} \right) y_{\omega,t}^{P+} \quad \forall \omega, t \quad (19)$$

$$1 \leq y_{\omega,t}^{P+} \quad \forall \omega, t \quad (20)$$

$$P_{\omega,t}^- + \Gamma_{\omega,t}^{P-} v_{\omega,t}^{P-} + \mu_{\omega,t}^{P-} \leq \frac{1}{2} \left(\overline{P}_t^{-, \max} + \underline{P}_t^{-, \max} \right) \quad \forall \omega, t \quad (21)$$

$$v_{\omega,t}^{P-} + \mu_{\omega,t}^{P-} \geq \frac{1}{2} \left(\overline{P}_t^{-, \max} - \underline{P}_t^{-, \max} \right) y_{\omega,t}^{P-} \quad \forall \omega, t \quad (22)$$

$$1 \leq y_{\omega,t}^{P-} \quad \forall \omega, t \quad (23)$$

$$E_{\omega,t} + \Gamma_{\omega,t}^{\text{Emax}} v_{\omega,t}^{\text{Emax}} + \mu_{\omega,t}^{\text{Emax}} \leq \frac{1}{2} (\bar{E}_t^{\text{max}} + \underline{E}_t^{\text{max}}) \quad \forall \omega, t \quad (24)$$

$$v_{\omega,t}^{\text{Emax}} + \mu_{\omega,t}^{\text{Emax}} \geq \frac{1}{2} (\bar{E}_t^{\text{max}} - \underline{E}_t^{\text{max}}) y_{\omega,t}^{\text{Emax}} \quad \forall \omega, t \quad (25)$$

$$1 \leq y_{\omega,t}^{\text{Emax}} \quad \forall \omega, t \quad (26)$$

$$-E_{\omega,t} + \Gamma_{\omega,t}^{\text{Emin}} v_{\omega,t}^{\text{Emin}} + \mu_{\omega,t}^{\text{Emin}} \leq -\frac{1}{2} (\bar{E}_t^{\text{min}} + \underline{E}_t^{\text{min}}) \quad \forall \omega, t \quad (27)$$

$$v_{\omega,t}^{\text{Emin}} + \mu_{\omega,t}^{\text{Emin}} \geq \frac{1}{2} (\bar{E}_t^{\text{min}} - \underline{E}_t^{\text{min}}) y_{\omega,t}^{\text{Emin}} \quad \forall \omega, t \quad (28)$$

$$1 \leq y_{\omega,t}^{\text{Emin}} \quad \forall \omega, t \quad (29)$$

$$v_{\omega,t}^{P+}, \mu_{\omega,t}^{P+}, y_{\omega,t}^{P+}, v_{\omega,t}^{P-}, \mu_{\omega,t}^{P-}, y_{\omega,t}^{P-}, v_{\omega,t}^{\text{Emax}}, \mu_{\omega,t}^{\text{Emax}}, y_{\omega,t}^{\text{Emax}}, v_{\omega,t}^{\text{Emin}}, \mu_{\omega,t}^{\text{Emin}}, y_{\omega,t}^{\text{Emin}} \geq 0 \quad \forall \omega, t, \quad (30)$$

where variables in sets $\Theta_{\omega,t} = \{P_{\omega,t}, P_{\omega,t}^+, P_{\omega,t}^-, E_{\omega,t}\}$, $\forall \omega, t$, and $\Theta_{\omega,t}^R = \{v_{\omega,t}^{P+}, \mu_{\omega,t}^{P+}, v_{\omega,t}^{P-}, y_{\omega,t}^{P+}, \mu_{\omega,t}^{P-}, y_{\omega,t}^{P-}, v_{\omega,t}^{\text{Emax}}, \mu_{\omega,t}^{\text{Emax}}, y_{\omega,t}^{\text{Emax}}, v_{\omega,t}^{\text{Emin}}, \mu_{\omega,t}^{\text{Emin}}, y_{\omega,t}^{\text{Emin}}\}$, $\forall \omega, t$, are the optimization variables of problem (13)-(30). Variables in sets $\Theta_{\omega,t}^R$, $\forall \omega, t$, are auxiliary variables needed to derive the robust problem [11].

Problem (13)-(30) is an LP problem. LP problems can be solved using available optimization software, e.g., CPLEX [29], and its convergence to the optimum is guaranteed [30].

The objective function (13) represents the minimization of the expected costs of participating in the DA market. Note that, as in problem (11)-(12), $P_{\omega,t}$ can also take both positive (if charging) and negative (if discharging) values. In the latter, terms $\lambda_{\omega,t} P_{\omega,t} \Delta t$ represent the minus income achieved by the EV aggregator. As a difference with the objective function of the deterministic problem (11), here the costs for each scenario are multiplied by the corresponding probability.

Constraints (14)-(16) are similar to constraints (2)-(4) of the deterministic problem; however, these constraints are considered for each scenario in this case. Constraints (17) ensure that the bidding curves decrease monotonically. Constraints (18)-(20), (21)-(23), (24)-(26), and (28)-(30) are robust constraints that allow us to consider the uncertainty in the power and energy bounds, i.e.,

in random variables $P_t^{+, \max}$, $P_t^{-, \max}$, $E_t^{\max}$, and $E_t^{\min}$, respectively. Additional details on how to obtain these constraints are provided in the Appendix of this paper and in [11].

In model (13)-(30) we include control parameters $\Gamma_{\omega, t}^{P+}$, $\Gamma_{\omega, t}^{P-}$, $\Gamma_{\omega, t}^{E\max}$, and $\Gamma_{\omega, t}^{E\min}$ in constraints (18), (21), (24), and (28), respectively. These parameters can take values between 0 and 1, and allow us to control the level of robustness in the problem. If one of these parameters is fixed to 0, then we ignore the uncertainty in the random variable of the corresponding constraint. However, if it is fixed to 1, then we consider that the uncertain variable in that constraint can take any value within the considered confidence bound and, thus, we obtain a comparatively more robust solution. The output of model (13)-(30), i.e., the bidding strategy of the EV aggregator, depends on the values considered for these parameters.

Note that in problem (13)-(30), we decide the charging / discharging power $P_{\omega, t}$ for each scenario, i.e., variable $P_{\omega, t}$ depends on the scenario realization ω . However, note that these decisions must be made before the EV aggregator knows the actual scenario realization. Nevertheless, we relax the non-anticipativity constraints that should impose $P_{\omega, t} = P_{\tilde{\omega}, t}$, $\forall \omega, \tilde{\omega}$, and use these scenario-dependent variables and the corresponding market price for each scenario and time period to build the bidding curves to be submitted by the EV aggregator to the market operator.

Finally, note that the bidding strategy problem proposed in this paper may be applied not only by an EV aggregator but also by other entities, e.g., the EMS in charge of a VPP, if the corresponding components are modeled within the proposed approach.

7. Case Study

The stochastic robust model (13)-(30) is analyzed in this section using a realistic case study.

367 *7.1. Data*

368 The bidding strategy is decided for a planning horizon of 24 hours divided
369 into hourly time steps.

370 We consider an aggregation of EVs whose main characteristics are provided
371 in Table 1. For the sake of simplicity, we assume that all EVs in the aggregation
372 have the same characteristics. Therefore, the charging and discharging efficien-
373 cies of the EV aggregation are equal to the individual charging and discharging
374 efficiencies provided in Table 1.

Table 1: Data of EVs in the aggregation	
Number of EVs	1200
Capacity of batteries	20 kWh
Maximum charging power	5 kW
Maximum discharging power	5 kW
Charging efficiency	90%
Discharging efficiency	90%
Initial energy content in the virtual battery	18000 kWh

375 The driving patterns of this EV aggregation are based on the data provided
376 in [8]. Figs. 2 and 3 provide the energy and power bounds of the virtual battery,
377 respectively. Note that in these figures we represent the confidence bounds
378 needed to solve the stochastic robust model (13)-(30). To obtain these bounds,
379 we take the data provided in [8] and consider that power and energy limits vary
380 within $\pm 5\%$ of their expected values. Finally, the energy contributions / drops
381 of EVs arriving to / departing from the virtual battery at each time step are
382 provided in Fig. 4. These data are also based on the data provided in [8].

383 We consider that the EV aggregator participates in the New England elec-
384 tricity market [31]. We use historical data of DA market prices and the ARIMA
385 model proposed in [28] to generate 20 scenarios for the 24 hours of May 4,
386 2011. These scenarios are represented in Fig. 5 and all of them have the same

Figure 2: Energy bounds.

probability.

7.2. Results

First, we fix the value of the control parameters $\Gamma_{\omega,t}^{P+}$, $\Gamma_{\omega,t}^{P-}$, $\Gamma_{\omega,t}^{E\max}$, and $\Gamma_{\omega,t}^{E\min}$ to 1, $\forall \omega, t$. This allows us to obtain a conservative solution since we take into account that the uncertain power and energy bounds can take any value within the considered confidence bounds.

We solve problem (13)-(30) and obtain the bidding curves in hours 3, 11, and 21, which are depicted in Figs. 6, 7, and 8, respectively.

Market prices in hour 3 (during night) are comparatively low. Thus, the EV aggregator participates in the DA market as a demand in that hour, i.e., it is willing to charge the batteries of EVs. However, note that the bid power is 0 if the price is higher than \$39.2/MWh in that hour. A different result can be observed in the bidding curve for hour 11. In this case, depending on the price, the EV aggregator participates in the DA market as a demand (positive values of power) or as a generating unit (negative values of power). This way, the aggregator takes advantage of price differences to sell part of the energy stored in the virtual battery. Finally, we observe in Fig. 8 that the EV aggregator behaves as an inelastic generating unit in hour 21. If we look at the market

Figure 3: Power bounds.

price scenarios in Fig. 5, we observe that the market prices in hour 21 are comparatively higher than those prices in the previous and following hours. Thus, the EV aggregator is willing to inject energy into the network, i.e., to discharge the EV fleet, in this hour. This results in an inelastic offer curve that indicates that the EV aggregator is willing to sell 4222 kW to the DA market independently of the price.

7.3. Impact of Robust Control Parameters

Next, we analyze the impact of the control parameters on the bidding strategy. We solve problem (13)-(30) again for different values of these control parameters. In particular, we consider three different values, namely, 0, 0.5, and 1, which represent different levels of risk and robustness in the solution. The bidding curves for different values of the control parameters and for hours 3 and 20 are provided in Figs. 9 and 10, respectively.

The bidding strategy of the EV aggregator depends on the selected values of the control parameters. Fixing these parameters to 1 results in comparatively more robust solutions since it takes into account that the uncertain variables can take any value within the considered confidence bounds, while lower values of these parameters increase the risk of the bidding strategies.

Considering different levels of robustness in the problem has also an impact

Figure 4: Energy contribution/drop of EVs arriving to/departing from the virtual battery.

on the expected operation costs. These costs for different values of the control parameters are provided in Table 2.. These costs are obtained as the average of the 20 scenarios.

Table 2: Expected operation costs for different values of control parameters [\$]

$\Gamma = 0$	$\Gamma = 0.5$	$\Gamma = 1$
366.41	381.46	396.89

Considering the more robust solution, i.e., fixing all control parameters to 1, results in an expected operation cost that is 8.32% higher than the expected cost if these parameters are fixed to 0, i.e., if a risky strategy is selected. However, note that if $\Gamma = 0$, we ignore the uncertainty in the power and energy bounds, which are assumed to take the average values of the considered confidence bounds. Then, if a deviation in these power and energy bounds occurs, the EV aggregation should compensate it by participating in other market mechanisms, e.g., the balancing market, which also has a cost.

Figure 5: Price scenarios.

7.4. Comparison of Different Charging Strategies

Next, the results are compared with two benchmark charging strategies:

1. Unidirectional charging (no V2G): we solve problem (13)-(30) by considering that $P_{\omega,t}^- = 0, \forall \omega, t$, i.e., we assume that the EV aggregator cannot sell the energy of the virtual battery in the market.
2. Conventional charging: instead of solving an optimization problem, we assume that the EV aggregator charges the virtual battery as much as possible, i.e., the virtual battery is charged at $P_t^{+,max}$ given that $E_{\omega,t} \leq E_t^{max}$. This corresponds to a conventional charging, when EVs start charging as soon as they are parked and until the battery is full or they depart for the next trip.

The bidding curves for hour 15 and for the different charging strategies are depicted in Fig. 11. We assume that the control parameters are fixed to 1.

Note that the bidding curves are different depending on the strategy selected. The larger charging power corresponds to the conventional charging. We also observe that for prices below \$55.5/MWh, considering the V2G technology results in a larger charging power than without the V2G technology. This can be

Figure 6: Bid curve in hour 3.

Figure 7: Bid curve in hour 11.

452 explained by the fact that, if the V2G technology is available, the EV aggregator
 453 prefers charging a larger energy in hour 15 in order to sell it in the following
 454 hours, when the prices are comparatively higher.

455 The expected costs for the different charging strategies are provided in Table
 456 3.

457 Note that the conventional charging results in the highest expected operation
 458 cost. This is so because EVs in the fleet start charging as soon as they are
 459 parked, independently of the market prices. On the other hand, if the V2G is
 460 available and the EV aggregator takes advantage of it, the expected operation

Figure 8: Bid curve in hour 21.

Figure 9: Bid curves in hour 3 for different values of Γ .

461 cost is reduced by 25.03% in comparison with the case without V2G. Considering
 462 the V2G technology, the EV aggregator can sell energy to the DA market in
 463 hours with comparatively high market prices (and is paid for that) and buy
 464 energy from the DA market in hours with comparatively low prices.

465 7.5. Computation Burden of the Problem

466 The LP model (13)-(30) was solved using CPLEX 12.2.0.1 [29] under GAMS
 467 [32] on an Intel-Core i3 processor clocking at 3.4 GHz and 4 GB of RAM. The
 468 computation time required to obtain the optimal solution for different values
 469 of the control parameters is less than 1 s, which is compatible with the time

Figure 10: Bid curves in hour 20 for different values of Γ .

Table 3: Expected operation costs for different charging strategies [\\$]

Proposed strategy	Without V2G	Conventional charging
396.89	529.38	851.08

requirements for deciding the bidding strategy in the DA market.

Finally, we show in Figure 12 the dependency of the expected profit with the number of scenarios. To do so, we run the model again but considering an increasing number of scenarios. In this case, we consider that the control parameters are fixed to 0.5.

We observe that there are important changes in the expected profit if the number of scenarios is low. However, the expected profit becomes approximately stable as we increase this number. Nevertheless, note that increasing the number of scenarios to run the model would not be a big deal. The resulting problem is an LP problem and there are many solvers that run this type of problems with a very large number of variables and constraints in almost no time. If needed, scenario reduction methods may be also considered [33].

8. Conclusions

A stochastic robust approach has been proposed for the bidding strategy problem of an EV aggregator. The conclusions below are in order:

Figure 11: Bid curves in hour 15 for different charging strategies.

1. An accurate modeling of the uncertainties in market prices and driving requirements is needed to obtain informed bidding decisions for the EV aggregator. In this sense, the proposed stochastic robust approach constitutes an appropriate tool.
2. It is possible to control the level of robustness in the solution by using the proposed approach.
3. The use of the V2G technology allows the EV aggregator to reduce its expected operation cost with respect to the unidirectional charging case.
4. Possible deviations should be compensated in other market mechanisms, e.g., in the balancing market. The incorporation of such market mechanisms in the proposed approach is subject of future research.

Appendix A. Robust Optimization

This appendix briefly describes how the robust model (13)-(30) described in Section 6 can be derived from the deterministic model (11)-(12) provided in Section 5.

Figure 12: Evolution of the expected profit with the number of scenarios.

500 Problem (11)-(12) can be written in a compact form as the following LP
 501 model:

$$\min_{x_j, \forall j}$$

$$\sum_j c_j x_j \quad (\text{A.1})$$

subject to

$$\sum_j a_{ij} x_j \leq b_i \quad \forall i \quad (\text{A.2})$$

$$\sum_j d_{kj} x_j \leq e_k \quad \forall k \quad (\text{A.3})$$

$$\sum_j f_{lj} x_j = g_l \quad \forall l \quad (\text{A.4})$$

$$x_j \geq 0 \quad \forall j. \quad (\text{A.5})$$

502 Problem (A.1)-(A.5) has some uncertain coefficients. In particular, these
 503 coefficients are the independent terms b_i in the inequality constraints (A.2).
 504 In problem (11)-(12), these uncertain coefficients are $P_t^{+, \max}$, $P_t^{-, \max}$, $E_t^{\min}$,
 505 and $E_t^{\max}$, i.e., the independent terms in constraints (4)-(6). We assume that
 506 these coefficients take values within known coefficient bounds, i.e., we know that
 507 $b_i \in [\underline{b}_i, \bar{b}_i], \forall i$.

508 In order to solve the LP problem (A.1)-(A.5) subject to uncertain coefficients
 509 within inequality constraints, we use the robust model proposed in [11], whose
 510 formulation is provided below:

$$\min_{x_j, \forall j; v_i, \mu_i, y_i, \forall i}$$

$$\sum_j c_j x_j \quad (\text{A.6})$$

subject to

$$\sum_j a_{ij} x_j + \Gamma_i v_i + \mu_i \leq \frac{1}{2} (\bar{b}_i + \underline{b}_i) \quad \forall i \quad (\text{A.7})$$

$$v_i + \mu_i \geq \frac{1}{2} (\bar{b}_i - \underline{b}_i) y_i \quad \forall i \quad (\text{A.8})$$

$$1 \leq y_i \quad \forall i \quad (\text{A.9})$$

$$v_i, \mu_i, y_i \geq 0 \quad \forall i \quad (\text{A.10})$$

$$\sum_j d_{kj} x_j \leq e_k \quad \forall k \quad (\text{A.11})$$

$$\sum_j f_{lj} x_j = g_l \quad \forall l \quad (\text{A.12})$$

$$x_j \geq 0 \quad \forall t. \quad (\text{A.13})$$

511 Problem (A.6)–(A.13) is obtained by applying duality and linearization prop-
 512 erties [11]. Parameters Γ_i in constraints (A.7) are included to control the level
 513 of robustness in the model against the uncertain coefficients in each constraint
 514 i . Each control parameter Γ_i can take values between 0 and the number of
 515 uncertain coefficients in the corresponding constraint. Since we only consider
 516 the uncertainty in the independent terms, then Γ_i can take values within $[0, 1]$.
 517 If Γ_i is fixed to 0, then we ignore the uncertainty in the uncertain coefficient b_i ,
 518 which takes its mean value $\frac{1}{2} (\bar{b}_i + \underline{b}_i)$, i.e., this can be interpreted as adopting a
 519 risky strategy since we assume that we know in advance the value of coefficient
 520 b_i . On the other hand, if Γ_i is fixed to 1, then we take into account that the
 521 uncertain coefficient b_i can take any value within the confidence bound $[\underline{b}_i, \bar{b}_i]$,
 522 i.e., we consider a comparatively more robust solution.

523 Acknowledgements

524 The work of L. Baringo has been partially funded by the Ministry of Science
525 of Spain under CICYT Project ENE2015-63879-R (MINECO/FEDER, UE), the
526 Junta de Comunidades de Castilla–La Mancha under Project POII-2014-012-P,
527 and the Universidad de Castilla–La Mancha under Grant GI20163388.

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