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Alberto Pliego Marugán
Ingenium Research Group, Universidad de Castilla-La Mancha
Alberto.Pliego@uclm.es

Fausto Pedro García Márquez
Ingenium Research Group, Universidad de Castilla-La Mancha
FaustoPedro.Garcia@uclm.es

Benjamin Lev
College of Business, Drexel University, Philadelphia, USA

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Optimal Decision Making via Binary Decision Diagrams for Investments under a Risky Environment

Alberto Pliego Marugán¹, Fausto Pedro García Márquez¹, Benjamin Lev²

¹Ingenium Research Group, ETSII Ciudad Real. University of Castilla-La Mancha, Spain

alberto.pliego@uclm.es; faustopedro.garcia@uclm.es;

²Trustee Professor, Drexel University, LeBow College of Business, USA.

b355@drexel.edu

Abstract

This paper presents two methods for supporting investments and resource allocation in a constrained risky environment. These methods are based on the application of logical decision trees and binary decision diagrams as an approach that allows quantitative analysis of a qualitative study. The scenario considered in this paper is a decision making process under risk environment, where stochastic variables are considered. The two novel procedures are introduced to facilitate the resource allocation as the objective of the decision making process. The first procedure uses the analytic expression provided by binary decision diagrams as an objective function of a non-linear programming model. The second procedure introduces an importance measure that takes into account some external constraints, unlike the classical importance measures that only consider the topology of the tree. The first technique will optimize the outcomes and the second will provide a good approximation of the outcomes using simpler calculations.

Keywords: Decision Making, Binary Decision Diagrams, Optimal Investment, Resource Allocation

1. Decision Making Process Background

Decision Making (DM) processes can involve a large number of variables, increasing the complexity and difficulty of qualitative and quantitative analysis. The main issues considered in this paper for the DM processes are the decision maker, the main problem (MP), the scenario, the constraints and the consequences of the decision.

The decision maker is the person, system or organization that makes a decision [61]. ~~The perspective of the decision maker will influence All decisions and assessments will be influenced by the decision makers.~~ Decision maker should possess some personal skills (experience, good judgment, creativity) and other skills supported by existing methods and DM support tools essential skills including: experience, good judgment, creativity and quantitative knowledge. ~~The first three skills are personal, and the final skill is supported by existing methods and support systems for DM [44][44].~~ These DM support systems are used in order aid decision makers in choosing between several alternatives and, consequently, to help the decision maker to decide what alternative is the best [20], [25], [43]. This paper presents and describes two quantitative methods to support the DM process.

The DM process described in this paper is focused on a MP, which represents an undesired event whose occurrence probability needs to be minimized. The logical structure of the MP is approached by a logical decision tree (LDT). Different scenarios can be considered in function of the information available in the DM process: ~~The following main scenarios can be distinguished according to the information available in the DM process:~~

- *DM under certainty:* This scenario implies that the decision maker has a complete information of the problem. The causes, consequences and all the variables of the problem are known. ~~The problem is entirely known and all possible states for all the variables are known and any consequences of each decision can be completely achieved.~~
- *DM under risk:* A risk environment is considered when some of the information available is stochastic. ~~Implies partial information and some information to the problem is stochastic.~~ This will be the scenario considered in this paper.
- *DM under uncertainty:* In this case, the decision maker has not a complete information of the problem, and/or ~~Information about the MP and its causes is not complete and~~ part of the information is missing [12][42].

The scenario developed in this paper corresponds to a DM under risk, where probabilistic values are assigned to those events or causes that can lead to the occurrence of the MP, called basic causes (BC). These probabilistic values are assumed by the following functions:

- *Classic probability:* It can be defined as “a priori” probability, based on a rationalist point of view and calculated deductively without the need to conduct an experiment [1]. For any event A it can be obtained by:

$$P(A) = \frac{m}{n},$$

where A is satisfied by exactly m of n possible outcomes. This approach is not recommended to be applied when outcomes are not equally likely or when all possible outcomes cannot be taken into account.

- *Frequentist probability:* It can be considered as “a posteriori” probability and related to an empiricist perspective. While classic probability focuses on deductive reasoning, frequentist probability focuses on inductive reasoning [1]. It is defined for a generic event A as follows:

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

where m is the number of times that A has been satisfied, and n is the number of times that the process has been performed.

- *Subjective probability:* This probability is also called Bayesian probability (degree of belief mapped onto the unit interval $[0, 1]$)[2], representing a mode of judgment. Therefore, it is closely related to the experience, beliefs, feelings and interests of the person who estimates the probability, i.e. Bayes’ theorem constitutes a learning mechanism to approach unknown quantities of interest. This probability allows for the addition of new information to the ‘a priori’ available probability in order to actualize the probability estimation once new data have been obtained. It is defined, where A and B are two events and $P(B) > 0$, as follows:

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

where $P(A|B)$ is the probability for A to be true if B is already true.

All the decisions are usually constrained by multitude of Every DM process has a number of constraints or requirements to consider, e.g. existing available resources, social or ethics issues, economic factor available budgets, environmental precautions, social issues, legal provisions, etc. [54][54]. In most of the cases, Generally, these constraints are not exogenous and not subject to mathematical or empirical models [3][3][49][49][50][50]. In order to consider them into a global model I, it is possible to carry out an An endogenization of constraints to include them into a global model might be carried out in order to consider them. For this purpose, The endogenization This endogenization can be done by reformulating the constraints into objectives for the DM. This idea, i.e. it transforms the DM process into a multi-objective task. involves

conceptualizing the constraints as goals of the decision maker, making it been possible to reformulate a constraint as an objective.

This paper is focused on expected-utility DM under constraints. The main ~~goals~~ objectives of this process are:

- ~~to that can be considered as a process that provides a solution with two objectives:~~
~~to satisfy the constraints; and~~
- ~~to rule out~~discard unfeasible solutions ~~and~~;
- ~~to~~ optimize objective functions among the surviving options [15][15][25][25].

For a ~~quantitative stochastic DM under risk environment~~-ease, the objective function provides a value that ~~determines~~ the goodness of the solution considered. ~~In this paper, this function will be~~ It is formulated with the help of ~~as an analytical expression gathered from a graphical tool called~~ a binary decision diagram (BDD). The most suitable solutions will be those that satisfy better all the constraints, thresholds and requirements. ~~Some thresholds or constraints might be established in order to determine the solutions that are most suitable.~~ Those alternatives remaining outside of those thresholds can be directly ruled out.

In general, DM processes are not completely reliable due to their inability to take into account the total range of events involved in the solution. The goodness of a decision can only be known in an ‘a posteriori’ evaluation of consequences. ~~The consequences of the decisions must be evaluated because they can affect the very structure of the DM. This evaluation provides an useful~~ Particularly, the evaluation of consequences is essential for improving those DM processes with data from forecasting studies. The results derived from a decision can affect the very structure of the problem or modify some features of the constraints and requirements. ~~Feedback is necessary in order to that allows for determining the quality of the decision. If the system responds as it is expected, then, the decision has been adequate~~adequate, otherwise, the DM process need to be reformulated ~~because the decision maker must know if the system responds as expected.~~ This feedback is the only way for the decision maker to know whether the method brings the problem close to the reality or not. ~~It is also an essential information when periodical decisions are required because a comparison with the preceding decision can be done.~~ Moreover, there are some decisions that need to be made periodically and ~~feedback is useful in order to improve the quality of the new decision, compared to the preceding decision.~~ Figure 1 shows how decision making should be carried out using LDTs and BDDs.

Figure 1. DM process [\[61\]](#).

2. Decision Making Approach

An optimal investment, subject to the limitation of resources, is the main objective of the DM process hereby considered. This paper introduces two novel strategies in order to support the resource allocation when a MP is given. Figure 2 shows the three common steps taken before the use of the novel methods presented in this paper.

Figure 2. Proposed optimization method process

Many studies present different models for decision making, e.g. cost-benefit analysis; elementary methods [45] such as pros and cons analysis, maximin and maximax methods, conjunctive and disjunctive methods, lexicographic method; simple multiattribute rating technique, generalized means, the analytic hierarchy process, Outranking methods; ELECTRE and PROMETHEE [45]; the fuzzy preference relations [46]; cognitive decision-making models [47]; large group decision making methods [48] [53]; etc. Moreover, there are a large amount of studies about decision making processes under risk contexts. Some of the most important models are collected in references [28], [29], [30], [31], [32] and [33].

The methods proposed in this paper attend to a new way of solving decision making problems through the linkage of graphical and mathematical tools. The first method (mathematical optimization approach) uses the expression provided by a BDD as an object function for a programming problem. This is a novel use of a Boolean function. Regarding the second method (Birnbbaum-Cost Measure Method), the novelty lies in the adaptation of the Birnbbaum measure in order to consider possible decision variables such as costs. Both methods are limited by the possibility of building a LDT that represents the problem with a correct accuracy.

It is important to remark that these methods can be applied to other general decision problems [40], e.g. at the design stage of products [37][42][51][52], to elaborate preventive or predictive maintenance plans [36][38][39].

The methods can aid to solve more specific problems, e.g. to maximize the total profit of end-of-line computer manufacturing [55], to carry out maintenance in a deteriorating manufacturing system [56], costs allocation in inventory pool of spare parts [57], planning optimization model with demand and yield uncertainty [58]. The methods also allow for making pricing decisions under different constraints [59].

There are two main gaps between the literature and the methods proposed in this paper. In first place, the flexibility of the first method proposed allows for considering any number of constraints finding more accurate outcomes. For example, reference [60] establishes a quantitative measure for prioritisation of items based on penalty incurred to their non-availability. Secondly, the optimization approach presented in this research work leads to perform a prioritisation based on different constraints. The simplicity of this method can improve the computational cost. In general, the new methods proposed are adaptable to any problem that can be logically defined, and several alternatives can be considered.

2.1 Obtaining Logical Decision Tree

The LDT is employed in this paper in order to establish a logical structure of the MP. These structures allow for the establishment of a logical interrelation among several single events [6]. These events, alone or by combination of them, are the responsibility of the MP, called basic causes (BC). The interrelation of BCs has been implemented in this paper using logical gates “AND $\square$ ” and “OR $\triangle$ ” (see Figure 3); therefore, a qualitative analysis is necessary to establish the aforementioned interrelation between the BCs.

Figure 3. Structure of a LDT.

The main elements for a LDT are:

- *BCs*. They are events that are part of the MP that cannot be broken down into other causes.
- *Non-Basic Causes*. The only difference between these causes and BCs is that they can be broken down into other causes.
- *Logical gates*. They are in charge of defining the interrelations between causes. In this paper, only AND and OR gates are used.
- *Top event*. It is the main event and it is defined as the MP in the DM process.

Figure 4 presents a simple example of the structure of a LDT. This LDT can correspond to the analysis of a delivery business that desires to improve the timeliness of the service. The business is seeking to reduce the “delay in the orders”. The LDT is composed by 32 basic causes and 26 non-basic causes. One of the branches of this LDT has been zoomed in order to show some of the causes in detail.

The following procedure is suggested when a certain problem has to be faced by a business and a LDT has to be built:

- Define properly the MP and its scope.
- Detect which are the BCs related with the MP.
- Study precisely the interrelation between mentioned BCs and the MP.
- Obtain the probability data for each BC.

The generation of a LDT is not a very hard task due to it is a graphical tool. However, an adequate knowledge of the problem is required.

The generation of basic causes or non-basic ones depends on the degree of detail in the information. Those cause that cannot be broken down into other causes will be considered as basic causes.

Figure 4. Example of a LDT.

2.2 Conversion from LDT into BDD

The conversion from LDT to BDD provides more -some advantages in terms of efficiency and accuracy for to the quantitative analysis. When the LDT has a large amount of basic causes, the direct analysis of the decision tree is often impossible. In these cases, it is necessary to use some truncation techniques and consequently, a loss of accuracy is produced [10]. BDDs provide an exact analytical expression of the occurrence probability of the MP. Therefore, the main reason that leads to hereby study to convert LDT to BDD and its subsequent BDD data handling is due to the ability to deal a large number of BCs using both computational time and resource management in a reasonably computational cost.

BDD is graphical tool that represents a Boolean function. The main advantage is that the a directed graph representation of a Boolean function, where equivalent Boolean subexpressions are uniquely represented [8][8][11][44]. It is a directed graph (with no cycles) where different events are and is composed of some interconnected by nodes that collect all the possible states. Each node or vertex is followed by two branches that determine the occurrence or non-occurrence of the corresponding event. There are two types of vertex: in a way that each node has two vertices. Each vertex can either be a terminal or and non-terminal vertex. BDD is a graph-based data structure whereby the occurrence probability of a certain problem in a DM can be achieved. Terminal vertex are followed by two terminal states Each single variable has two branches: (0-value that corresponds to the non-occurrence of the event, and 1-value branches that is associated corresponds to the cases where the variable is 0 event occurs; 1-branch cases are those where the event occurs and corresponds when the variable is 1. The transformation from LDT to BDD is achieved by applying some mathematical algorithms such as the Rauzy method (used in this paper) [7] or the simple component-connection method [41]. In this study, the conditional control sentence, the ITE (If-Then-Else) conditional expression are is considered for building the one of the BDD's cornerstones [7][9] (see Figure 5):

Figure 5. ITE applied to BDD [62].

Figure 5 could be described as: “If BC_i variable occurs, then f_i , else f_2 ” [7]. The solid line always belongs to the 1-branches and the dashed lines to the 0-branches. Taking into account Shannon’s theorem [35], it can be obtained by the following expression:

$$f = BC_i \cdot f_1 + \overline{BC_i} \cdot f_2 = ITE(BC_i, f_1, f_2)$$

It is necessary to establish a correct sequence of BCs in the transformation from LDT to BDD (morefurther detailed information about of the conversion and variable ordering methods can be found in [4][4], [5][5] and [7][7]). Figure 6 shows the conversion from LDT to the corresponding BDD. BCs are in the following sequence: BC_1, BC_2, BC_3, BC_4

Figure 6. Conversion from LDT to BDD [62].

Once the conversion from LDT to BDD is done, ~~it is possible to obtain an accurate expression of the~~ exact occurrence probability of the MP (Q_{MP}) ~~can be easily obtained.~~ For this purpose, a probability value has to be assigned ~~to by assigning a probability value to~~ each BC.

2.3 Obtaining Cut-Sets and Analytical Expression from BDD

~~The occurrence probability of the MP can be obtained from Cut-Sets. turn into an important concept when referring to BDDs.~~ They are the paths “from the top to the ones” and each one of them provides a different scenario ~~in which where~~ the MP would occur [62]. The following Cut-Sets have been obtained from the BDD in Figure 3.

$$\begin{aligned} CS_1 &= \{BC_1, BC_3\} \\ CS_2 &= \{BC_1, \overline{BC_3}, BC_4\} \\ CS_3 &= \{\overline{BC_1}, BC_2, BC_3\} \\ CS_4 &= \{\overline{BC_1}, BC_2, \overline{BC_3}, BC_4\} \end{aligned}$$

The MP probability can be achieved because the different paths (Cut-Sets) are independent (mutually exclusive). ~~Therefore, all the Cut-Sets together will represent all the possible scenarios, and the sum of their probabilities correspond to the global occurrence probability of the MP. and may be expressed as the sum of probabilities of all the BDD paths, i.e. an analytic expression consisting of the sum of each analytic expression that forms the Cut Sets [34][34].~~ This expression will represent the utility function in the DM process. Therefore, the analytic expression that provides Q_{MP} is:

$$Q_{MP} = \sum_{i=1}^N P(CS_i),$$

Where:

- N : is the total number of Cut-Sets
- $P(CS_i)$: is the probability of occurrence of the i^{th} Cut-Set

Therefore, the occurrence probability of the MP in Figure 6 is provided by the following expression:

$$Q_{MP} = P(BC_1) \cdot P(BC_3) + P(BC_1) \cdot (1 - P(BC_3)) \cdot P(BC_4) + (1 - P(BC_1)) \cdot P(BC_2) \cdot P(BC_3) + (1 - P(BC_1)) \cdot P(BC_2) \cdot (1 - P(BC_3)) \cdot P(BC_4)$$

2.4 Mathematical Optimization Approach

Once the function Q_{MP} is achieved [7], the goal is to minimize the occurrence probability of the MP. It will be assumed that LDT will be stable and therefore the reduction of the Q_{MP} will be performed by taking corrective actions (using additional resources) on the different BCs.

The nature of problems that can be examined via DM processes can be very different, but the presented method requires a probability assignment in all cases. This probability assignment, from a frequentist point of view, only depends on the frequency of occurrence of different BCs. The more frequent a BC is, the higher the occurrence probability it has. The position of a BC within the LDT and its own frequency of occurrence are two relevant factors in an optimized DM process.

The MP probability can be minimized by minimizing the probabilities of its causes. Therefore, Given a BC, the goal objective is to determine the investment on it each basic cause in order to reduce its the occurrence probability [61]. of occurrence, considering all the probabilities of BCs and the total investment. The objective function seeks to minimize the probability of occurrence of the top event Q_{MP} . A new vector **Imp (BC)** that consider will consider the reduction of probability for each BCs this reduction is defined, therefore as:

$$\mathbf{Imp}(\mathbf{BC}) = [Imp(BC_1), Imp(BC_2), \dots Imp(BC_i) \dots Imp(BC_n)]$$

The i^{th} component of **Imp(BC)** provides the reduction of the probability of the BC_i of occurrence when some resources are allocated on the i^{th} -BC. Each component of **Imp(BC)** corresponds to an optimization variable whose value is proportional to the resources allocated on it. In addition, the occurrence probabilities are collected by a probability a probability vector **P(BC)** is defined as:

$$\mathbf{P}(\mathbf{BC}) = [P(BC_1), P(BC_2), \dots P(BC_i) \dots P(BC_n)]$$

The i^{th} component of **P(BC)** provides the probability of occurrence of the i^{th} -BC. Once the resources are allocated BCs have been improved, the new probability assignment P^* will be the difference between its the original probability of occurrence P and the Imp :

$$\begin{aligned} \mathbf{P}^*(\mathbf{BC}) &= [P^*(BC_1), P^*(BC_2), \dots P^*(BC_i) \dots P^*(BC_n)] \\ &= [P(BC_1) - Imp(BC_1), P(BC_2) - Imp(BC_2), \dots, P(BC_i) - Imp(BC_i), \dots, P(BC_n) - Imp(BC_n)] \end{aligned}$$

The new vector ~~The BDD evaluated using $\mathbf{P}(\mathbf{BC})$ provides the value of Q_{MP} . If it is being evaluated using $\mathbf{P}^*(\mathbf{BC})$, it will be used to evaluate the MP. These new probabilities will generate a new MP probability that will be e data obtained will be defined as Q_{MP}^* . The goal objective of this process is to obtain It would be desirable to have the following inequality $-Q_{MP} \geq Q_{MP}^*$, otherwise, the optimization procedure generates incorrect results is not correct.~~

The analytic expression provided by BDD becomes an optimization function when it is evaluated employing $\mathbf{P}^*(\mathbf{BC})$. The optimization function will be defined as $Q_{MP}(Imp)$.

This paper assumes that all the BCs can be improved but not necessarily corrigible, i.e. there can be some BCs whose occurrence probabilities cannot be reduced to 0. Therefore, each **Imp** will be between 0 and a certain threshold, a . This constitutes the first constraint, which is defined as:

$$0 \leq Imp(BC_i) \leq a_i,$$

where a_i determines the maximum improvement that can be implemented in the i^{th} BC. This parameter is subject to:

$$0 \leq a_i \leq P(BC_i)$$

Therefore, $a_i = 0$ indicates that the i^{th} BC is capable of being totally corrected because the BC allows its own probability of occurrence to be 0 (in this case BC_i will not continue contributing to the MP occurrence). If $a_i = P(BC_i)$, then the improvements in the i^{th} BC are not possible.

An improvement cost vector, **IC (BC)**, is defined for each BC, where a high IC for a BC means a large amount of resources must be invested to reduce the probability of occurrence of such BC. IC refers to marginal improvement costs, given by:

$$\mathbf{IC}(\mathbf{BC}) = [IC(BC_1), IC(BC_2) \dots IC(BC_i) \dots IC(BC_n)],$$

where $IC(BC_i)$ indicates the ~~amount of resources invested cost in BC_i offer~~ reducing the probability of occurrence of BC_i from 1 to 0.

The total amount of resources at the time of the investment operation is given by the Budget (Bg), obtaining the following constraint:

$$\sum_{i=1}^N IC(BC_i) \cdot Imp(BC_i) \leq Bg$$

Considering the abovementioned constraints, the optimization problem is defined in its standard form as:

$$\text{minimize } Q_{MP}(Imp)$$

$$\text{subject to } \sum_{i=1}^N IC(BC_i) \cdot Imp(BC_i) \leq Bg$$

$$Imp(BC_i) - a_i \leq 0$$

$$-Imp(BC_i) \leq 0$$

This method allows establishing preferences among possible solutions. This could be done by defining a ponderation between the different constraints. The constraints can be associated with a certain value that can give a numerical importance to them. The solution that best meets the more important constraints will be the chosen one.

The resulting non-linear programming problem is an NP-hard problem, where the optimization function is non-linear. The complexity of the problem depends on the number of variables and the structure of the programming problem (objective function and constraints). Therefore, the necessary conditions of optimality are defined by the Karush-Khun-Tucker (KKT) conditions [23].

The procedure employed in this paper allows connecting the LDT analysis to any traditional optimization approach including mathematical optimization algorithms (e.g. Newton's method and Gradient Descent) and direct search methods (e.g. Simplex method and the Nelder-Mead method), but the complexity of the problem could require the use of unconventional optimization algorithms such as heuristics (e.g. Simulated Annealing, Deterministic Annealing, Tabu Search, Genetic Algorithms, Ant Systems or Neural Networks, etc.) [14], [16], [17], [19], [21], [22] .

2.5 Birnbaum-Cost Measure Method

The importance measures are employed in order to analyze the influence of the BCs in the LDT. Employing importance measures makes it possible to show the events that have more effect on the probability of the top event. There are several methods for computing the IMs, the most important being the Fussel-Vesely, Birnbaum and Criticality [27], [24], [26]. These methods are based on the determination of the structural importance of each BC, and quantitatively analyzing the weight of each BC over the MP, i.e. it provides an index of the contribution of each BC with regard to the whole MP. The main limitation of the importance measures is that they only consider the occurrence probability and the location within the tree of the BCs. None of the methods are able to consider some constraints, such as the improvement cost considered in this paper.

The Birnbaum measure has been chosen to calculate the importance of the BCs due to advantages over the rest of the methods [13]. This measurement provides the value associated with the direct relation between Q_{MP} and the corresponding BC, but it does

not consider the probability of the BC. This can lead to major importance being given to more rare BCs than they should have. The Criticality method takes into account the probability of occurrence of the BC itself [24]; however, for the objective treated in this paper the Birnbaum has been used instead of the Criticality method because it allows for the quantification of the reduction of Q_{MP} depending of the BCs improved.

$$I_i^{Birm} = \frac{\partial Q_{MP}}{\partial P(BC_i)},$$

where:

I_i^{Birm} is the importance measure given by Birnbaum method of the i^{th} BC

$P(BC_i)$ is the probability assigned to the i^{th} BC

Q_{MP} is the MP probability

A new parameter is introduced to obtain the marginal cost by reducing the probability defined as Birnbaum-Cost Measure importance (BCM), given by the following expression:

$$BCM_i = \frac{I_i^{Birm}}{IC} = \frac{1}{IC} \cdot \frac{\partial Q_{MP}}{\partial P(BC_i)}$$

The DM based on Birnbaum-Cost Measure would allocate resources on the BC with the highest Birnbaum-Cost Measure value until it reaches its improvement threshold, then on the second one and so on, until all resources have been used.

3. Case Study

This section shows a case study in order to clarify the proposed procedure and some results are presented. Figure 7 shows a LDT that corresponds to a simplified version of the branch zoomed in Figure 4. Therefore, the MP considered in this case study is “Lack of notification”.

Figure 7. LDT optimization example.

The following Q_{MP} has been obtained by converting the LDT to BDD [7], [7].

$$Q_{MP} = P(BC_6) + (1 - P(BC_6)) \cdot P(BC_3) \cdot P(BC_1) + (1 - P(BC_6)) \cdot P(BC_3) \cdot (1 - P(BC_1)) \cdot P(BC_2) + (1 - P(BC_6)) \cdot P(BC_3) \cdot (1 - P(BC_1)) \cdot (1 - P(BC_2)) \cdot P(BC_4) \cdot P(BC_5) + (1 - P(BC_6)) \cdot (1 - P(BC_3)) \cdot P(BC_4) \cdot P(BC_5)$$

Considering the following probabilities $P(BC) = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6]$, the Q_{MP} will be:

$$Q_{MP}(P(BC)) = 0.890$$

The initial parameters are given in Table I.

TABLE I. Optimization Example Parameters

Basic Cause	P	a	IC (€)	Bg (€)
BC_1	0.1	0.05	1000	1800
BC_2	0.2	0.05	2000	
BC_3	0.3	0.10	3000	
BC_4	0.4	0.15	4000	
BC_5	0.5	0.05	5000	
BC_6	0.6	0.20	6000	

Once all the parameters have been defined and the analytical expression of Q_{MP} has been provided, it is possible to use the introduced methods.

3.1 Mathematical Optimization Approach

The objective function would be:

$$Q_{MP}(Imp(BC)) = (0.6 - Imp(BC_6)) + (1 - (0.6 - Imp(BC_6))) \cdot (0.3 - Imp(BC_3)) \cdot (0.1 - Imp(BC_1)) + (1 - (0.6 - Imp(BC_6))) \cdot (0.3 - Imp(BC_3)) \cdot (1 - (0.1 - Imp(BC_1))) \cdot (0.2 - Imp(BC_2)) + (1 - (0.6 - Imp(BC_6))) \cdot (0.3 - Imp(BC_3)) \cdot (1 - (0.1 - Imp(BC_1))) \cdot (1 - (0.2 - Imp(BC_2))) \cdot (0.4 - Imp(BC_4)) \cdot (0.5 - Imp(BC_5)) + (1 - (0.6 - Imp(BC_6))) \cdot (1 - (0.3 - Imp(BC_3))) \cdot (0.4 - Imp(BC_4)) \cdot (0.5 - Imp(BC_5))$$

The approach to the optimization problem will be:

$$\text{minimize } QMP(Imp(BC))$$

$$\begin{aligned}
& \text{subject to } (100 \cdot \text{Imp}(BC_1) + 200 \cdot \text{Imp}(BC_2) + 300 \cdot \text{Imp}(BC_3) + 400 \cdot \text{Imp}(BC_4) \\
& \quad + 500 \cdot \text{Imp}(BC_5) + 600 \cdot \text{Imp}(BC_6)) \cdot 10 \leq 1800 \\
& \text{Imp}(BC_1) - 0.05 \leq 0; \quad -\text{Imp}(BC_1) \leq 0 \\
& \text{Imp}(BC_2) - 0.05 \leq 0; \quad -\text{Imp}(BC_2) \leq 0 \\
& \text{Imp}(BC_3) - 0.1 \leq 0; \quad -\text{Imp}(BC_3) \leq 0 \\
& \text{Imp}(BC_4) - 0.15 \leq 0; \quad -\text{Imp}(BC_4) \leq 0 \\
& \text{Imp}(BC_5) - 0.05 \leq 0; \quad -\text{Imp}(BC_5) \leq 0 \\
& \text{Imp}(BC_6) - 0.2 \leq 0; \quad -\text{Imp}(BC_6) \leq 0
\end{aligned}$$

Table II shows all the results for all the optimization parameters once the neural network is applied.

Table II. Optimization Example Results

Basic Cause	P	Imp	P^*	Investment (€)	Q_{MP}	Q_{MP}^*	% Q_{MP} Reduction
BC1	0.10	0	0.10	0	0.89	0.79	12%
BC2	0.20	0	0.20	0			
BC3	0.30	0	0.30	0			
BC4	0.40	0.16	0.24	634			
BC5	0.50	0.11	0.39	534			
BC6	0.60	0.11	0.50	632			

3.2 Birnbaum-Cost Measure Method

Figure 8 shows a comparison between Birnbaum and Birnbaum-Cost measures based on the case study presented in Figure 7. The Birnbaum Measure and the Birnbaum-Cost Measure have been calculated according to the expressions in section 2.5.

Figure 8. Birnbaum Measure vs. Birnbaum-Cost Measure

The results provided by the Birnbaum Measure method show that the investment order should be $BC_6, BC_5, BC_4, BC_3, BC_2, BC_1$, but using the Birnbaum-Cost Measure method the order obtained is $BC_5, BC_4, BC_6, BC_1, BC_2, BC_3$.

Table III shows where some monetary resources must be allocated according to the proposed analysis. It shows the resource allocation, given the same conditions as the previous section example, i.e. the budget is 1800.

Table III. Birnbaum-Cost Measure Method. Case Study Results

Basic Cause	P	Imp	P^*	Investment (€)	Q_{MP}	Q_{MP}^*	% Q_{MP} Reduction
BC1	0.10	0	0.10	0	0.90	0.81	9%
BC2	0.20	0	0.20	0			
BC3	0.30	0	0.30	0			
BC4	0.40	0	0.40	0			
BC5	0.50	0.36	0.14	1800			
BC6	0.60	0	0.50	0			

4. Comparison between the methods, considering different availabilities of resources

A comparison between the aforementioned methods is carried out in this section. The aim of the following analysis is to show the results of each method when different budgets are considered for the investments. The first step is to calculate the maximum investment according to the constraints established by the parameter ‘ a ’. The maximum investment corresponds to:

$$Invest_{max} = \sum_{i=1}^N (P(BC_i) - a_i) * IC(BC_i)$$

In this case, taking into account the data in Table I, the maximum investment is:

$$Invest_{max} = (0.1 - 0.05) * 1000 + (0.2 - 0.05) * 2000 + (0.3 - 0.1) * 3000 + (0.4 - 0.15) * 4000 + (0.5 - 0.05) * 5000 + (0.6 - 0.2) * 6000 = 6600$$

Table IV shows the investments suggested by the optimization method for different budgets from 600 € to the maximum investment of 6600 €, with an increment of 600 € for each step.

Table IV. Optimization Approach. Results with Different Budgets

	BC 1	BC 2	BC 3	BC 4	BC 5	BC 6
Budget 600 €	0	0	0	232	137	231
Budget 1200€	0	0	0	434	333	433
Budget 1800 €	0	0	0	634	534	632
Budget 2400 €	0	0	0	834	733	833
Budget 3000 €	0	0	0	1000	950	1050
Budget 3600 €	0	0	0	1000	1250	1350
Budget 4200 €	50	0	0	1000	1525	1625
Budget 4800 €	50	0	0	1000	1825	1925
Budget 5400 €	50	61	0	1000	1889	2400
Budget 6000 €	50	300	0	1000	2250	2400
Budget 6600 €	50	300	600	1000	2250	2400

Table V shows the outcomes of the method based on Birnbaum-Cost Measure considering the same budgets as in Table IV.

Table V. Birnbaum-Cost Measure Method. Results with Different Budgets

	BC 1	BC 2	BC 3	BC 4	BC 5	BC 6
Budget 600 €	0	0	0	0	600	0
Budget 1200€	0	0	0	0	1200	0
Budget 1800 €	0	0	0	0	1800	0
Budget 2400 €	0	0	0	150	2250	0
Budget 3000 €	0	0	0	750	2250	0
Budget 3600 €	0	0	0	1000	2250	350
Budget 4200 €	0	0	0	1000	2250	950
Budget 4800 €	0	0	0	1000	2250	1550
Budget 5400 €	0	0	0	1000	2250	2150
Budget 6000 €	50	300	0	1000	2250	2400
Budget 6600 €	50	300	600	1000	2250	2400

Finally, the probability of occurrence of the MP (Q_{MP}) is evaluated for each considered budget. Figure 9 shows the comparison between the Q_{MP} using each of the methods.

Figure 9. Optimization vs. Birnbaum-Cost Measure Method. Results with Different Budgets

As can be observed in Figure 9, the optimization approach provides better results than the Birnbaum-Cost one; however, the optimization approach could require a higher computational cost. Therefore, both methods could be useful according to the desired accuracy.

5. Conclusions

This paper has introduced two different methods (optimization approach and Birnbaum-Cost Measure) for decision making regarding resource allocation under a risk environment. There must be a main problem that is generated by the occurrence of multiple causes which, in turn, can be broken down into basic causes and whose interrelationships can be depicted by a graphical tool named logical decision trees. The first step is to apply the presented methods to obtain an analytical expression that will constitute, in probabilistic terms, the occurrence probability of the main problem according to the occurrence probabilities of the BCs. This analytical expression has been used in the first method as an objective function that will have to be optimized under certain constraints to minimize the probability of occurrence of the undesired main problem. Moreover, this analytical expression has also been used in the Birnbaum-Cost Measure method to assess the participation of each BC in the main problem.

The case study analysed in this paper corresponds to an ideal case of investments in which all BCs are known. This is not typical in a real decision making process, because most problems are not completely known or probabilities are not entirely reliable. In that case, an analysis under uncertainty should be done. The expression obtained from the BDD is able to provide a threshold value of error that could be useful to rule out those causes that are not important enough. Therefore, it is possible to provide a good solution

employing this method when there are missing data with little contribution to the main problem.

The Birnbaum-Cost Measure method based on a modified Birnbaum importance measurement is proposed in order to facilitate the choice of which resources should be invested for those cases in that optimization process is very complex.

A comparison between the two methods has been carried out, taking into account different availabilities of resources. The optimization approach has provided better results than the Birnbaum-Cost Measure. However, the method based on the Birnbaum-Cost Measure requires less computational cost and is a viable option when the optimization approach is too complex.

The optimisation approach presented in this paper can be too complex to be solved through conventional optimization approaches. In these cases, can be necessary to employ heuristics methods. It should be taken into account for further research the relation between the degree of detail required (number of BCs considered, number of constraints, non-linearity of functions, etc.) and the accuracy of the solution.

The Birnbaum-Cost measure proposed in this paper is only an example of the way in which an exogenous variable can be introduced in an endogenous measure. The main limitation of this method is that only one constraint is taken into account (improvement cost). It is suggested to consider more constraints in future works.

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