On the Solution of Revenue- and Network-Constrained Day-Ahead Market Clearing under Marginal Pricing—Part I: An Exact Bilevel Programming Approach

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Abstract—The first of this two-paper series addresses a practical day-ahead auction model where generation revenue constraints are explicitly incorporated in the problem formulation, as routinely done in several national electricity markets across Europe. The revenue-constrained market-clearing procedure includes the effect of the transmission network, inter-temporal constraints associated with generation scheduling, demand-side bidding, and marginal pricing. This auction design is an instance of price-based market clearing which features two major complicating factors. First, locational marginal prices become decision variables of the optimization process. In addition, producer revenues are formulated as bilinear and highly nonconvex products of power outputs and market-clearing prices. The resulting problem is formulated as a mixed-integer nonlinear bilevel program with bilinear terms for which available solution techniques rely on heuristics, approximations, or modeling simplifications. This paper presents a novel and exact methodology whereby the original problem is recast as an equivalent single-level mixed-integer linear program. As a consequence, finite convergence to optimality is guaranteed and the use of standard commercial software is allowed. The proposed transformation is based on duality theory of linear programming, Karush-Kuhn-Tucker optimality conditions, and integer algebra results. In the second part of this two-paper series, numerical results from several case studies illustrate the effective performance of the proposed solution approach.

Index Terms—Bilevel programming, locational marginal pricing, mixed-integer linear equivalent, pool-based electricity market, price-based market clearing, revenue-constrained auction design.

NOMENCLATURE

For quick reference, the symbols used throughout this paper and its companion [1] are listed below.

A. Indices

b Demand bid block index.

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h Time period index.
 i Generating unit index.
 j Consumer index.
 l Transmission line index.
 n Bus index.
 o Generation offer block index.
 t Time period index.

B. Sets

$\mathcal{B}_j$ Index set of the demand bid blocks of consumer j .
 $\mathcal{F}_{it}$ Feasibility set for the scheduling variables associated with unit i in period t .
 $\mathcal{I}$ Index set of generating units.
 $\mathcal{I}_n$ Index set of the generating units located at bus n .
 $\mathcal{J}$ Index set of consumers.
 $\mathcal{J}_n$ Index set of the consumers located at bus n .
 $\mathcal{L}$ Index set of transmission lines.
 $\mathcal{N}$ Index set of buses.
 $\mathcal{O}_i$ Index set of the generation offer blocks of unit i .
 $\mathcal{T}$ Index set of time periods.

C. Functions

$\mathcal{R}_i(\cdot)$ Function representing the right-hand side of the revenue constraint of unit i .

D. Constants

C_{bjt}^d Price of the b th demand block bid by consumer j in period t .
 C_{oit}^g Price of the o th generation block offered by unit i in period t .
 C_{it}^{nl} No-load price offered by unit i in period t .
 C_{it}^{sd} Shut-down price offered by unit i in period t .
 C_{it}^{su} Start-up price offered by unit i in period t .
 DT_i Minimum down time of unit i .
 $fr(l)$ Origin bus of line l .
 H Sufficiently large positive constant.

M_i^f	Coefficient of the fixed term of the minimum revenue function offered by unit i .
M_{oit}^v	Slope of the o th block of the minimum revenue function offered by unit i in period t .
$n(i)$	Bus where generating unit i is located.
$n(j)$	Bus where consumer j is located.
n_{B_j}	Cardinality of $\mathcal{B}_j$.
n_I	Number of generating units.
n_i^{DT}	Number of periods during which unit i must be initially scheduled off due to its minimum down time constraint.
n_i^{UT}	Number of periods during which unit i must be initially scheduled on due to its minimum up time constraint.
n_{i0}^{off}	Number of periods during which unit i has been scheduled off prior to the first period of the time span (end of period 0).
n_{i0}^{on}	Number of periods during which unit i has been scheduled on prior to the first period of the time span (end of period 0).
n_J	Number of consumers.
n_L	Number of transmission lines.
n_N	Number of buses.
n_{O_i}	Cardinality of $\mathcal{O}_i$.
n_T	Number of periods of the time span.
$\bar{P}_{bjt}^d$	Upper bound for p_{bjt}^d .
$\bar{P}_{jt}^d$	Upper bound for p_{jt}^d .
$\underline{P}_{jt}^d$	Lower bound for p_{jt}^d .
$\bar{P}_l^f$	Power flow capacity of line l .
P_{i0}^g	Initial power output of unit i .
$\bar{P}_{it}^g$	Upper bound for p_{it}^g .
$\underline{P}_{it}^g$	Lower bound for p_{it}^g .
$\bar{P}_{oit}^g$	Upper bound for p_{oit}^g .
R_i^{dn}	Ramp-down rate of unit i .
R_i^{sd}	Shut-down ramp rate of unit i .
R_i^{su}	Start-up ramp rate of unit i .
R_i^{up}	Ramp-up rate of unit i .
$to(l)$	Destination bus of line l .
UT_i	Minimum up time of unit i .
V_{i0}	Initial on/off status of unit i .
x_l	Reactance of line l .
$\underline{\gamma}_{it}^{lo}$	Lower bound for dual variable γ_{it}^{lo} .
$\bar{\gamma}_{it}^{up}$	Upper bound for dual variable γ_{it}^{up} .
$\bar{\epsilon}_{it}$	Upper bound for dual variable ϵ_{it} .
$\bar{\xi}_{it}$	Upper bound for dual variable ξ_{it} .
$\underline{\sigma}_{it}$	Lower bound for dual variable σ_{it} .
$\underline{\Psi}_i$	Lower bound for Ψ_i .

E. Variables

a_{it}	Auxiliary variable equal to the product $v_{it}\gamma_{it}^{lo}$.
b_{it}	Auxiliary variable equal to the product $v_{it}\gamma_{it}^{up}$.
c_{it}	Auxiliary variable equal to the product $v_{it}\xi_{it}$.
d_{it}	Auxiliary variable equal to the product $v_{it-1}\xi_{it}$.
e_{it}	Auxiliary variable equal to the product $v_{it}\epsilon_{it}$.
k_{it}	Auxiliary variable equal to the product $v_{it+1}\epsilon_{it}$.
p_{bjt}^d	Consumption level awarded to block b bid by consumer j in period t .
p_{jt}^d	Power consumption of consumer j in period t .
p_{lt}^f	Power flow of line l in period t .
p_{it}^g	Power output of unit i in period t .
p_{oit}^g	Generation level awarded to block o offered by unit i in period t .
q_{it}	Auxiliary variable equal to the product $v_{it}\sigma_{it}$.
r_i	Total revenue earned by unit i .
r_i^{min}	Minimum revenue that unit i is willing to earn.
s_{it}^{sd}	Shut-down offer cost of unit i in period t .
s_{it}^{su}	Start-up offer cost of unit i in period t .
v_{it}	Binary variable that is equal to 1 if unit i is scheduled on in period t , being 0 otherwise.
w_{it}	Auxiliary variable equal to the product $v_{it-1}\sigma_{it}$.
z_i	Variable that is equal to 0 if unit i is scheduled off along the time span, being 1 otherwise.
δ_{nt}	Voltage phase angle at bus n in period t .
Δr_i	Revenue margin of unit i .
λ_t^{av}	Load-weighted average locational marginal price for energy in period t .
Ψ_i	Declared profit of unit i .

F. Dual Variables

α_{it}	Dual variable associated with the definition of p_{it}^g as the sum of the generation levels awarded to the offer blocks.
β_{oit}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{oit}^g .
β_{oit}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{oit}^g .
γ_{it}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{it}^g .
γ_{it}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{it}^g .
ϵ_{it}	Dual variable associated with the shut-down ramp rate constraint of unit i in period t .
θ_{jt}	Dual variable associated with the definition of p_{jt}^d as the sum of the consumption levels awarded to the demand bid blocks.
λ_{nt}	Locational market-clearing price for energy at bus n in period t .
μ_{bjt}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{bjt}^d .

μ_{bjt}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{bjt}^d .
ν_{lt}	Dual variable associated with the definition of p_{lt}^f in terms of δ_{nt} .
ξ_{it}	Dual variable associated with the constraint modeling the ramp-up and start-up ramp rates of unit i in period t .
ρ_{jt}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{jt}^d .
ρ_{jt}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{jt}^d .
σ_{it}	Dual variable associated with the ramp-down rate constraint of unit i in period t .
ϕ_{lt}^{lo}	Dual variable associated with the constraint imposing the lower bound for p_{lt}^f .
ϕ_{lt}^{up}	Dual variable associated with the constraint imposing the upper bound for p_{lt}^f .

I. INTRODUCTION

REVENUE sufficiency is a desirable property in electricity markets by which revenues earned from awarded offers should cover the offer costs [2], [3]. Unfortunately, currently implemented auction designs relying on both unit commitment models and marginal-pricing-based remuneration [4], [5] do not guarantee the fulfillment of revenue sufficiency [2], [6]. Under such market designs, revenues from selling energy are determined by the products of the generation levels and marginal market-clearing prices for energy resulting from an economic dispatch problem with scheduling variables fixed at their optimal values. Due to the presence of nonconvexities associated with power system operation, such as no-load, start-up, and shut-down offers, as well as indivisibilities in power generation and consumption, offer costs may exceed revenues. This issue may be stressed by the growing production levels from intermittent renewable-based generating units, which are characterized by essentially zero marginal costs but require the scheduling of conventional plants to cope with unexpected network congestions and ramping events.

The revenue insufficiency of nonconvex electricity markets under marginal pricing can be addressed in two different ways:

- 1) By adopting an alternative remuneration scheme leading to a new market settlement [7]–[19] either by 1) uplifting the payments resulting from locational marginal pricing, 2) using other types of uplift, 3) relying on a pricing scheme different from marginal pricing, or 4) combining some of them. The strategy adopted in US electricity markets belongs to this class and consists in the implementation of out-of-market financial mechanisms as lump-sum monetary transfers between market agents [7], [8], [11], also known as offer cost compensations or make-whole payments, under locational marginal pricing. More specifically, producers that, as a result of the market-clearing process, incur economic losses, i.e., negative declared profits, are paid *ex post* the difference between the offer costs and the revenues earned. Additionally, in the presence of opportunity costs, uplift

payments incentivize the attainment of an equilibrium solution.

- 2) Through a tighter formulation of the market-clearing procedure explicitly setting bounds on either the revenues or the offer-based profits earned by producers [16], [17], [20]–[31], thereby resulting in a revenue-constrained auction model. The strategy adopted in national electricity markets across Europe falls within this category and consists in considering a marginal pricing scheme and extending the constraint set of the auction model to include terms related to revenues from selling energy that are referred to as minimum income conditions. Relevant examples are the Iberian electricity market [20] and APX [21]. Moreover, such revenue constraints are also explicitly accounted for in a somewhat related day-ahead auction model dealing with cross-border trading at a pan-European level [32]. Such an auction model is a previous step towards the creation of a single electricity market replacing the national ones, as a consequence of the increasing commitment to a common European energy policy [33]. It should be noted that the resulting pan-European market will be greater than those of Midwest ISO and PJM.

An important policy issue that is beyond the scope of this paper is the discussion on which alternative is the best option. Whether guaranteeing revenue sufficiency through alternative remuneration schemes such as the make-whole payments currently implemented in US markets, or through the extension of the constraint set of the auction models with revenue constraints, as done in Europe, has raised a long-standing debate in industry and academia due to the pros and cons of both approaches. Revenue-unconstrained models with make-whole payments are far simpler from a mathematical perspective and yield the maximum attainable level of social welfare. In contrast, the required compensations may give rise to price discrimination among market participants, may provide inadequate economic signals, and require the identification of the agents paying for such uplifts, which may be controversial [11], [15], [16]. On the other hand, revenue-constrained models are not discriminatory and maintain the desirable economic properties associated with marginal pricing. In addition, offers including too high bounds on revenues or profits increase the possibility of not being scheduled. Therefore, some researchers argue that honest offering by suppliers is promoted [34] so that generation offers reflect actual costs. Unfortunately, the attainment of such benefits requires sacrificing social welfare and transparency, solving a mathematically challenging optimization problem for which no exact solution methodology is currently available, and facing the potential exercise of market power by producers. The implementation of instances of both alternatives in current industry practice reveals the lack of consensus on this debate.

Motivated by the undeniable practical interest and the modeling and methodological challenges posed, this paper is focused on an instance of the alternative adopted in Europe, namely revenue- and network-constrained market clearing for energy in a day-ahead pool-based electricity market under marginal pricing. Market-clearing procedures for energy are

implemented by system operators to determine the set of awarded generation offers and demand bids for energy maximizing the level of declared social welfare. In addition, locational market-clearing prices for energy also result from the optimization process. Here, market clearing is implemented under the standard marginal pricing scheme [35] while considering demand-side bidding, the effect of the transmission network, as well as generation-related features such as production limits, ramp rates, minimum up and down times, and revenue constraints. The modeling framework is general to incorporate the aforementioned two instances of revenue constraints, thereby being suitable for day-ahead electricity markets currently implemented worldwide. Similar to previous related works [16], [17], [22]–[31], the analysis of market power abuse is beyond the scope of this two-paper series.

Unlike revenue-unconstrained auction designs, such as those implemented in US electricity markets, which are typically formulated as instances of mixed-integer linear programming [4], [36], the revenue-constrained models routinely solved in Europe [20], [21] belong to the class of price-based market clearing [37] where market-clearing prices become decision variables of the optimization. This aspect drastically complicates the solution of the resulting problem in two respects: 1) market-clearing prices may themselves result from an optimization process, as is the case under marginal pricing; and 2) revenue constraints include bilinear and hence highly nonconvex terms involving products of two continuous decision variables, namely power outputs and market-clearing prices, thereby leading to mixed-integer nonlinear optimization models for which exact solutions are in general unavailable.

Such challenges have triggered extensive research on this topic [16], [17], [20]–[31]. However, most works may be of reduced practical applicability given the use of an optimization goal different from the widely accepted declared social welfare maximization [22]–[25], [28], [31]. Therefore, within the context of current industry practice where the optimization process is driven by declared social welfare maximization, relevant works on revenue-constrained auction models are [16], [17], [26], [27], [29], [30].

In [26], [27], Motto and Galiana formulated a single-period and single-bus version of revenue-constrained market clearing as a mathematical program with equilibrium constraints [38]. A mixed-integer linear equivalent was derived by using Karush-Kuhn-Tucker (KKT) optimality conditions and a linearization scheme requiring additional binary variables. Moreover, bilinear revenue terms were equivalently linearized by introducing the concepts of augmented cost and augmented revenue. Unfortunately, the solution approach is not suitable for auction models incorporating transmission network constraints or inter-temporal operational constraints.

In [29], [30], García-Bertrand *et al.* dealt with a competitive market near-equilibrium including minimum profit conditions. The proposed solution approach consisted in appending the revenue-related constraints to the set of optimality conditions associated with the corresponding revenue-unconstrained auction. Subsequently, bilinear revenue terms were addressed by either 1) an iterative procedure based on a successive over-relaxation [29] or 2) as done in [30], a computationally

expensive approximation based on Schur's decomposition [39] requiring additional binary variables. As recognized by the authors, both approaches may yield suboptimal or even infeasible solutions.

In [16], Ruiz *et al.* proposed a revenue-constrained auction in a single-period setting wherein the presence of non-convexities was addressed by using a procedure based on a primal-dual transformation. This approach features two shortcomings. First, this methodology may lead to solutions different from those yielded by an economic dispatch problem and, as a consequence, the resulting locational market-clearing prices may be inconsistent with the theory of marginal pricing. Moreover, the binary expansion approach [40] used to linearize revenue terms is a computationally expensive approximation requiring extra binary variables.

In [17], discretely constrained, mixed linear complementarity problems were solved by relaxing both integrality and complementarity to build feasible problems. Within this framework, Gabriel *et al.* addressed a revenue- and network-constrained day-ahead auction model including price uplifts and a binary-expansion-based approach for bilinear revenue terms. Similar to [16], the methodology relies on an inexact and computationally inefficient approximation and the desirable consistency with the widely adopted economic-dispatch-based marginal pricing is not guaranteed.

Regarding industry practice, several real-life electricity markets deal with revenue constraints in day-ahead market clearing [20], [21]. A relevant approach for this instance of price-based auction can be found in the operational rules of the Iberian day-ahead electricity market, where an *ad-hoc* heuristic is described [20]. However, this approach does not guarantee optimality and may even terminate without attaining feasibility, as shown in the companion paper [1] and in [27].

The thrust of this paper is the proposal of an original and exact approach based on bilevel programming [41] to effectively address the real-life instances of revenue-constrained day-ahead market clearing under marginal pricing that have been implemented in several European electricity markets [20], [21] for over a decade and that are currently solved without even ensuring the attainment of feasibility [20]. Moreover, the proposed approach improves upon previously reported solutions for revenue-constrained market clearing [16], [17], [26], [27], [29], [30], which rely on simplified models or either inexact or computationally expensive methodologies.

The benefits of using bilevel programming are threefold. First, locational marginal prices, i.e., sensitivities, can be properly characterized without requiring any modeling simplifications. Furthermore, this sound mathematical-programming-based approach relies on well-grounded properties related to optimality and feasibility. In addition, the particular conditions featured by the proposed bilevel program allow its transformation to a single-level equivalent suitable for commercially available software.

Based on the findings of [37], the schedule of generating units is determined in the upper-level problem, whereas the lower-level problem is associated with dispatching decisions including levels of awarded offers and bids, network-related variables, as well as the corresponding locational marginal

1
2 prices. In addition, primal and dual information from the
3 lower level is used to compute producer revenues from selling
4 energy. The proposed bilevel programming framework gives
5 rise to relevant distinctive modeling aspects over previous
6 works [16], [17], [26], [27], [30]. First, this paper differs
7 from [26], [27] due to the explicit consideration of the effect
8 of the transmission network. Unlike [16], [17], the econom-
9 ically desirable uplift-free remuneration scheme relying on
10 economic-dispatch-based marginal pricing [35] is preserved.
11 Moreover, in contrast to [16], [17], [26], [27], [30], inter-
12 temporal operational constraints such as minimum up and
13 down times as well as ramping rates are explicitly accounted
14 for. As another salient modeling feature, the proposed bilevel
15 programming framework allows accommodating any practical
16 instance of generation revenue constraints such as minimum
17 revenue conditions and minimum profit constraints.

18
19 The resulting problem is formulated as a mixed-integer non-
20 linear bilevel program with bilinear revenue terms for which
21 no exact solution technique is currently available. The key idea
22 of the proposed solution approach is to convert the original
23 problem into an equivalent single-level mixed-integer linear
24 program. To that end, a two-step procedure is proposed. First,
25 based on practical modeling aspects that render the lower-
26 level problem an instance of linear programming, an effective
27 primal-dual transformation [42] is applied to recast the original
28 bilevel program as a single-level equivalent with bilinear
29 terms. Subsequently, and as a major distinctive methodological
30 feature over [16], [17], [26], [27], [29], [30], bilinear terms are
31 handled by a novel, exact, and computationally inexpensive
32 linearization scheme without requiring the addition of extra
33 binary variables. This linearization is based on duality the-
34 ory of linear programming, KKT optimality conditions, and
35 integer algebra results. The resulting single-level equivalent
36 is an instance of mixed-integer linear programming. Hence,
37 finite convergence to optimality is guaranteed while providing
38 a measure of the distance to optimality along the optimization
39 process [43]. Moreover, off-the-shelf software based on the
40 standard branch-and-cut algorithm is readily available [44],
41 which is beneficial for practical implementation purposes.

42
43 Thus, this paper features advantageous modeling and
44 methodological novelties with respect to the heuristic used
45 in industry practice [20] and previous relevant works [16],
46 [17], [26], [27], [29], [30]. The modeling novelties are 1) the
47 joint consideration of marginal pricing, network constraints,
48 demand-side bidding, and a multiperiod setting precisely in-
49 corporating inter-temporal operational constraints; and 2) the
50 flexibility to characterize both minimum revenue conditions
51 and minimum profit constraints. The methodological novelty
52 is the transformation of the original mixed-integer nonlinear
53 bilevel problem to a computationally effective mixed-
54 integer linear equivalent. This conversion, aside from using a
55 sound primal-dual transformation, relies on a new linearization
56 scheme for bilinear revenue terms involving the products
57 of two continuous decision variables that, unlike available
58 linearization schemes [39], [40], is exact and does not require
59 additional binary variables.

60 The main contributions of this paper are twofold:

- 1) From a modeling perspective, a novel formulation is pre-
sented to account for generation revenue constraints in
pool-based day-ahead market-clearing procedures under
marginal pricing, such as those currently implemented
across Europe. The resulting model, which includes the
effect of the transmission network, demand-side bidding,
and inter-temporal operational constraints, is formulated
as a mixed-integer nonlinear bilevel program with bi-
linear terms. It is worth emphasizing that explicitly
formulating time-varying locational marginal prices as
decision variables of the optimization problem is of
utmost importance because it permits the development
of mathematical-programming-based solutions to this
complex problem, such as the one reported here.
- 2) From a methodological perspective, this paper provides,
for the first time in the technical literature, an effective
solution approach based on a primal-dual transformation
and the exact conversion of bilinear revenue terms into
equivalent linear expressions, without requiring extra
binary variables. As a result, the original mixed-integer
nonlinear bilevel program with bilinear terms is recast
as a single-level mixed-integer linear equivalent that
overcomes the shortcomings of available solutions in
industry practice and in the technical literature.

It should be noted that the tool presented here, aside from
addressing the unresolved challenges posed in practice by
revenue-constrained market clearing, provides valuable infor-
mation for the debate on the best way to ensure revenue
sufficiency in nonconvex electricity markets under marginal
pricing. As an example, the proposed approach may help
system operators and regulators to accurately monitor market
participants' behavior and to precisely examine the long-term
implications of this trading scheme. Such analyses may require
dynamic game theoretical models or equilibrium-based models
wherein the proposed tool would play a key role.

The remainder of this paper is organized as follows. Section
II presents the problem formulation. Section III describes
the solution approach. Relevant conclusions are provided
in Section IV. Finally, for the sake of completeness, well-
known linear expressions for minimum up and down times,
shutdowns, and start-ups are formulated in the Appendix.
Furthermore, results from several case studies are presented
and discussed in the companion paper [1]. It should be noted
that the adoption of a two-part setting is intended to clearly
convey the methodological contributions on a step-by-step
basis. This layout also allows comprehensively illustrating
the numerical behavior of the proposed approach and its
superiority over existing methodologies from the perspectives
of both solution quality and computational burden.

II. PROBLEM FORMULATION

The incorporation of revenue constraints in day-ahead
market clearing requires the selection of a pricing scheme
to characterize market-clearing prices. Based on worldwide
industry practice [4], [5] and relevant references on revenue-
constrained market clearing [26], [27], [29], [30], a marginal
pricing scheme relying on a single nodal price for energy in

each period [35] is adopted. Thus, locational market-clearing prices are hereinafter referred to as locational marginal prices.

It is worth emphasizing that the theory of marginal pricing relies on the assumption of convexity of the optimization. Unfortunately, convexity is not held in market clearing due to the presence of binary variables, which are required to model minimum power outputs different from zero, minimum up and down times, shutdowns, and start-ups. This issue has triggered extensive research on nonconvex pricing for generation scheduling problems [12], [14], [16]. A widely adopted solution, however, is to derive marginal prices from a multiperiod economic dispatch or optimal power flow driven by declared social welfare maximization wherein binary scheduling variables have been fixed at their optimal values [4], [5]. Thus, as prescribed in [35], locational energy prices for a given feasible schedule are characterized in terms of the Lagrange multipliers or dual variables associated with the nodal power balance equations. In other words, the definition of the vector of market-clearing prices under marginal pricing is an optimization problem itself. This optimization also provides the optimal generation levels associated with the optimal generation schedule. Therefore, since locational marginal prices and generation levels are both dependent on the set of binary scheduling variables, the above-described practical procedure to obtain marginal prices in nonconvex auctions is also suitable for the nonconvexity of bilinear generation revenue terms involving products of generation levels and prices.

Hence, based on [37], the proposed revenue- and network-constrained day-ahead auction design can be formulated as the following mixed-integer nonlinear bilevel programming problem:

$$\begin{aligned}
 & \text{Maximize}_{r_i, s_{it}^{sd}, s_{it}^{su}, v_{it}, z_i} \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \\
 & \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\sum_{o \in \mathcal{O}_i} C_{oit}^g p_{oit}^g + s_{it}^{sd} + s_{it}^{su} + C_{it}^{ml} v_{it} \right) \quad (1) \\
 & \text{subject to:} \\
 & s_{it}^{sd}, s_{it}^{su}, v_{it} \in \mathcal{F}_{it}; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (2) \\
 & v_{it} \in \{0, 1\}; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (3) \\
 & 0 \leq z_i \leq 1; \forall i \in \mathcal{I} \quad (4) \\
 & z_i \geq v_{it}; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (5) \\
 & z_i \leq \sum_{t \in \mathcal{T}} v_{it}; \forall i \in \mathcal{I} \quad (6) \\
 & r_i = \sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t}; \forall i \in \mathcal{I} \quad (7) \\
 & r_i \geq \mathcal{R}_i(p_{oit}^g, s_{it}^{sd}, s_{it}^{su}, v_{it}, z_i); \forall i \in \mathcal{I} \quad (8)
 \end{aligned}$$

where optimal consumption levels p_{bjt}^d , generation levels p_{oit}^g and p_{oit}^g , and dual variables λ_{nt} are obtained from the following multiperiod dc optimal power flow:

$$\begin{aligned}
 & \text{Maximize}_{p_{bjt}^d, p_{jt}^d, p_{lt}^f, p_{oit}^g, p_{oit}^g, \delta_{nt}} \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \quad (9)
 \end{aligned}$$

subject to:

$$\begin{aligned}
 \sum_{j \in \mathcal{J}_n} p_{jt}^d &= \sum_{i \in \mathcal{I}_n} p_{it}^g + \sum_{l|to(l)=n} p_{lt}^f \\
 &- \sum_{l|fr(l)=n} p_{lt}^f : (\lambda_{nt}); \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \quad (10)
 \end{aligned}$$

$$p_{lt}^f = \frac{1}{x_l} (\delta_{fr(l)t} - \delta_{to(l)t}) : (\nu_{lt}); \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (11)$$

$$-\bar{P}_l^f \leq p_{lt}^f \leq \bar{P}_l^f : (\phi_{lt}^{lo}, \phi_{lt}^{up}); \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (12)$$

$$\underline{P}_{it}^g v_{it} \leq p_{it}^g \leq \bar{P}_{it}^g : (\gamma_{it}^{lo}, \gamma_{it}^{up}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (13)$$

$$\underline{P}_{jt}^d \leq p_{jt}^d \leq \bar{P}_{jt}^d : (\rho_{jt}^{lo}, \rho_{jt}^{up}); \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (14)$$

$$p_{it}^g = \sum_{o \in \mathcal{O}_i} p_{oit}^g : (\alpha_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (15)$$

$$0 \leq p_{oit}^g \leq \bar{P}_{oit}^g : (\beta_{oit}^{lo}, \beta_{oit}^{up}); \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (16)$$

$$p_{jt}^d = \sum_{b \in \mathcal{B}_j} p_{bjt}^d : (\theta_{jt}); \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (17)$$

$$0 \leq p_{bjt}^d \leq \bar{P}_{bjt}^d : (\mu_{bjt}^{lo}, \mu_{bjt}^{up}); \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (18)$$

$$\begin{aligned}
 p_{it}^g &\leq p_{it-1}^g + R_i^{up} v_{it-1} + R_i^{su} (v_{it} - v_{it-1}) \\
 &+ \bar{P}_{it}^g (1 - v_{it}) : (\xi_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 p_{it}^g &\geq p_{it-1}^g - R_i^{dn} v_{it} - R_i^{sd} (v_{it-1} - v_{it}) \\
 &- \bar{P}_{it}^g (1 - v_{it-1}) : (\sigma_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 p_{it}^g &\leq R_i^{sd} (v_{it} - v_{it+1}) + \bar{P}_{it}^g v_{it+1} : (\epsilon_{it}); \\
 &\forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1. \quad (21)
 \end{aligned}$$

For the sake of consistency, hourly time periods are considered; $\sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d = \bar{P}_{jt}^d, \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$; and $\sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g = \bar{P}_{it}^g, \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$.

The bilevel model (1)–(21) comprises two optimization levels, namely the upper level (1)–(8) and the lower level (9)–(21). Upper-level decision variables are producer revenues r_i , shut-down offers s_{it}^{sd} , start-up offers s_{it}^{su} , on/off statuses of generating units v_{it} , and variables z_i associated with revenue constraints. Lower-level decision variables are power flows p_{lt}^f , voltage phase angles δ_{nt} , power outputs p_{oit}^g , power consumptions p_{jt}^d , and awarded levels of generation offer and demand bid blocks p_{oit}^g and p_{bjt}^d , respectively, whereas dual variables λ_{nt} are an outcome of the lower-level problem.

The goal of the upper-level problem is to maximize the revenue-constrained declared social welfare (1) evaluated at the optimal values of consumption levels p_{bjt}^d , generation levels p_{oit}^g and p_{oit}^g , and dual variables λ_{nt} . The objective function maximized in (1) comprises five terms respectively related to 1) demand bids, 2) generation energy offers, 3) generation shut-down offers, 4) generation start-up offers, and 5) generation no-load offers. As is customary in industry [20], [21], stepwise linear bids and offers for energy are considered.

The feasibility set $\mathcal{F}_{it}$ in (2) includes minimum up and down times, shutdowns, and start-ups, which are modeled using mixed-integer linear programming [45], as formulated

in the Appendix. Expressions (3) impose the integrality of scheduling variables v_{it} .

Expressions (4)–(6) characterize variables z_i , which identify those generators whose revenue constraints must be enforced, i.e., those with awarded offers along the time span. Thus, z_i is equal to 1 if unit i is scheduled on in at least one period and, hence, the corresponding revenue constraint is imposed. On the other hand, z_i is equal to 0 for those units that are scheduled off along the scheduling horizon, for which the corresponding revenue constraints are disregarded. Therefore, z_i is a binary-valued variable equal to the maximum of v_{it} over all periods. Note, however, that z_i can be equivalently formulated as a continuous variable lying in the interval $[0, 1]$, as modeled in (4), that is greater than or equal to every v_{it} along the time span (5), and that is less than or equal to the summation of v_{it} over all time periods (6). If unit i is scheduled off in all periods, i.e., $v_{it} = 0, \forall t \in \mathcal{T}$, $z_i = \max\{0, \dots, 0\} = 0$. For this case, (4) and (6) would yield the same result, as desired. In contrast, if some v_{it} is equal to 1 for unit i , $z_i = \max_{t \in \mathcal{T}}\{v_{it}\} = 1$, which results from (4) and (5).

The bilinear revenue terms from selling energy are defined in (7), where both primal and dual variables of the lower-level problem are involved. Expressions (8) represent generation revenue constraints. Note that $\mathcal{R}_i(\cdot)$ may adopt different forms to accommodate any instance of revenue-constrained auction designs such as those incorporating minimum revenue conditions or minimum profit constraints, as described next:

- 1) Minimum revenue conditions impose that the total revenue earned by each scheduled unit must be greater than or equal to a level determined by the corresponding minimum revenue function submitted as part of its generation offer. Thus, the minimum revenue function of unit i , which corresponds to $\mathcal{R}_i(\cdot)$ in (8), yields the minimum revenue this unit is willing to earn for each awarded energy offer. Based on current industry practice [20], minimum revenue conditions can be incorporated in (8) as follows:

$$r_i \geq M_i^f z_i + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} M_{oit}^v p_{oit}^g; \forall i \in \mathcal{I}. \quad (22)$$

For each unit i , the right-hand side of (22) is the minimum revenue function, which comprises two terms: 1) a fixed term characterized by parameter M_i^f , and 2) an energy-dependent variable term where parameters M_{oit}^v are the slopes related to the energy offer blocks. Note that if $z_i = 0$, and hence $p_{oit}^g = 0, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T}$, the minimum revenue is equal to 0, i.e., the revenue condition does not constrain the optimization.

- 2) Minimum profit constraints impose that the declared profit of each scheduled unit i , Ψ_i , must be greater than or equal to a lower bound $\underline{\Psi}_i$. The declared profit of unit i is defined as the revenue earned by this unit minus its generation offer cost:

$$\begin{aligned} \Psi_i = r_i - \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \\ - \sum_{t \in \mathcal{T}} (s_{it}^{sd} + s_{it}^{su} + C_{it}^m v_{it}); \forall i \in \mathcal{I}. \end{aligned} \quad (23)$$

Thus, minimum profit constraints are modeled by replacing (8) as follows:

$$\begin{aligned} r_i - \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g - \sum_{t \in \mathcal{T}} (s_{it}^{sd} \\ + s_{it}^{su} + C_{it}^m v_{it}) \geq \underline{\Psi}_i z_i - H(1 - z_i); \forall i \in \mathcal{I} \end{aligned} \quad (24)$$

where H is a sufficiently large positive constant used to discard minimum profit constraints for those units that are scheduled off throughout the time span.

It is worth pointing out that both (22) and (24) are mixed-integer linear expressions.

The lower-level problem is a multiperiod optimal power flow particularized for the optimal values of the scheduling variables v_{it} . This optimization problem is also driven by the maximization of the revenue-constrained declared social welfare (9). Note that the shut-down, start-up, and no-load terms included in (1) are dropped in the objective function (9) because those terms become constant for given values of v_{it} . Expressions (10) are the nodal power balance equations, where each associated Lagrange multiplier or dual variable λ_{nt} represents the sensitivity of the optimal revenue-constrained declared social welfare with respect to a small perturbation in the corresponding power balance constraint for the values of scheduling variables v_{it} determined in the upper level. Thus, according to [35], λ_{nt} is the marginal price for energy at bus n in period t . Using a dc load flow model, constraints (11) characterize line power flows in terms of voltage phase angles. Constraints (12) impose the transmission capacity limits across the transmission network. Production and consumption limits are respectively set in (13) and (14). The block structures of generation offers and demand bids are respectively characterized through constraints (15)–(16) and (17)–(18). Finally, ramping rates are modeled by (19)–(21) as described in [45].

Problem (1)–(21) is a mixed-integer bilevel program with two particular features, namely 1) bilinear terms involving products of two continuous decision variables are included in (7), and 2) the lower-level problem (9)–(21) is parameterized in terms of the upper-level binary variables v_{it} in such a way that this optimization is a linear program.

III. SOLUTION APPROACH

The proposed solution approach consists in reformulating the mixed-integer nonlinear bilevel program with bilinear terms (1)–(21) as a single-level mixed-integer linear equivalent according to the following two-step procedure.

A. Step 1: Single-Level Equivalent with Bilinear Terms

Based on the linearity of the lower level (9)–(21), the original bilevel program can be converted into a mixed-integer nonlinear equivalent by using results from duality theory of linear programming [46]. As shown in [42], the lower-level problem can be equivalently replaced with its primal feasibility constraints (10)–(21), its dual feasibility constraints, and the equality corresponding to the strong duality theorem in which the lower-level primal and dual objective functions are equated. This primal-dual transformation yields the following single-level mixed-integer program including bilinear terms:

$$\begin{aligned}
& \text{Maximize} \quad \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d \\
& p_{bjt}^d, p_{jt}^d, p_{it}^f, p_{it}^g, \\
& p_{oit}^g, r_i, s_{it}^{sd}, s_{it}^{su}, \\
& v_{it}, z_i, \alpha_{it}, \beta_{oit}^{lo}, \\
& \beta_{oit}^{up}, \gamma_{it}^{lo}, \gamma_{it}^{up}, \delta_{nt}, \\
& \epsilon_{it}, \theta_{jt}, \lambda_{nt}, \mu_{bjt}^{lo}, \\
& \mu_{bjt}^{up}, \nu_{it}, \xi_{it}, \rho_{jt}^{lo}, \\
& \rho_{jt}^{up}, \sigma_{it}, \phi_{it}^{lo}, \phi_{it}^{up} \\
& + C_{it}^{ml} v_{it} \quad (25)
\end{aligned}$$

subject to:

$$\text{Constraints (2)–(6) and (8)} \quad (26)$$

$$r_i = \sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t}; \forall i \in \mathcal{I} \quad (27)$$

$$\text{Constraints (10)–(21)} \quad (28)$$

$$\begin{aligned}
& -\lambda_{n(i)t} + \gamma_{it}^{lo} + \gamma_{it}^{up} + \xi_{it} - \xi_{it+1} + \sigma_{it} - \sigma_{it+1} \\
& + \epsilon_{it} + \alpha_{it} = 0; \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (29)
\end{aligned}$$

$$\begin{aligned}
& -\lambda_{n(i)n_T} + \gamma_{in_T}^{lo} + \gamma_{in_T}^{up} + \xi_{in_T} + \sigma_{in_T} \\
& + \alpha_{in_T} = 0; \forall i \in \mathcal{I} \quad (30)
\end{aligned}$$

$$-\alpha_{it} + \beta_{oit}^{lo} + \beta_{oit}^{up} = -C_{oit}^g; \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (31)$$

$$\lambda_{n(j)t} + \rho_{jt}^{lo} + \rho_{jt}^{up} + \theta_{jt} = 0; \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (32)$$

$$-\theta_{jt} + \mu_{bjt}^{lo} + \mu_{bjt}^{up} = C_{bjt}^d; \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (33)$$

$$\nu_{lt} + \phi_{lt}^{lo} + \phi_{lt}^{up} + \lambda_{fr(l)t} - \lambda_{to(l)t} = 0; \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (34)$$

$$\begin{aligned}
& - \sum_{l|fr(l)=n} \frac{1}{x_l} \nu_{lt} + \sum_{l|to(l)=n} \frac{1}{x_l} \nu_{lt} = 0; \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \quad (35)
\end{aligned}$$

$$\gamma_{it}^{lo}, \sigma_{it} \leq 0, \gamma_{it}^{up}, \xi_{it} \geq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (36)$$

$$\epsilon_{it} \geq 0; \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (37)$$

$$\rho_{jt}^{lo} \leq 0, \rho_{jt}^{up} \geq 0; \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (38)$$

$$\phi_{lt}^{lo} \leq 0, \phi_{lt}^{up} \geq 0; \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (39)$$

$$\beta_{oit}^{lo} \leq 0, \beta_{oit}^{up} \geq 0; \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (40)$$

$$\mu_{bjt}^{lo} \leq 0, \mu_{bjt}^{up} \geq 0; \forall j \in \mathcal{J}, \forall b \in \mathcal{B}_j, \forall t \in \mathcal{T} \quad (41)$$

$$\begin{aligned}
& \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g = \\
& \sum_{l \in \mathcal{L}} \sum_{t \in \mathcal{T}} \bar{P}_l^f (\phi_{lt}^{up} - \phi_{lt}^{lo})
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(P_{it}^g v_{it} \gamma_{it}^{lo} + \bar{P}_{it}^g v_{it} \gamma_{it}^{up} + \sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g \beta_{oit}^{up} \right) \\
& + \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \left(P_{jt}^d \rho_{jt}^{lo} + \bar{P}_{jt}^d \rho_{jt}^{up} + \sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d \mu_{bjt}^{up} \right)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \xi_{i1} + R_i^{up} V_{i0} \xi_{i1} + R_i^{su} (v_{i1} \xi_{i1} - V_{i0} \xi_{i1}) \right. \\
& \quad \left. + \bar{P}_{i1}^g (\xi_{i1} - v_{i1} \xi_{i1}) \right]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{up} v_{it-1} \xi_{it} + R_i^{su} (v_{it} \xi_{it} - v_{it-1} \xi_{it}) \right. \\
& \quad \left. + \bar{P}_{it}^g (\xi_{it} - v_{it} \xi_{it}) \right]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \sigma_{i1} - R_i^{dn} v_{i1} \sigma_{i1} - R_i^{sd} (V_{i0} \sigma_{i1} - v_{i1} \sigma_{i1}) \right. \\
& \quad \left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right]
\end{aligned}$$

$$\begin{aligned}
& - \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{dn} v_{it} \sigma_{it} + R_i^{sd} (v_{it-1} \sigma_{it} - v_{it} \sigma_{it}) \right. \\
& \quad \left. + \bar{P}_{it}^g (\sigma_{it} - v_{it-1} \sigma_{it}) \right] \\
& + \sum_{i \in \mathcal{I}} \sum_{t=1}^{n_T-1} \left[R_i^{sd} (v_{it} \epsilon_{it} - v_{it+1} \epsilon_{it}) + \bar{P}_{it}^g v_{it+1} \epsilon_{it} \right] \quad (42)
\end{aligned}$$

where (25)–(27) correspond to the upper optimization level (1)–(8), whereas (28)–(42) equivalently replace the lower optimization level (9)–(21). The lower-level primal feasibility constraints are included in (28), expressions (29)–(41) represent the lower-level dual feasibility constraints, and (42) is the equality associated with the strong duality theorem.

B. Step 2: Sorting Out Bilinear Terms

Bilinear terms in the single-level equivalent (25)–(42) can be categorized as: 1) products of power outputs p_{it}^g and locational marginal prices λ_{nt} in the definition of producer revenues (27), and 2) products of binary variables v_{it} and continuous dual variables in the strong duality equality (42).

As described next, bilinear products in (27) can be equivalently transformed into linear expressions based on KKT optimality conditions.

First, the summation of bilinear terms in the right-hand side of (27) can be expressed as follows:

$$\sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t} = \sum_{t=1}^{n_T-1} p_{it}^g \lambda_{n(i)t} + p_{in_T}^g \lambda_{n(i)n_T}; \forall i \in \mathcal{I}. \quad (43)$$

Using (29) and (30), expression (43) becomes:

$$\begin{aligned}
& \sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t} = \sum_{t \in \mathcal{T}} (p_{it}^g \gamma_{it}^{lo} + p_{it}^g \gamma_{it}^{up}) + p_{i1}^g \xi_{i1} \\
& + \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \xi_{it} + \sum_{t=1}^{n_T-1} p_{it}^g \epsilon_{it} + p_{i1}^g \sigma_{i1} \\
& + \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \sigma_{it} + \sum_{t \in \mathcal{T}} p_{it}^g \alpha_{it}; \forall i \in \mathcal{I}. \quad (44)
\end{aligned}$$

Furthermore, the last summation in the right-hand side of (44) can be recast by using (15):

$$\sum_{t \in \mathcal{T}} p_{it}^g \alpha_{it} = \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g \alpha_{oit}; \forall i \in \mathcal{I}. \quad (45)$$

From (31), α_{it} can be replaced in the right-hand side of (45) as follows:

$$\begin{aligned}
& \sum_{t \in \mathcal{T}} p_{it}^g \alpha_{it} = \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} (C_{oit}^g p_{oit}^g + p_{oit}^g \beta_{oit}^{lo} + p_{oit}^g \beta_{oit}^{up}); \\
& \forall i \in \mathcal{I}. \quad (46)
\end{aligned}$$

Therefore, using (46) in (44) yields:

$$\begin{aligned} \sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t} &= \sum_{t \in \mathcal{T}} (p_{it}^g \gamma_{it}^{lo} + p_{it}^g \gamma_{it}^{up}) + p_{i1}^g \xi_{i1} \\ &+ \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \xi_{it} + \sum_{t=1}^{n_T-1} p_{it}^g \epsilon_{it} + p_{i1}^g \sigma_{i1} \\ &+ \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \sigma_{it} + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g \\ &+ \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g \beta_{oit}^{lo} + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g \beta_{oit}^{up}; \forall i \in \mathcal{I}. \end{aligned} \quad (47)$$

In the right-hand side of (47) there are still terms involving products of continuous variables. Such bilinear terms can be sorted out by using the following complementary slackness conditions, which are associated with inequalities (13), (16), and (19)–(21) characterizing generation energy offers:

$$(p_{it}^g - P_{it}^g v_{it}) \gamma_{it}^{lo} = 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (48)$$

$$(p_{it}^g - \bar{P}_{it}^g v_{it}) \gamma_{it}^{up} = 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (49)$$

$$(p_{oit}^g - 0) \beta_{oit}^{lo} = 0; \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (50)$$

$$(p_{oit}^g - \bar{P}_{oit}^g) \beta_{oit}^{up} = 0; \forall i \in \mathcal{I}, \forall o \in \mathcal{O}_i, \forall t \in \mathcal{T} \quad (51)$$

$$\begin{aligned} &\left[p_{i1}^g - \left(P_{i0}^g + R_i^{up} V_{i0} + R_i^{su} (v_{i1} - V_{i0}) \right) \right. \\ &\left. + \bar{P}_{i1}^g (1 - v_{i1}) \right] \xi_{i1} = 0; \forall i \in \mathcal{I} \end{aligned} \quad (52)$$

$$\begin{aligned} &\left[p_{it}^g - p_{it-1}^g - \left(R_i^{up} v_{it-1} + R_i^{su} (v_{it} - v_{it-1}) \right) \right. \\ &\left. + \bar{P}_{it}^g (1 - v_{it}) \right] \xi_{it} = 0; \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \end{aligned} \quad (53)$$

$$\begin{aligned} &\left[p_{i1}^g - \left(P_{i0}^g - R_i^{dn} v_{i1} - R_i^{sd} (V_{i0} - v_{i1}) \right) \right. \\ &\left. - \bar{P}_{i1}^g (1 - V_{i0}) \right] \sigma_{i1} = 0; \forall i \in \mathcal{I} \end{aligned} \quad (54)$$

$$\begin{aligned} &\left[p_{it}^g - p_{it-1}^g - \left(-R_i^{dn} v_{it} - R_i^{sd} (v_{it-1} - v_{it}) \right) \right. \\ &\left. - \bar{P}_{it}^g (1 - v_{it-1}) \right] \sigma_{it} = 0; \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \end{aligned} \quad (55)$$

$$\begin{aligned} &\left[p_{it}^g - \left(R_i^{sd} (v_{it} - v_{it+1}) + \bar{P}_{it}^g v_{it+1} \right) \right] \epsilon_{it} = 0; \\ &\forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1. \end{aligned} \quad (56)$$

From (48)–(56), the following equalities can be derived:

$$\sum_{t \in \mathcal{T}} p_{it}^g \gamma_{it}^{lo} = \sum_{t \in \mathcal{T}} P_{it}^g v_{it} \gamma_{it}^{lo}; \forall i \in \mathcal{I} \quad (57)$$

$$\sum_{t \in \mathcal{T}} p_{it}^g \gamma_{it}^{up} = \sum_{t \in \mathcal{T}} \bar{P}_{it}^g v_{it} \gamma_{it}^{up}; \forall i \in \mathcal{I} \quad (58)$$

$$\sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g \beta_{oit}^{lo} = 0; \forall i \in \mathcal{I} \quad (59)$$

$$\sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g \beta_{oit}^{up} = \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} \bar{P}_{oit}^g \beta_{oit}^{up}; \forall i \in \mathcal{I} \quad (60)$$

$$\begin{aligned} p_{i1}^g \xi_{i1} &= \left[P_{i0}^g + R_i^{up} V_{i0} + R_i^{su} (v_{i1} - V_{i0}) \right. \\ &\left. + \bar{P}_{i1}^g (1 - v_{i1}) \right] \xi_{i1}; \forall i \in \mathcal{I} \end{aligned} \quad (61)$$

$$\begin{aligned} \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \xi_{it} &= \sum_{t=2}^{n_T} \left[R_i^{up} v_{it-1} \right. \\ &\left. + R_i^{su} (v_{it} - v_{it-1}) + \bar{P}_{it}^g (1 - v_{it}) \right] \xi_{it}; \forall i \in \mathcal{I} \end{aligned} \quad (62)$$

$$\begin{aligned} p_{i1}^g \sigma_{i1} &= \left[P_{i0}^g - R_i^{dn} v_{i1} - R_i^{sd} (V_{i0} - v_{i1}) \right. \\ &\left. - \bar{P}_{i1}^g (1 - V_{i0}) \right] \sigma_{i1}; \forall i \in \mathcal{I} \end{aligned} \quad (63)$$

$$\begin{aligned} \sum_{t=2}^{n_T} (p_{it}^g - p_{it-1}^g) \sigma_{it} &= \sum_{t=2}^{n_T} \left[-R_i^{dn} v_{it} \right. \\ &\left. - R_i^{sd} (v_{it-1} - v_{it}) - \bar{P}_{it}^g (1 - v_{it-1}) \right] \sigma_{it}; \forall i \in \mathcal{I} \end{aligned} \quad (64)$$

$$\begin{aligned} \sum_{t=1}^{n_T-1} p_{it}^g \epsilon_{it} &= \sum_{t=1}^{n_T-1} \left[R_i^{sd} (v_{it} - v_{it+1}) \right. \\ &\left. + \bar{P}_{it}^g v_{it+1} \right] \epsilon_{it}; \forall i \in \mathcal{I}. \end{aligned} \quad (65)$$

Note that the bilinear terms in the right-hand side of (47) correspond to the left-hand sides of (57)–(65). Thus, replacing them in (47) results in:

$$\begin{aligned} \sum_{t \in \mathcal{T}} p_{it}^g \lambda_{n(i)t} &= \sum_{t \in \mathcal{T}} \left(P_{it}^g v_{it} \gamma_{it}^{lo} + \bar{P}_{it}^g v_{it} \gamma_{it}^{up} \right) \\ &+ \left[P_{i0}^g \xi_{i1} + R_i^{up} V_{i0} \xi_{i1} + R_i^{su} (v_{i1} \xi_{i1} - V_{i0} \xi_{i1}) \right. \\ &\left. + \bar{P}_{i1}^g (\xi_{i1} - v_{i1} \xi_{i1}) \right] \\ &+ \sum_{t=2}^{n_T} \left[R_i^{up} v_{it-1} \xi_{it} + R_i^{su} (v_{it} \xi_{it} - v_{it-1} \xi_{it}) \right. \\ &\left. + \bar{P}_{it}^g (\xi_{it} - v_{it} \xi_{it}) \right] \\ &+ \left[P_{i0}^g \sigma_{i1} - R_i^{dn} v_{i1} \sigma_{i1} - R_i^{sd} (V_{i0} \sigma_{i1} - v_{i1} \sigma_{i1}) \right. \\ &\left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right] \\ &- \sum_{t=2}^{n_T} \left[R_i^{dn} v_{it} \sigma_{it} + R_i^{sd} (v_{it-1} \sigma_{it} - v_{it} \sigma_{it}) \right. \\ &\left. + \bar{P}_{it}^g (\sigma_{it} - v_{it-1} \sigma_{it}) \right] \\ &+ \sum_{t=1}^{n_T-1} \left[R_i^{sd} (v_{it} \epsilon_{it} - v_{it+1} \epsilon_{it}) + \bar{P}_{it}^g v_{it+1} \epsilon_{it} \right] \\ &+ \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g C_{oit}^g + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} \bar{P}_{oit}^g \beta_{oit}^{up}; \forall i \in \mathcal{I}. \end{aligned} \quad (66)$$

As a consequence, the bilinear terms involving products of continuous decision variables are expressed as sums of terms including products of binary decision variables and continuous dual variables. Such products are similar to those in the strong duality equality (42) resulting from the primal-dual transformation. Using previously reported integer algebra results [47], such bilinear terms can be equivalently recast as:

$$\gamma_{it}^{lo} v_{it} \leq a_{it} \leq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (67)$$

$$\gamma_{it}^{lo} (1 - v_{it}) \leq \gamma_{it}^{lo} - a_{it} \leq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (68)$$

$$0 \leq b_{it} \leq \bar{\gamma}_{it}^{up} v_{it}; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (69)$$

$$0 \leq \gamma_{it}^{up} - b_{it} \leq \bar{\gamma}_{it}^{up} (1 - v_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (70)$$

$$0 \leq c_{it} \leq \bar{\xi}_{it} v_{it}; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (71)$$

$$0 \leq \xi_{it} - c_{it} \leq \bar{\xi}_{it}(1 - v_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (72)$$

$$0 \leq d_{it} \leq \bar{\xi}_{it}v_{it-1}; \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (73)$$

$$0 \leq \xi_{it} - d_{it} \leq \bar{\xi}_{it}(1 - v_{it-1}); \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (74)$$

$$0 \leq e_{it} \leq \bar{\epsilon}_{it}v_{it}; \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (75)$$

$$0 \leq \epsilon_{it} - e_{it} \leq \bar{\epsilon}_{it}(1 - v_{it}); \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (76)$$

$$0 \leq k_{it} \leq \bar{\epsilon}_{it}v_{it+1}; \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (77)$$

$$0 \leq \epsilon_{it} - k_{it} \leq \bar{\epsilon}_{it}(1 - v_{it+1}); \forall i \in \mathcal{I}, \forall t = 1 \dots n_T - 1 \quad (78)$$

$$\underline{\sigma}_{it}v_{it} \leq q_{it} \leq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (79)$$

$$\underline{\sigma}_{it}(1 - v_{it}) \leq \sigma_{it} - q_{it} \leq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (80)$$

$$\underline{\sigma}_{it}v_{it-1} \leq w_{it} \leq 0; \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (81)$$

$$\underline{\sigma}_{it}(1 - v_{it-1}) \leq \sigma_{it} - w_{it} \leq 0; \forall i \in \mathcal{I}, \forall t = 2 \dots n_T \quad (82)$$

where a_{it} , b_{it} , c_{it} , d_{it} , e_{it} , k_{it} , q_{it} , and w_{it} represent $v_{it}\gamma_{it}^{lo}$, $v_{it}\gamma_{it}^{up}$, $v_{it}\xi_{it}$, $v_{it-1}\xi_{it}$, $v_{it}\epsilon_{it}$, $v_{it+1}\epsilon_{it}$, $v_{it}\sigma_{it}$, and $v_{it-1}\sigma_{it}$, respectively. Expressions (67)–(68), (69)–(70), (71)–(72), (73)–(74), (75)–(76), (77)–(78), (79)–(80), and (81)–(82) model the respective linearizations.

It should be noted that the above linearizations require setting bounds on lower-level dual variables γ_{it}^{lo} , γ_{it}^{up} , ϵ_{it} , ξ_{it} , and σ_{it} . As described in [37], these bounding parameters may be selected based on the values of the corresponding dual variables resulting from the optimal solution to the associated revenue-unconstrained auction model.

C. Single-Level Mixed-Integer Linear Equivalent

The single-level equivalent of problem (1)–(21) is formulated as the following mixed-integer linear program:

$$\begin{aligned} & \text{Maximize} \\ & a_{it}, b_{it}, c_{it}, d_{it}, e_{it}, \\ & k_{it}, p_{ijt}^d, p_{ijt}^g, p_{ijt}^f, p_{ijt}^g, \\ & p_{oit}^g, q_{it}, r_i, s_{it}^{sd}, s_{it}^{su}, \\ & v_{it}, w_{it}, z_i, \alpha_{it}, \beta_{oit}^{lo}, \\ & \beta_{oit}^{up}, \gamma_{it}^{lo}, \gamma_{it}^{up}, \delta_{nt}, \epsilon_{it}, \\ & \theta_{jt}, \lambda_{nt}, \mu_{b_{jt}}^{lo}, \mu_{b_{jt}}^{up}, \\ & \nu_{lt}, \xi_{it}, \rho_{jt}^{lo}, \rho_{jt}^{up}, \\ & \sigma_{it}, \phi_{lt}^{lo}, \phi_{lt}^{up} \end{aligned} \quad \begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d \\ & - \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\sum_{o \in \mathcal{O}_i} C_{oit}^g p_{oit}^g + s_{it}^{sd} + s_{it}^{su} \right. \\ & \left. + C_{it}^{ml} v_{it} \right) \end{aligned} \quad (83)$$

subject to:

$$\text{Constraints (2)–(6) and (8)} \quad (84)$$

$$\begin{aligned} r_i &= \sum_{t \in \mathcal{T}} \left(\underline{P}_{it}^g a_{it} + \bar{P}_{it}^g b_{it} \right) \\ &+ \left[P_{i0}^g \xi_{i1} + R_i^{up} V_{i0} \xi_{i1} + R_i^{su} (c_{i1} - V_{i0} \xi_{i1}) \right. \\ &\quad \left. + \bar{P}_{i1}^g (\xi_{i1} - c_{i1}) \right] \\ &+ \sum_{t=2}^{n_T} \left[R_i^{up} d_{it} + R_i^{su} (c_{it} - d_{it}) + \bar{P}_{it}^g (\xi_{it} - c_{it}) \right] \\ &+ \left[P_{i0}^g \sigma_{i1} - R_i^{dn} q_{i1} - R_i^{sd} (V_{i0} \sigma_{i1} - q_{i1}) \right. \\ &\quad \left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right] \\ &- \sum_{t=2}^{n_T} \left[R_i^{dn} q_{it} + R_i^{sd} (w_{it} - q_{it}) + \bar{P}_{it}^g (\sigma_{it} - w_{it}) \right] \end{aligned}$$

TABLE I
DIMENSION OF PROBLEM (83)–(89)

# of constraints	# of binary variables	# of real variables
$2n_T(n_N + 2n_L + 2n_J)$		$2n_T(n_N + 2n_L + 2n_J)$
$+ n_I(44n_T - 10)$		$+ n_I(17n_T - 3)$
$+ 3n_T \sum_{i \in \mathcal{I}} n_{O_i}$	$n_I n_T$	$+ 3n_T \sum_{i \in \mathcal{I}} n_{O_i}$
$+ 3n_T \sum_{j \in \mathcal{J}} n_{B_j} + 2$		$+ 3n_T \sum_{j \in \mathcal{J}} n_{B_j}$

$$\begin{aligned} & + \sum_{t=1}^{n_T-1} \left[R_i^{sd} (e_{it} - k_{it}) + \bar{P}_{it}^g k_{it} \right] \\ & + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} p_{oit}^g C_{oit}^g + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} \bar{P}_{oit}^g \beta_{oit}^{up}; \forall i \in \mathcal{I} \end{aligned} \quad (85)$$

$$\text{Constraints (10)–(21)} \quad (86)$$

$$\text{Constraints (29)–(41)} \quad (87)$$

$$\begin{aligned} & \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{t \in \mathcal{T}} C_{bjt}^d p_{bjt}^d - \sum_{i \in \mathcal{I}} \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} C_{oit}^g p_{oit}^g = \\ & \sum_{l \in \mathcal{L}} \sum_{t \in \mathcal{T}} \bar{P}_l^f (\phi_{lt}^{up} - \phi_{lt}^{lo}) \end{aligned}$$

$$\begin{aligned} & + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\underline{P}_{it}^g a_{it} + \bar{P}_{it}^g b_{it} + \sum_{o \in \mathcal{O}_i} \bar{P}_{oit}^g \beta_{oit}^{up} \right) \\ & + \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \left(\underline{P}_{jt}^d \rho_{jt}^{lo} + \bar{P}_{jt}^d \rho_{jt}^{up} + \sum_{b \in \mathcal{B}_j} \bar{P}_{bjt}^d \mu_{bjt}^{up} \right) \\ & + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \xi_{i1} + R_i^{up} V_{i0} \xi_{i1} + R_i^{su} (c_{i1} - V_{i0} \xi_{i1}) \right. \\ & \quad \left. + \bar{P}_{i1}^g (\xi_{i1} - c_{i1}) \right] \end{aligned}$$

$$\begin{aligned} & + \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{up} d_{it} + R_i^{su} (c_{it} - d_{it}) + \bar{P}_{it}^g (\xi_{it} - c_{it}) \right] \\ & + \sum_{i \in \mathcal{I}} \left[P_{i0}^g \sigma_{i1} - R_i^{dn} q_{i1} - R_i^{sd} (V_{i0} \sigma_{i1} - q_{i1}) \right. \\ & \quad \left. - \bar{P}_{i1}^g (1 - V_{i0}) \sigma_{i1} \right] \\ & - \sum_{i \in \mathcal{I}} \sum_{t=2}^{n_T} \left[R_i^{dn} q_{it} + R_i^{sd} (w_{it} - q_{it}) + \bar{P}_{it}^g (\sigma_{it} - w_{it}) \right] \\ & + \sum_{i \in \mathcal{I}} \sum_{t=1}^{n_T-1} \left[R_i^{sd} (e_{it} - k_{it}) + \bar{P}_{it}^g k_{it} \right] \end{aligned} \quad (88)$$

$$\text{Constraints (67)–(82)} \quad (89)$$

where (83)–(84) correspond to (1)–(6) and (8), expressions (85) are the linear revenue terms, expressions (86)–(87) respectively include the lower-level primal and dual feasibility constraints, expression (88) is the linear equality associated with the strong duality theorem, and (89) models the linearization of the products of upper-level binary decision variables and lower-level continuous dual variables.

Table I provides the size of the single-level mixed-integer linear equivalent (83)–(89) in terms of the numbers of constraints, binary variables, and real variables. It is worth em-

phasizing that the number of minimum up and down time constraints depends on the initial statuses of generators. Therefore, the number of constraints listed in Table I is an upper bound on the corresponding actual number of constraints.

IV. CONCLUSION

This paper has presented an exact bilevel-programming-based approach to address revenue- and network-constrained market clearing within the context of pool-based day-ahead electricity markets under marginal pricing. For over a decade, this auction model, which constitutes the solution adopted in national electricity markets across Europe to ensure revenue sufficiency, has been routinely albeit inadequately solved by European system operators. The revenue-constrained auction design is a particular instance of price-based market clearing wherein locational marginal prices become decision variables of the optimization. Moreover, revenue terms are formulated as bilinear and hence highly nonconvex products of power outputs and locational marginal prices. Practical modeling aspects allow converting the resulting mixed-integer nonlinear bilevel program with bilinear terms into a single-level mixed-integer linear equivalent with no extra binary variables. This transformation applies results from duality theory of linear programming, Karush-Kuhn-Tucker optimality conditions, and integer algebra.

The proposed model and solution methodology feature relevant aspects for the purposes of practical implementation: 1) the resulting generation and consumption levels are consistent with an economic dispatch problem, 2) the resulting vector of locational market-clearing prices complies with marginal pricing, 3) price uplifts are not required, and 4) both minimum revenue conditions and minimum declared profits can be handled.

The second part of this two-paper series provides a detailed analysis of three case studies, namely 1) an illustrative five-bus example, 2) a case based on the IEEE Reliability Test System, and 3) a case based on the IEEE 118-bus system. Results from these case studies are compared with those achieved by available techniques in order to show the effectiveness of the proposed methodology. In addition, computational issues are discussed on the basis of the computing times required to attain high-quality solutions for a suite of benchmarks of increasing size including a case study based on the IEEE-300 bus system.

APPENDIX

The feasibility set $\mathcal{F}_{it}$ for variables s_{it}^{sd} , s_{it}^{su} , and v_{it} is characterized by the following set of linear constraints:

$$\begin{aligned} \sum_{t=1}^{n_i^{UT}} (1 - v_{it}) &= 0; \forall i \in \mathcal{I} \\ \sum_{h=t}^{t+UT_i-1} v_{ih} &\geq UT_i (v_{it} - v_{it-1}); \\ \forall i \in \mathcal{I}, \forall t &= n_i^{UT} + 1 \dots n_T - UT_i + 1 \end{aligned} \quad (90)$$

$$\begin{aligned} \sum_{h=t}^{n_T} [v_{ih} - (v_{it} - v_{it-1})] &\geq 0; \\ \forall i \in \mathcal{I}, \forall t &= n_T - UT_i + 2 \dots n_T \end{aligned} \quad (92)$$

$$\begin{aligned} \sum_{t=1}^{n_i^{DT}} v_{it} &= 0; \forall i \in \mathcal{I} \\ \sum_{h=t}^{t+DT_i-1} (1 - v_{ih}) &\geq DT_i (v_{it-1} - v_{it}); \\ \forall i \in \mathcal{I}, \forall t &= n_i^{DT} + 1 \dots n_T - DT_i + 1 \end{aligned} \quad (93)$$

$$\begin{aligned} \sum_{h=t}^{n_T} [1 - v_{ih} - (v_{it-1} - v_{it})] &\geq 0; \\ \forall i \in \mathcal{I}, \forall t &= n_T - DT_i + 2 \dots n_T \end{aligned} \quad (94)$$

$$s_{it}^{sd} \geq C_{it}^{sd} (v_{it-1} - v_{it}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (96)$$

$$s_{it}^{sd} \geq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (97)$$

$$s_{it}^{su} \geq C_{it}^{su} (v_{it} - v_{it-1}); \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (98)$$

$$s_{it}^{su} \geq 0; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (99)$$

where $n_i^{DT} = \min \left\{ n_T, \left(DT_i - n_{i0}^{off} \right) (1 - V_{i0}) \right\}$ and $n_i^{UT} = \min \left\{ n_T, (UT_i - n_{i0}^{on}) V_{i0} \right\}$.

Minimum up and down time constraints are respectively modeled in (90)–(92) and (93)–(95), whereas (96)–(97) and (98)–(99) respectively correspond to shut-down and start-up offers. The interested reader is referred to [45] for a detailed description of these linear constraints.

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