A Bi-level Approach to Distribution Network and Renewable Energy Expansion Planning considering Demand Response

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Abstract—This paper presents a bi-level model for distribution network and renewable energy expansion planning under a Demand Response (DR) framework. The role of DR has recently attracted an increasing interest in power systems. However, previous models have not been completely adapted in order to treat DR on an equal footing. The target of the distribution network and generation planner, modeled through the upper-level problem, is to minimize generation and network investment cost while meeting the demand. This upper-level problem is constrained by the lower-level problem, stressing the importance of integrating DR to time-varying prices into those investment models. The objective function considered for the lower level is the minimization of overall payment faced by the consumers. Using the Karush Kuhn Tucker (KKT) complementarity constraints, the proposed bi-level model is recast as a mixed-integer linear programming problem, which is solvable using efficient off-the-shelf branch-and-cut solvers. Detailed results from an insular case study (La Graciosa, Canary Islands, Spain) are presented.

Index Terms—DR, Renewable Energy Sources (RES) expansion planning, distribution network expansion planning, bi-level programming, KKT complementarity constraints.

NOMENCLATURE

Indices and Sets

b	Index for substation load levels
B	Set of substation load levels for the calculation of the substation price
i, j	Indices for nodes
l	Index for feeder types
L	Set of feeder types, $L = \{EFF, ERF, NRF, NAF\}$ where EFF, ERF, NRF and NAF denote existing fixed feeders, existing replaceable feeders, new replacement feeders, and newly added feeders, respectively
ll, lb	Indices for load levels
LL, LB	Sets of load levels
p	Index for generator types
P	Set of generator types, $P = \{W, \Theta\}$ where W and Θ denote wind and photovoltaic generators, respectively
t	Index for time stages

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T	Set of time stages
tr	Index for transformer types
TR	Set of transformer types, $TR = \{ET, NT\}$ where ET and NT denote existing transformers and newly added transformers, respectively
w	Index for scenarios
γ^l	Set of branches with feeders of type l
Ω_i^l, Ω^{LN}	Sets of nodes connected to node i by a feeder l ,
Ω^N, Ω^p	load nodes, system nodes, candidate nodes for
Ω^{SS}	DG, and substation nodes
Parameters	
C_b	Costs for substation load level b [\$/MWh]
$C_{i,j}^{l,l}$	Investment cost of feeder l in branch $i-j$ [\$/km]
$C_{i,j}^{M,l}$	Maintenance cost of feeder l in branch $i-j$ [\$/]
$C_i^{l,NT}$	Investment cost of adding a new transformer at substation node i [\$/]
$C_i^{l,p}$	Investment cost of generator p at node i [\$/MW]
$C_i^{M,p}$	Maintenance cost of generator p at node i [\$/]
$C_i^{M,tr}$	Maintenance cost of transformer tr at substation node i [\$/]
$C^{E,p}$	Generation costs of generator type p [\$/MWh]
$\bar{C}_{t,ll,w}^{SS}$	Average substation cost at stage t for load level ll in scenario w considering adjacent blocks [\$/MWh]
$\bar{D}_{i,t}$	Fictitious nodal demand at substation node i at stage t
$D_{i,t,ll,w}$	Expected demand at node i at stage t for load level ll in scenario w [MVA]
$\bar{F}_{i,j}^l$	Maximum capacity of feeder l in branch $i-j$ [MVA]
$\bar{G}_i^{tr}$	Upper limit for energy supplied by transformer tr at substation node i [MVA]
$\bar{G}^p$	Maximum capacity of generator type p [MVA]
I	Annual investment rate
IB_t	Investment budget for stage t [\$/]
$\ell_{i,j}$	Feeder length [km]
L_b^{min}	Lower limit of substation load level b [MVA]
L_b^{max}	Upper limit of substation load level b [MVA]
n_{DG}, n_T	Number of candidate nodes for distributed generation, number of time stages
pf	System power factor
RR^l, RR^{NT}	Capital recovery rates for investment in feeders, new transformers, generators and substations
RR^p, RR^{SS}	
$Z_{i,j}^l$	Unitary impedance magnitude of feeders [Ω]

Δ_{ll}, Δ_{lb}	Duration of each load level ll [h]
ε	Penetration limit for distributed generation [%]
$\xi_{ll, ll}, \xi_{lb, ll}$	Own and cross price elasticities
$\pi_{ll, w}$	Probability of scenario w of load level ll
v^l, v^{NT}, v^p, v^{ss}	Lifetimes of feeders, new transformers, generators and substation assets other than transformers [yr]
<i>Variables</i>	
$d_{i,t, ll, w}$	Demand at node i at stage t for load level ll in scenario w when considering DR [MVA]
$d_{i,t, ll, w}^U$	Unserved power at node i at stage t for load level ll at scenario w [MVA]
$dr_{i,t, ll, w}$	DR at node i stage t for load level ll in scenario w [MVA]
$dr_{i,t, ll, w}^+, dr_{i,t, ll, w}^-$	Upper and lower DR limits at node i at stage t for load level ll in scenario w [MVA]
$f_{i,j,t, ll, w}^l$	Current flow through feeder l installed in branch $i-j$ at stage t , load level ll in scenario w , measured at node i , which is greater than 0 if node i is the supplier and 0, otherwise [MVA]
$\tilde{f}_{i,j,t}^l$	Fictitious current flow through feeder l in branch $i-j$ at stage t , measured at node i , which is greater than 0 if node i is the supplier and 0, otherwise
$g_{i,t, ll, w}^p$	Energy supplied by generator p at node i at stage t for load level ll in scenario w [MVA]
$g_{i,t, ll, w}^{tr}$	Energy supplied by transformer tr at substation node i at stage t for load level ll in scenario w [MVA]
$p_{t, ll, w}$	Demand payment at stage t for load level ll in scenario w [MVA]
$v_{i,t, ll, w}$	Nodal voltage at node i at stage t , load level ll in scenario w
$x_{i,j,t}^l$	Binary variable for the installation of feeder l installed in branch $i-j$ at stage t
$x_{i,t}^{NT}$	Binary variable for the installation of new transformers at substation node i at stage t
$x_{i,t}^p$	Binary variable for the installation of generator type p at nodes i at stage t
$x_{i,t}^{ss}$	Binary variable for the reinforcement of existing substations or construction of new ones at substation node i at stage t
$y_{i,j,t}^l, y_{i,t}^p, y_{i,t}^{tr}$	Binary utilization variables for feeders, generators and transformers
$z_{t, ll, w}^{dr}$	Binary variable associated with DR at stage t for load level ll in scenario w
$z_{t,b, ll, w}$	Binary variable associated with substation prices at substation load level b stage t for load level ll in scenario w
$\lambda_{i,t, ll, w}^1, \lambda_{i,t, w}^2$	Lagrange multipliers
$\mu_{i,t, ll, w}^1, \mu_{i,t, ll, w}^2$	
$\theta_{t,b, ll, w}$	Variable associated with substation demand at stage t for substation load level b and load level ll in scenario w
$\rho_{t,b, ll, w}$	Variable associated with substation demand at stage t for substation load level b and load level

 ll in scenario w

I. INTRODUCTION AND CONTRIBUTIONS

DEMAND response and dynamic pricing are often considered as means of mitigating the uncertainties and intermittencies of renewable generation and improving the system's efficiency with respect to economic and environmental metrics. The idea is to allow the consumers to adjust their consumption in response to a signal that reflects the wholesale market conditions, possibly the real-time prices. Characterized by passing on the real-time wholesale electricity prices to the end consumers, real-time pricing creates a closed-loop feedback system between the physical layer and the market layer of the system. In recent years, a large number of research works have been carried out on generation and distribution expansion planning. These problems have been investigated with different views, methods, constraints and objectives. However, these models have not been completely adapted in order to treat price-dependent resources, such as DR, on an equal footing. We present a long-term distribution network and renewable energy expansion planning model, that considers short-term real time pricing demand response, i.e., as the electricity price increases, consumers reduce their demand and vice versa. Electricity demand can often no longer be assumed to be a fixed quantity, because demand response programs allow consumers to respond to electricity prices and adapt their electricity consumption accordingly. In economic theory, this is equivalent to a downward sloping demand function to represent the bidding behavior of electricity consumers. Several research works have been devoted to outline the advantages of integrating renewable technologies in the distribution network. The proposed approach bases on the findings of [1], where a model to obtain the optimal allocation and sizing of distributed generation units and distribution network assets while minimizing network investments is presented. The model described in [2] proposes an integrated methodology for distribution network expansion planning incorporating distributed generation considering reliability. The methods in [1] and [2] present a comprehensive literature review on previous works performed, including [3]–[7]. The distribution network expansion problem is analyzed through a heuristic algorithm in [3]. Authors in [4] present a probabilistic approach to distribution system planning. Authors in [5] present a mixed-integer quadratically-constrained model to solve the distribution system expansion planning problem. The formulation presented in [6] presents the mathematical model of considering DG on the expansion planning of a sub-transmission system. A practical method is proposed in [7], combining Successive Linear Programming with Gauss-Seidel iteration to co-optimize AC and DC transmission and generation capacities in a linearized DC network. The effects of integrating short-term DR into long-term generation investment planning are investigated in [8]. Authors in [9] present an integrated methodology for planning distribution networks in which the operation of distributed generators (DGs) and cross-connections (CCs) is optimally planned. The effects of integrating DR into long-term generation and transmission network investment planning are investigated in [10] through a probabilistic multi-objective function. Authors in [11] present an analytical framework to incorporate DR in

long-term resource planning. A symmetric treatment of load and generation creates the strongest possible incentive for final consumers to actively participate in the wholesale electricity market. The proposed approach translates this argument to medium- and long-term distribution and generation expansion planning.

Bi-level programming is considered an appropriate framework to model optimization problems in which one or several constraints are optimization problems themselves [12], [13]. Therefore the objective function of the upper-level problem is optimized considering that the lower-level problem optimizes its own objective function. Bi-level programming is proposed as a suitable solution methodology for the generation and distribution expansion planning problem, considering the expansion planning in the upper level and decisions associated to the flexibility of the demand in the lower level. Unlike the formulation proposed in [14], in which the operation of electric power systems where the energy purchase from DG and substations is minimized, the proposed approach minimizes the payment of the consumers in a long-term planning problem. Unlike previous works [1]–[10], the proposed programming formulation explicitly takes into consideration the minimization of consumers' payment (lower-level problem), incorporating DR into the long-term expansion planning. An extensive study about elasticities is performed in [15]. Based on [15] and [16], sufficiently accurate estimations for consumers' behavior have been included, quantifying the relationship between demand and market prices. Several methods have been proposed in the literature targeting the aforementioned problems. However, in our opinion, none of the existing works sufficiently stresses the relevance of DR in the expansion planning modeling. The proposed approach is based on the work performed in [17], enhancing the generation and distribution network expansion planning incorporating the bi-level formulation in order to adequately account for consumer's flexibility. Table I summarizes the main differences between this paper and the state of the art on multi-stage co-optimized expansion planning [1]–[10]. According to the presented state of the art, the following contributions can be listed:

- A novel multi-stage model is proposed to accurately include DR in the joint distribution and generation expansion planning problem through a bi-level formulation. Timing, location, and sizing of DG units and distribution network assets are modeled. The model proposes a bi-level formulation to incorporate price-dependent resources.
- The present model stresses the importance of integrating DR to time-varying prices (real-time prices) into those investment models. Own-price and cross-price elasticities are included in order to incorporate consumers' willingness to adjust the demand profile in response to price changes. Own price elasticity is defined as the change in the load of a certain level with respect to a change in price in the same load level ($\xi_{ll,u}$). Analogously, cross-price elasticity represents the change in the load of a certain level with respect to a change in price in other load levels ($\xi_{lb,u}$).
- Threats experienced by isolated systems as a consequence of increasing DG penetration are even higher than those experienced by interconnected systems since they cannot

depend on the smoothing effect of a large balancing area. The novelty of the proposed approach is to stress the relevance of DR in isolated systems.

- A scenario-based stochastic programming framework is proposed to model the correlated uncertainty characterizing demand and renewable-based power generation. The associated deterministic equivalent is formulated as a mixed-integer linear program suitable for commercially available software.

The rest of this paper is organized as follows. Section II describes the proposed formulation, including substation price calculation and demand payment. In Section III, the test system of La Graciosa and the corresponding results are presented as a validation of the proposed model. Finally, some relevant conclusions are drawn in Section IV.

II. PROBLEM FORMULATION

The modeling of uncertainty in a distribution system for each single time block is based on the load, wind speed and solar irradiation duration curves. Hourly historical data are expressed as per-unit factors. Then, demand factor data are arranged from higher to lower values keeping the correlation between the different hourly data of wind speed and solar irradiation. The load duration curve is approximated using time blocks. In order to accurately include peak demand in the model, the length of the time blocks varies along the load duration curve. For each time block, wind speed and solar irradiation levels are arranged from higher to lower values (Fig. 2). In order to reduce the loss of information on the sequence of individuals when composing the load duration curve, demand, wind speed, and solar irradiation curves have been split into 48 different blocks based on quarter (4), weekend/working-day (2), day/night (2) and time blocks (3) criteria. For each time block, all combinations of demand, wind speed and solar irradiation levels are considered. Each combination is assigned a probability within the time block, $\pi_{ll,w}$, equal to the probability of the demand factor times the probability of the wind speed factor and the solar irradiation factor. For the present paper, 3 factor levels for demand, 3 factor levels for wind power and 3 factor levels for solar irradiation have been considered within each time block leading to 1,296 operating conditions per stage.

This procedure is based on the work proposed in [18]. The approach described in [18], and extended in [1] and [19], has revealed an effective performance to model the relation between demand, wind speed and solar irradiation. The methodology can be described in five steps:

Step 1) Firstly, historical values are separated into four groups of seasonal data, where working day/weekend and day/night separation has been performed in order to adequately represent the considered scenarios, resulting in 16 different sets of data that will be separately arranged following the next steps. Demand, wind speed, and solar irradiation are not statistically independent magnitudes. For each of the 16 data sets in a year the statistical interrelations are specifically accounted for while maintaining the correlation among them. Each of the considered data sets is divided by annual peak demand, maximum annual wind speed, and maximum annual solar irradiation, respectively, to obtain the factorized data.

TABLE I
COMPARISON OF MULTI-STAGE CO-OPTIMIZED EXPANSION PLANNING MODELS

Approach	Topological changes	Renewable generation	Correlation	Demand Response	Bilevel Formulation
[1]	✓	✓	✓	×	×
[2], [6]	✓	✓	×	×	×
[3], [5]	✓	×	×	×	×
[4], [9]	×	✓	✓	×	×
[7], [8]	×	✓	✓	✓	×
[10]	✓	✓	✓	×	×
Proposed approach	✓	✓	✓	✓	✓

Fig. 1. Structure of the bi-level problem considered.

Step 2) For every data set, the demand curve is built sorting data in descending order keeping the hourly correlation between demand, wind speed and solar irradiation. Triplets of hourly factors for demand, wind speed, and solar irradiation are sorted by demand factor in descending order thereby creating a load duration curve for each data set, thus losing the chronology of the considered data.

Step 3) Time blocks are set to model the impact of the demand values.

Step 4) Wind speed and solar irradiation are ordered from highest to lowest in each time block. Then, for each time block and type of data (demand, wind speed, and solar irradiation), a cumulative distribution function is calculated. The proposed formulation provides a sufficiently accurate representation of the demand, wind speed and solar irradiation data within the considered blocks.

Step 5) The distribution function obtained is divided into segments. As depicted in Fig. 2, a total of three segments are considered. Each segment has an associated probability obtained from the corresponding cumulative distribution function. The system scenarios are formed by combining the levels obtained when modeling data uncertainty within each time block. Wind speed and solar irradiation values are then converted to power output values for wind and PV technologies.

Uncertainty sources, namely demand, wind speed, and solar irradiation are characterized by a set of scenarios that are generated on the basis of a previously reported approach relying on historical data [18]. Authors in [1] and [19] have extended the work described in [18] solving the multi-stage expansion planning problem. The approach proposed in the present formulation enhances [1] and [19] in order to adequately integrate time-dependent resources in the

expansion planning. In this sense, the annual pattern is repeated for every block considered in the formulation. Therefore, the probability and level of each scenario in every stage is considered constant. According to [20], although the variability of the wind speed is site specific, it tends to be similar and can reasonably be assumed to be about 6%. Similar results are provided in [21] accounting for the inter-annual variability of solar irradiation. Based on these claims, the assumption of constant probability for the considered scenarios among the different stages is deemed sufficient in order to construct the scenario tree for all stages. As formulated, the model is very flexible and can be easily extended to interconnected systems.

Fig. 2. Load, wind speed and solar irradiation duration curves for the summer/working-day/day scenario.

For each load level ll , each scenario w comprises an average demand factor $\mu_{ll,w}^D$, an average level of wind speed $\mu_{ll,w}^W$, and an average level of solar irradiation $\mu_{ll,w}^{\theta}$. Nodal demands in

each scenario are equal to the product of the forecasted values and the demand factor $\mu_{il,w}^D$ throughout the planning horizon giving rise to $D_{i,t,ll,w}$. Moreover, for each scenario w , $\mu_{il,w}^W$ and $\mu_{il,w}^\theta$ are converted to maximum levels of wind and photovoltaic power generation, $\hat{G}_{i,t,ll,w}^W$ and $\hat{G}_{i,t,ll,w}^\theta$, respectively. For the sake of simplicity, we consider that $\mu_{il,w}^W$ and $\mu_{il,w}^\theta$ are identical for all candidate nodes and all considered time stages.

The distribution network and renewable generation expansion planning can be formulated in a compact way as:

$$\min_{x,y} F(x,y) \quad (A1)$$

subject to:

$$x \in X \quad (A2)$$

$$G^e(x,y) = 0 \quad (A3)$$

$$G^i(x,y) \geq 0 \quad (A4)$$

where vector x represents the upper-level variables, whereas vector y characterizes the reaction of final consumers to time-varying prices. The objective function (A1) comprises the total cost, including investment, maintenance, energy purchase from substations and DG production and unserved power. Expressions (A2), exclusively involving investment variables, include network, generation, and investment constraints. Constraints (A3) denote the equalities associated to the model including Kirchhoff's law constraints. Constraints (A4) model all inequalities involving variables such as power flow limits and generation limits.

The minimization of the consumer's payment is an optimization problem by itself (A5)–(A7) which can be formulated for the optimal vector of investment variables x^* :

$$\min_y f(x^*, y) \quad (A5)$$

subject to:

$$G^E(x^*, y) = 0: (\lambda) \quad (A6)$$

$$G^I(x^*, y) \geq 0: (\mu) \quad (A7)$$

where the Lagrange multipliers associated with constraints (A6) and (A7) are in parentheses. Thus, the distribution network and renewable generation expansion planning considering DR formulated in (A1)–(A4) can be recast as the following bi-level programming problem:

$$\min_x F(x, y^*) \quad (A8)$$

subject to:

$$x \in X \quad (A9)$$

$$H(x, y^*) \geq 0 \quad (A10)$$

where y^* is obtained from

$$\min_y f(x^*, y) \quad (A11)$$

subject to:

$$G^e(x^*, y) = 0: (\lambda) \quad (A12)$$

$$G^i(x^*, y) \geq 0: (\mu) \quad (A13)$$

The upper level controls the upper-level decision variables x . Lower-level decision variables y characterize the reaction of final consumers to time-varying prices. The objective of the upper level is the minimization of the net present value of the total cost (including investment, maintenance, energy purchase from substations and DG

production and unserved power), evaluated at the optimal values of the lower-level variables. Substation prices can be accurately approximated by a set of price levels (Fig. 3). The proposed formulation assumes incremental substation energy demand and incremental energy prices. Therefore, the binary variable $z_{t,b,ll,w}$ is required in order to adequately account for the demand in the substation. Variable $z_{t,b,ll,w}$ takes value of 1 for all substation steps b required to cover the energy demanded in the substation. The proposed approach is consistent with the traditional methodology where higher prices are charged to higher substation consumptions, following the rules of the marginal pricing setting. This approximation yields piecewise linear functions, which for practical purposes are indistinguishable from the nonlinear models if enough blocks are used. The lower-level objective function (A11) is linear, since DR only depends on the price difference between adjacent load levels. Analogously, constraints (A12) and (A13), including power balance equations as well as DR limits, are modeled as linear expressions. Hence, the lower-level problem (A11)–(A13) is parameterized in terms of the upper-level decision vector in such a way that the lower-level problem is linear and thus convex.

Fig. 3. Substation prices.

Replacing the inner problem of the bi-level problem with the KKT optimality conditions and including the resultant system in the upper-level problem is a well-established technique to solve bi-level problems. Incorporating the lower-level problem into the upper one can be achieved either by applying Karush–Kuhn–Tucker (KKT) optimality conditions or by using duality theory. According to [22], both methodologies are equivalent from a rigorous mathematical point of view. The application of KKT conditions yields nonlinearities related to the product of dual and primal variables in the complementarity conditions. For the linearization of such nonlinearities, bounds for dual variables are required, which is customarily set based on experience. The proposed co-optimization avoids the weaknesses associated with the functional decoupling of distribution network planning and distributed generation planning that may lead to economic losses and a suboptimal use of resources.

The following sections provide a comprehensive description of the formulation used in both upper and lower levels.

A. Upper-Level Objective Function and Constraints

As previously described, the total cost in (1) consists of four terms: (2) investment, (3) maintenance, (4) energy purchased from substations and DG production and (5) unserved power.

$$cost = \min \left\{ \sum_t \frac{(1+I)^{-(t-1)}}{I} c_t^I + \sum_t [(1+I)^{-(t-1)} (c_t^M + c_t^E + c_t^U)] + \frac{(1+I)^{-(n_T-1)}}{I} (c_{n_T}^M + c_{n_T}^E + c_{n_T}^U) \right\} \quad (1)$$

where:

$$c_t^I = \sum_{l \in \{NRF, NAF\}} RR^I \sum_{(i,j) \in Y^I} C_{i,j}^{I,l} \ell_{i,j} x_{i,j,t}^I + RR^{SS} \sum_{i \in \Omega^{SS}} C_i^{I,SS} x_{i,t}^{SS} + RR^{NT} \sum_{i \in \Omega^{SS}} C_i^{I,NT} x_{i,t}^{NT} + \sum_{p \in P} RR^P \sum_{i \in \Omega^P} C_i^{I,p} pf \bar{G}^p x_{i,t}^p; \quad \forall t \in T \quad (2)$$

$$c_t^M = \sum_{l \in L} \sum_{(i,j) \in Y^l} C_{i,j}^{M,l} (y_{i,j,t}^l + y_{j,i,t}^l) + \sum_{tr \in TR} \sum_{i \in \Omega^{SS}} C_i^{M,tr} y_{i,t}^{tr} + \sum_{p \in P} \sum_{i \in \Omega^P} C_i^{M,p} y_{i,t}^p; \quad \forall t \in T \quad (3)$$

$$c_t^E = \sum_{w \in \Pi} \sum_{ll \in LL} \pi_{ll,w} pf \Delta_{ll} (\sum_{tr \in TR} \sum_{i \in \Omega^{SS}} C_{i,ll,w}^{SS} g_{i,t,ll,w}^{tr} + \sum_{p \in P} \sum_{i \in \Omega^P} C_{i,ll,w}^{E,p} g_{i,t,ll,w}^p); \quad \forall t \in T \quad (4)$$

$$c_t^U = \sum_{\omega \in \Pi} \sum_{ll \in LL} \pi_{ll,w} pf \Delta_{ll} \sum_{i \in \Omega^{LN}} C_{i,ll,w}^U d_{i,t,ll,w}^U; \quad \forall t \in T \quad (5)$$

Constraint (6) represents the nodal current balance equations, i.e., Kirchhoff's current law. Fulfillment of Kirchhoff's voltage law is secured by constraint (7). The nonlinear constraint reformulation is presented in Appendix section A. Upper and lower limits for nodal voltages are defined by constraint (8). Maximum capacity of the feeders' constraints current flow are formulated in (9). Following the description provided in [23] and successfully implemented in [1], the linearized network model is an adapted version of the dc model that is based on two assumptions: (i) all current injections and flows have the same power factor, and (ii) the per-unit voltage drop across a branch is equal to the difference between the per-unit magnitudes of the nodal voltages at both ends of the branch. Assumption (i) allows expressing Kirchhoff's current law as a set of linear scalar equalities in terms of current magnitudes. In addition, assumption (ii) allows formulating Kirchhoff's voltage law for branches in use as a linear expression relating the magnitudes of currents, nodal voltages, and branch impedances.

$$\sum_{l \in L} \sum_{j \in \Omega_j^l} (f_{i,j,t,ll,w}^l - f_{j,i,t,ll,w}^l) = \sum_{tr \in TR} g_{i,t,ll,w}^{tr} + \sum_{p \in P} g_{i,t,ll,w}^p - d_{i,t,ll,w} + d_{i,t,ll,w}^U; \quad \forall i \in \Omega^N, \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (6)$$

$$y_{i,j,t}^l [Z_{i,j}^l \ell_{i,j} f_{i,j,t,ll,w}^l - (v_{i,t,ll,w} - v_{j,t,ll,w})] = 0; \quad \forall l \in L, \forall i \in \Omega_j^l, \forall j \in \Omega^N, \forall t \in T, \forall ll \in LL, \forall \omega \in \Pi \quad (7)$$

$$\underline{V} \leq v_{i,t,ll,w} \leq \bar{V}; \quad \forall i \in \Omega^N, \forall t \in T, \forall ll \in LL, \forall \omega \in \Pi \quad (8)$$

$$0 \leq f_{i,j,t,ll,w}^l \leq y_{i,j,t}^l \bar{F}_{i,j}^l; \quad \forall l \in L, \forall i \in \Omega_j^l, \forall j \in \Omega^N, \forall t \in T, \forall ll \in LL, \forall \omega \in \Pi \quad (9)$$

The upper bound for the current that can be supplied by the transformers in use is defined by constraint (10). Renewable generators' outputs are limited by the maximum power availability depending on the technology and the meteorology

(11). Doing so, renewable generators may be curtailed by the operator. Unserved power, $d_{i,t,ll,w}^U$, is defined as continuous and non-negative (12). Constraint (13) limits the level of penetration of DG as a fraction of the demand.

$$0 \leq g_{i,t,ll,w}^{tr} \leq y_{i,t}^{tr} \bar{G}_i^{tr}; \quad \forall tr \in TR, \forall i \in \Omega^{SS}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (10)$$

$$0 \leq g_{i,t,ll,w}^p \leq y_{i,t}^p \bar{G}_i^p; \quad \forall p \in P, \forall i \in \Omega^P, \forall t \in T, \forall ll \in LL, \forall \omega \in \Pi \quad (11)$$

$$0 \leq d_{i,t,ll,w}^U \leq d_{i,t,ll,w}; \quad \forall i \in \Omega_j^l, \forall t \in T, \forall ll \in LL, \forall \omega \in \Pi \quad (12)$$

$$\sum_{p \in P} \sum_{i \in \Omega^P} g_{i,t,ll,w}^p \leq \varepsilon \sum_{i \in \Omega^{LN}} d_{i,t,ll,w}; \quad \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (13)$$

Feeders' use is characterized by constraints (14)–(16), explicitly considering the direction of current flows. The utilization of new transformers is formulated in (17) and the use of installed generators is modeled in (18). Only one investment is allowed for each component along the planning horizon (19)–(22). Constraints (23) guarantee that new transformers can only be added at substations that have been previously expanded or built. The overall budget available for investment at each stage is limited by constraint (24).

$$y_{i,j,t}^{EFF} + y_{j,i,t}^{EFF} = 1; \quad \forall (i,j) \in Y^{EFF}, \forall t \in T \quad (14)$$

$$y_{i,j,t}^l + y_{j,i,t}^l = \sum_{\tau=1}^t x_{i,j,\tau}^l; \quad \forall l \in \{NRF, NAF\}, \forall (i,j) \in Y^l, \forall t \in T \quad (15)$$

$$y_{i,j,t}^{ERF} + y_{j,i,t}^{ERF} = 1 - \sum_{\tau=1}^t x_{i,j,\tau}^{NRF}; \quad \forall (i,j) \in Y^{ERF}, \forall t \in T \quad (16)$$

$$y_{i,t}^{NT} \leq \sum_{\tau=1}^t x_{i,\tau}^{NT}; \quad \forall i \in \Omega^{SS}, \forall t \in T \quad (17)$$

$$y_{i,t}^p \leq \sum_{\tau=1}^t x_{i,\tau}^p; \quad \forall i \in \Omega^P, \forall t \in T \quad (18)$$

$$\sum_{t \in T} x_{i,j,t}^l \leq 1; \quad \forall l \in \{NRF, NAF\}, \forall (i,j) \in Y^l \quad (19)$$

$$\sum_{t \in T} x_{i,t}^{SS} \leq 1; \quad \forall i \in \Omega^{SS} \quad (20)$$

$$\sum_{t \in T} x_{i,t}^{NT} \leq 1; \quad \forall i \in \Omega^{SS} \quad (21)$$

$$\sum_{t \in T} x_{i,t}^p \leq 1; \quad \forall p \in P, \forall i \in \Omega^P \quad (22)$$

$$x_{i,t}^{NT} \leq \sum_{\tau=1}^t x_{i,\tau}^{SS}; \quad \forall i \in \Omega^{SS}, \forall t \in T \quad (23)$$

$$\sum_{l \in \{NRF, NAF\}} \sum_{(i,j) \in Y^l} C_{i,j}^{I,l} \ell_{i,j} x_{i,j,t}^l + \sum_{i \in \Omega^{SS}} C_i^{I,SS} x_{i,t}^{SS} + \sum_{i \in \Omega^{SS}} C_i^{I,NT} x_{i,t}^{NT} + \sum_{p \in P} \sum_{i \in \Omega^P} C_i^{I,p} pf \bar{G}^p x_{i,t}^p + \sum_{p \in P} \sum_{i \in \Omega^{ST}} C_i^{I,ST} pf \bar{G}^{ST} x_{i,t}^{ST} \leq IB_t; \quad \forall t \in T \quad (24)$$

Radial operation of the distribution networks is guaranteed by constraints (25)–(33). Constraint (25) imposes nodes to have a single input flow while expression (26) sets a maximum of one input flow for the remaining nodes. Authors in [24] have demonstrated that traditional radiality constraints (25)–(26) may fail to guarantee a radial operation of the distribution system when DG is considered. Following the approach presented in [24], a set of radiality constraints (27)–(33) has been added where a fictitious system with fictitious demands is modeled through (27)–(33) in order to avoid the existence of areas exclusively supplied by DG and thereby topologically disconnected from all substations. Fictitious demand at load

nodes that are candidate locations for DG installation is equal to 1 p.u., being 0 at the remaining nodes (34). These fictitious nodal demands can only be supplied by fictitious substations located at the original substation nodes which inject fictitious energy through the actual feeders. Constraints in (27) represent the fictitious current nodal balance equations. Additionally, constraints (28)–(33) limit the fictitious flows through the fictitious feeders and substations. Constraints (28)–(32) bound the fictitious flows through feeders. Finally, constraints (33) set the limits for the fictitious currents injected by the fictitious substations.

$$\sum_{l \in L} \sum_{i \in \Omega_j^L} y_{i,j,t}^l = 1; \quad \forall j \in \Omega_t^{LN}, \forall t \in T \quad (25)$$

$$\sum_{l \in L} \sum_{i \in \Omega_j^L} y_{i,j,t}^l \leq 1; \quad \forall j \notin \Omega_t^{LN}, \forall t \in T \quad (26)$$

$$\sum_{l \in L} \sum_{j \in \Omega_i^L} (\tilde{f}_{i,j,t}^l - \tilde{f}_{j,i,t}^l) = \tilde{g}_{i,t}^{SS} - \tilde{D}_{i,t}; \quad \forall i \in \Omega^N, \forall t \in T \quad (27)$$

$$0 \leq \tilde{f}_{i,j,t}^{EFF} \leq n_{DG}; \quad \forall i \in \Omega_j^{EFF}, \forall j \in \Omega^N, \forall t \in T \quad (28)$$

$$0 \leq \tilde{f}_{i,j,t}^{ERF} \leq n_{DG} (1 - \sum_{\tau=1}^t x_{ij\tau}^{NRF}); \quad \forall (i,j) \in Y^{ERF}, \forall t \in T \quad (29)$$

$$0 \leq \tilde{f}_{j,i,t}^{ERF} \leq n_{DG} (1 - \sum_{\tau=1}^t x_{ij\tau}^{NRF}); \quad \forall (i,j) \in Y^{ERF}, \forall t \in T \quad (30)$$

$$0 \leq \tilde{f}_{i,j,t}^l \leq n_{DG} \sum_{\tau=1}^t x_{ij\tau}^l; \quad (31)$$

$$\forall l \in \{NRF, NAF\}, \forall (i,j) \in Y^l, \forall t \in T$$

$$0 \leq \tilde{f}_{j,i,t}^l \leq n_{DG} \sum_{\tau=1}^t x_{ij\tau}^l; \quad (32)$$

$$\forall l \in \{NRF, NAF\}, \forall (i,j) \in Y^l, \forall t \in T$$

$$0 \leq \tilde{g}_{i,t}^{SS} \leq n_{DG}; \quad \forall i \in \Omega^{SS}, \forall t \in T \quad (33)$$

$$\tilde{D}_{i,t} = \begin{cases} 1; & \forall i \in ((\Omega^W \cup \Omega^0) \cap \Omega_t^{LN}), \forall t \in T \\ 0; & \forall i \notin ((\Omega^W \cup \Omega^0) \cap \Omega_t^{LN}), \forall t \in T \end{cases} \quad (34)$$

To adequately integrate price-dependent resources, the substation price has been defined as a function of the injected power in the distribution network (see Fig. 3). Equations (1), (4), (35) and (36) are affected by these variable substation costs. Substation prices are variables of the upper level.

The upper level determines the substation prices, which are the linkage between both levels. Based on substation prices, (35) and (36) define the maximum and minimum amount of shiftable load for every load level. As can be seen (35) and (36) are nonlinear expressions which can be linearized as explained in Appendix section B.

$$dr_{i,t,ll,w}^+ = \max \left\{ 0, \sum_{lb} \xi_{lb,ll} \frac{(D_{i,t,ll,w} \Delta_{ll} + D_{i,t,lb,w} \Delta_{lb}) (c_{t,ll,w}^{SS} - \overline{c_{t,ll,w}^{SS}})}{\Delta_{lb} + \Delta_{ll}} \right\} \quad (35)$$

$$dr_{i,t,ll,w}^- = \max \left\{ 0, \sum_{lb} \xi_{lb,ll} \frac{(D_{i,t,ll,w} \Delta_{ll} + D_{i,t,lb,w} \Delta_{lb}) (c_{t,ll,w}^{SS} - \overline{c_{t,ll,w}^{SS}})}{\Delta_{lb} + \Delta_{ll}} \right\} \quad (36)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$\overline{c_{t,ll,w}^{SS}} = \frac{\sum_{lb} c_{t,lb,w}^{SS} \Delta_{lb}}{\sum_{lb} \Delta_{lb}} \quad \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (37)$$

The aforementioned equation (4) models all production costs including substation production costs, which are dependent on the substation price for the considered load level and generation of the individual units. To do this, a binary expansion approach [16] has been used. The discrete substation price, $c_{t,ll,w}^{SS}$, has been defined by b steps (or blocks) between

L_b^{min} and L_b^{max} by adding binary variable $z_{t,b,ll,w}$ (38). Variable $\theta_{t,b,ll,w}$ represents the level of substation power injection for each considered step b (Fig. 3). The value of $\theta_{t,b,ll,w}$ is limited by the upper and lower limits of the considered step (39). The sequentiality of the active step b introduced in the formulation of the substation price is guaranteed in (40). Equation (41) guarantees the equivalence between the sum of the energy supplied by the different transformers and the considered power injection. The nonlinear constraint reformulation is presented in Appendix section C. These constraints are included in the upper-level formulation.

$$c_{t,ll,w}^{SS} = \sum_{b \in \Omega^B} C_b z_{t,b,ll,w} \quad \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (38)$$

$$z_{t,b,ll,w} L_b^{min} \leq \theta_{t,b,ll,w} \leq z_{t,b,ll,w} L_b^{max} \quad (39)$$

$$\forall b \in B, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$z_{t,b,ll,w} \geq z_{t,b+1,ll,w} \quad (40)$$

$$\forall b \in B, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$\sum_{i \in \Omega^{SS}} g_{i,t,ll,w}^{tr} = \sum_{b \in \Omega^B} z_{t,b,ll,w} \theta_{t,b,ll,w} \quad (41)$$

$$\forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

B. Lower-Level Objective Function and Constraints

The objective function of the lower level represents the total payment that the consumers have to afford over the considered time horizon. DR (42) has been introduced in the model considering elastic demand functions calibrated by load levels. Final demand levels are defined in (43). Constraint (44) guarantees that the overall positive and negative demand responses are equivalent. DR load shifting is limited by the maximum amount of shiftable load (defined as a function of the upper and lower limits $dr_{i,t,ll,w}^+$ and $dr_{i,t,ll,w}^-$), expressed as a function of the substation prices and the demanded energy, which is a parameter at this level (45)–(46). Lower-level variables are the final demand ($d_{i,t,ll,w}$) and demand modifications ($dr_{i,t,ll,w}$).

$$payment = \min \left\{ \sum_{t,ll,w} ((1+I)^{-(t-1)} + \frac{(1+I)^{-(n_T-1)}}{I}) [\pi_{ll,w} p_{t,ll,w}] \right\} \quad (42)$$

subject to:

$$d_{i,t,ll,w} = D_{i,t,ll,w} + dr_{i,t,ll,w} : \lambda^1_{i,t,ll,w} \quad (43)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$\sum_{ll} dr_{i,t,ll,w} * \Delta_{ll} = 0 : \lambda^2_{i,t,w} \quad (44)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall w \in \Pi$$

$$dr_{i,t,ll,w}^+ - dr_{i,t,ll,w}^- \geq 0 : \mu^1_{i,t,ll,w} \quad (45)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$dr_{i,t,ll,w}^+ + dr_{i,t,ll,w}^- \geq 0 : \mu^2_{i,t,ll,w} \quad (46)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

The price elasticity assumptions determine the slope of the demand function with own-price elasticities, $\xi_{ll,ll}$, and cross-price elasticities, $\xi_{lb,ll}$, being exogenously provided, based upon values from the literature. Own-price elasticity refers to how energy consumers react to every single load level, considering the average price of the incumbent load levels. Cross-price elasticity refers to the consumer's reaction to the

prices in other load levels. The addition of price elasticity results in demand function $d_{i,t,ll,w}$, which expresses the quantity demanded, $d_{i,t,ll,w}$, as function of the relative deviations of load level prices from the reference level. For simplicity reasons, both own and cross elasticities are considered to have the same value. Based on [15]–[16], the present paper analyses five different values for cross-price elasticity between day and night periods and between the levels corresponding to the same time block ranging from 0 to 4%. For each quarter (q) and working day/weekend (r), the energy can be exchanged between: blocks of the same day; blocks of the same night; blocks from day to night; blocks from night to day.

Incorporating the KKT conditions (as previously described), the lower-level problem is reformulated as:

$$\mu^1_{i,t,ll,w} \geq 0 \quad \forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (47)$$

$$\mu^2_{i,t,ll,w} \geq 0 \quad \forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (48)$$

$$\sum_{t,b,ll,w} \left((1+I)^{-(t-1)} + \frac{(1+I)^{-(n_T-1)}}{I} \right) [\pi_{ll,w} p f C_{b,z,t,b,ll,w}] - \sum_{i,t,ll,w} \lambda^1_{i,t,ll,w} = 0 \quad (49)$$

$$\lambda^1_{i,t,ll,w} + \mu^1_{i,t,ll,w} - \mu^2_{i,t,ll,w} - \sum_{ll} \lambda^2_{i,t,w} \Delta_{ll} = 0 \quad (50)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$(dr_{i,t,ll,w}^+ - dr_{i,t,ll,w}) \mu^1_{i,t,ll,w} = 0 \quad (51)$$

$$\forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi$$

$$(dr_{i,t,ll,w} - dr_{i,t,ll,w}^-) \mu^2_{i,t,ll,w} = 0 \quad (52)$$

Electricity customers in isolated electric power systems expect reasonable and non-discriminatory prices. Substation prices have been considered in the proposed methodology as an adequate non-discriminatory price signal for consumers. Equation (53) models the demand payment for each load level as a function of the substation price and the total volume of demand (Fig. 3). Total payment faced by the consumers is included in the objective function of the lower level (42). Due to the intrinsic nature of bilevel programming, overall payment of the consumers and its related constraints are replaced by its KKT conditions. Therefore, overall payment of the consumers can only be calculated ex-post in the proposed approach. Due to the previous considerations, the non-linear expression presented in (53) does not require a specific linearization.

$$p_{t,ll,w} = \sum_{i \in \Omega^N} c_{t,ll,w}^{SS} p f d_{i,t,ll,w} \quad \forall t \in T, \forall ll \in LL, \forall w \in \Pi \quad (53)$$

III. CASE STUDY

La Graciosa is a small island belonging to the Canary Islands located near Lanzarote and is connected to the Lanzarote–Fuerteventura electric power system, consisting of 26 nodes and 37 branches. La Graciosa is connected to the Lanzarote – Fuerteventura electric power system. The distribution network analyzed consists of 23 load nodes, 3 substation nodes, and 37 branches (Fig. 4). This section presents the results of the proposed methodology for La Graciosa system, including a comparative analysis based on different elasticities. The results of the proposed expansion-

planning algorithm after the considered time horizon are presented in Fig. 6-8. A baseline situation where no elasticity is considered ($\xi_{ll,ll} = \xi_{lb,ll} = 0\%$) has been defined in order to adequately present the results. Then, different scenarios increasing the elasticity are defined (ranging from 0% to 4%).

A. System Description

The planning horizon is 15 years subdivided into five periods of three years, following the approach proposed in [1]. For reproducibility purposes, data for nodal peak demands and branch lengths as well as the single line diagram for the considered system can be downloaded from [25].

Fig. 4. One-line diagram of the La Graciosa distribution network.

B. Results

Results presented in Tables II and III outline the impact of DR in the expansion planning problem, reducing costs for the generation and distribution company and reducing the overall payment for the consumers.

Costs	$\xi_{lb,ll}$ 0%	$\xi_{lb,ll}$ 1%	$\xi_{lb,ll}$ 2%	$\xi_{lb,ll}$ 3%	$\xi_{lb,ll}$ 4%
Investment	346.37	333.20	333.20	350.00	350.00
O&M	330.30	322.64	322.64	338.57	338.57
Energy purchased	5959.75	5863.06	5796.49	5646.86	5538.90
Unserved power	0.00	0.00	0.00	0.00	0.00
Total	6636.42	6518.91	6452.34	6335.43	6227.47

	$\xi_{lb,ll}$ 0%	$\xi_{lb,ll}$ 1%	$\xi_{lb,ll}$ 2%	$\xi_{lb,ll}$ 3%	$\xi_{lb,ll}$ 4%
Payment	7967.84	7892.81	7883.27	7785.43	7721.04

It can be noticed how a higher elasticity benefits both central planner and consumers, reducing the operating and investment costs and the payment from the consumers. Additionally, operating and investment costs reduction are bigger than the demand payment reduction, showing the potential benefit derived from the introduction of DR for the central planner under the considered framework. The long-

term benefit allowing demand to respond to time-varying price conditions is a flatter load shape, which should reduce the need for peak saving capacity and, in turn, reduce emissions and costs through the more effective and efficient use of the grid. Both generation and distribution companies and final consumers are benefited in long-term planning from the exposure of consumers to real-time prices. In order to adequately integrate the short-term reaction of the consumers in the short term into the long-term expansion planning proposed, and aiming at achieving the higher economic benefit for the consumers derived from its flexibility, the traditional formulation has been relaxed replacing the equality constraint by the definition of a maximum value, as presented in Eq. (35)–(37), where DR load shifting is limited by the maximum amount of shiftable load, expressed as a function of the substation prices and the demanded energy. In order to adequately integrate the short-term reaction of the consumers into the long-term expansion planning proposed, and aiming at achieving the higher economic benefit for the consumers derived from its flexibility, variables $dr_{i,t,il,w}^+$ and $dr_{i,t,il,w}^-$ only set the limits for the variation of demand, while demand can be freely modified in between them to suit the interests of the consumers. Therefore, the inherent flexibility of the consumers results in lower payment of the demand and higher costs for the generation and distribution company when the bi-level approach is proposed, as presented in Tables IV and V. For the same reason, the single-level formulation allows the generation and distribution planner to take advantage of this flexibility, resulting in lower operating and investment costs, as presented in Table V. In order to better understand the comparison presented in Table IV, a counterpart of Table II for the single-level formulation is presented in Table VI. Increasing the flexibility of the consumers result in a reduction of the total payment, since consumers would act driven by economic incentives taking advantage of their flexibility to reduce their payment. On the other hand, from the presented results it can be noticed how a higher elasticity benefits both central planner and consumers, reducing the operating and investment costs and the payment from the consumers.

TABLE IV
TOTAL OPERATING AND INVESTMENT COSTS [THOUSANDS OF €]

	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
Bi-level	6636.42	6518.91	6452.34	6335.43	6227.47
Single-level	6636.42	6502.65	6411.93	6269.82	6157.66

TABLE V
TOTAL PAYMENT OF THE DEMAND [THOUSANDS OF €]

	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
Bi-level	7967.84	7892.81	7883.27	7785.43	7721.04
Single-level	7967.84	7898.15	7889.04	7812.61	7760.95

TABLE VI
SINGLE-LEVEL TOTAL OPERATING AND INVESTMENT COSTS
[THOUSANDS OF €]

Costs	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
Investment	346.37	346.37	328.10	333.20	341.60
O&M	330.30	330.30	326.42	322.64	334.63
Energy purchased	5959.75	5825.98	5757.41	5613.98	5481.43
Unserviced power	0	0	0	0	0
Total	6636.42	6502.65	6411.93	6269.82	6157.66

Tables VII and VIII present the investment decisions for wind and PV technologies. Analyzing Tables VII and VIII, it is worth mentioning how DR modifies the deployment of renewable technologies in the distribution network.

TABLE VII
WIND GENERATION INVESTMENT DECISIONS PER STAGE

Stage	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
1	N8,N16	N8,N16	N8,N16	N8,N16	N8,N16
2	N7,N13	N13	N13	N7,N13	N7,N13
3		N10	N10		
4	N10	N7	N7	N10	N10
5		N11	N11	N11	N11

TABLE VIII
PHOTOVOLTAIC GENERATION INVESTMENT DECISIONS PER STAGE

Stage	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
1					
2		N14	N14		
3	N4,N8, N14	N4	N4	N4,N8, N14	N4,N8, N14
4	N13	N8	N8	N13	N13
5	N12,N16	N16	N16	N16	N16

A major advantage of DR is its potential for deferring investments in distribution network capacity driven by natural demand growth. Results presented in Table IX outline this effect, leading to a deferment in network investment when increasing the elasticity. In Table IX, the nomenclature n-n represents replaced feeders, while N-N represents added feeders.

TABLE IX: NETWORK INVESTMENT DECISIONS PER STAGE

Stage	$\xi_{ib,il}$ 0%	$\xi_{ib,il}$ 1%	$\xi_{ib,il}$ 2%	$\xi_{ib,il}$ 3%	$\xi_{ib,il}$ 4%
1	n1-n8, N3-N23, N8-N22, N22-N25	n1-n8, N3-N23, N8-N22, N22-N25	n1-n8, N3-N23, N8-N22, N22-N25	n2-n16, N3-N23, N8-N22	N3-N23, N8-N22
2				N7-N25, N16-N26	N7-N25
3	N16-N24, N16-N26	N16-N26	N16-N26		N16-N26
4		N16-N24	N16-N24	N16-N24	N16-N24
5					

The final situation of the distribution system of La Graciosa is presented in Figs. 5–7, showing the expansion of the distribution network to cover the expected demand increase and the installation of distributed generation within the distribution network for different levels of elasticity.

Fig. 5. One-line diagram of the La Graciosa distribution network with 0% elasticity.

It must be noted how the integration of demand response programs allowing consumers to respond to electricity prices and adapting their electricity consumption accordingly impacts the generation and distribution network expansion planning.

Fig. 6. One-line diagram of the La Graciosa distribution network with 2% elasticity.

Fig. 7. One-line diagram of the La Graciosa distribution network with 4% elasticity.

Simulations have been run on a Dell PowerEdge R910X64 with 4 Intel Xeon E7-4820 processors at 2 GHz and 768 GB of RAM using CPLEX 12.6 [26] under GAMS 24.2.3 [27]. The stopping criterion for the branch-and-cut algorithm of CPLEX used in the proposed model is based on an optimality gap equal to 0.5%. Under this stopping criterion, the attainment of each solution took 10 h on average. The results indicate the practical applicability of the proposed long-term distribution network expansion planning algorithm, especially considering the time frame of the problem addressed. The computational time is highly dependent on the optimality gap to ensure accuracy of the solution and the number of binary variables which are dependent on the considered network (replaceable lines and location of technologies). For bigger systems, the number of replaceable lines and location of new technologies can be reduced or fixed (which is a reasonable assumption) in order to make the problem tractable.

IV. CONCLUSIONS

DR is a promising subject in operation and planning of electrical power systems. However, its impact in joint generation and distribution expansion planning has not been fully analysed yet. Its relevance in short-term operation of power systems has been extensively investigated, where

symmetric treatment of demand and generation creates the strongest incentive for final consumers to participate actively in the wholesale electricity market. When considering medium- and long-term planning, demand responsiveness has an impact on reduction or deferment in distribution expansion planning. The present paper demonstrates how DR can contribute to adequately accommodate renewable generation in a joint expansion planning. Additionally, the potential of DR for deferring investments has been successfully demonstrated.

APPENDIX. MODEL LINEARIZATION

The optimization model for the joint generation and distribution expansion planning presented includes nonlinearities, which make the optimal solution hard to obtain. Instead of directly addressing the original problem of mixed-integer nonlinear programming, a mixed-integer linear formulation is proposed. These nonlinearities are recast as linear expressions by using a piecewise linear approximation and integer algebra results for the bilinear terms.

A. Kirchhoff's Law Linearization

Constraints in (7) model Kirchhoff's voltage law for all feeders in use. The equivalent integer linear reformulation is defined in (B1), where M is a large enough positive constant and its influence is similar to constraint (7). This non-linearity is modeled through binary variables:

$$\begin{aligned} -M(1 - y_{i,j,t}^l) &\leq Z_{i,j}^l f_{i,j,ll,w}^l - (v_{i,t,ll,w} - v_{j,t,ll,w}) \\ &\leq M(1 - y_{i,j,t}^l); \\ \forall l \in L, \forall i \in \Omega_j^l, \forall j \in \Omega^N, \forall t \in T, \forall l \in LL, \forall \omega \in \Pi \end{aligned} \quad (B1)$$

B. Demand Response Linearization

Constraints (35) and (36) represent positive and negative demand shifting, respectively. The equivalent integer linear reformulation is defined in (B2)–(B3). Variable $z_{t,ll,w}^{dr}$ is defined as a binary variable associated with DR at stage t for load level ll and scenario w . Its formulation is consequent with the standard formulation of the Big-M linearization procedure, as presented by Fortuny-Amat in [28].

$$\begin{aligned} -M(1 - z_{t,ll,w}^{dr}) &\leq dr_{i,t,ll,w}^+ - \\ &\sum_{lb} \xi_{lb,ll} \frac{(D_{t,i,ll} \Delta_{ll} + D_{t,i,lb} \Delta_{lb}) (c_{t,lb,w}^{SS} - c_{t,ll,w}^{SS})}{\Delta_{lb} + \Delta_{ll}} \leq M(1 - z_{t,ll,w}^{dr}) \end{aligned} \quad (B2)$$

$$\begin{aligned} z_{t,ll,w}^{dr} &\leq dr_{i,t,ll,w}^- - \\ &\sum_{lb} \xi_{lb,ll} \frac{(D_{t,i,ll} \Delta_{ll} + D_{t,i,lb} \Delta_{lb}) (c_{t,ll,w}^{SS} - c_{t,lb,w}^{SS})}{\Delta_{lb} + \Delta_{ll}} \leq M z_{t,ll,w}^{dr} \end{aligned} \quad (B3)$$

C. Substation Energy Demand Linearization

The non-linear product of binary and continuous variables defined in (41) can be equivalently reformulated as an integer equation (also known as Big-M reformulation) (B4). In constraint (B4) M is a large-enough positive constant.

$$\begin{aligned} -(1 - z_{t,b,ll,w})M &\leq \theta_{t,b,ll,w} - \sum_{i \in \Omega^{SS}} g_{i,t,ll,w}^{tr} \\ (1 - z_{t,b,ll,w})M &\leq \theta_{t,b,ll,w} \quad \forall i \in \Omega^{LN}, \forall t \in T, \forall ll \in LL, \forall w \in \Pi \end{aligned} \quad (B4)$$

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