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SIMULTANEOUS MEASUREMENT OF FIELD-DEPENDENCE ELASTIC MODULI BY LASER INTERFEROMETRY

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Abstract

A methodology is applied which allows the simultaneous determination of the Young's modulus and the shear modulus to evaluate elastic moduli variations with the magnetic field (ΔE and ΔG). The methodology followed is based on the simultaneous detection of the transverse and torsional natural frequencies of a slender magnetic bar found inside a solenoid. The resultant vibration is detected via a heterodyne interferometric optical system with a broad bandwidth. The detection and excitation systems of the vibration do not interact with the sample. The applicability of the method is shown characterizing 10mm-diameter nickel bars. The results are compared with those obtained from longitudinal vibrations.

Keywords: magnetostriction, elastic constants, resonance, interferometry, speckle
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1 Introduction

The elastic study of magnetic materials requires the knowledge of their state of magnetization, as this influences the elastic behavior of the solid. In this study, we are interested in the simultaneous measurement of the ΔE and ΔG effects, that is, the variation of the Young's modulus E and the shear modulus G of a material when it is subjected to an external magnetic field H .

There are various methods for measuring the field-dependence of elastic moduli that basically use electromagnetic sensors. A summary of such methods is given by Squire [1]. Normally, different experiments lead to the determination of E and G . The most usual procedures for measuring the ΔE effect are the resonance/anti-resonance method [2] and the classic method based on the determination of the ultrasonic wave velocity [3]. The most common methods of determining the ΔG effect are by using a torsional pendulum [4] and the methods based on the Widemann effect [5].

On the other hand, it is known that the dependency of elastic constants with magnetization can be evaluated by determining the natural frequencies of samples with a specific geometry [6]. One procedure based on sample vibration is the vibrating reed method which has been used to measure the elastic moduli of metallic ribbons [7]. The flexural or torsional resonant vibrations of a sample are excited by an alternating electric field and the vibration is measured via capacitive detectors.

In this study, we attempt to simultaneously measure ΔE and ΔG from the transverse and torsional vibration of a slender bar. To measure the vibration we have used an

interferometric laser system. E and G are calculated from the direct measurement of the displacements, instead of using indirect electromagnetic measurements. The method can be used for any H field and there is no interference with the magnetization state of the sample.

The proposed methodology has recently been successfully applied to elastically characterize non-magnetic isotropic samples [8]. In a previous study [9], a parallel method is described only to measure the ΔE effect, based on the detection of the longitudinal natural frequencies of a bar.

The optical method used is based on the heterodyne speckle interferometry. The advantages of this technique are that it allows the measurement of the displacement in a point of the material without having contact between the measuring system and the sample, it is immune to low-frequency vibrations, and it is quick and precise. The method also allows the detection of the two displacement components (in and out-of-plane) without electromagnetic excitation, which is particularly important for the case that we are dealing with. Optical methods normally imply certain restrictions on the sample surface. In this case the only condition is that the sample ought to have a scattering surface, and by simply scratching the surface the vibration can be detected on the sample. The basic idea for studying the dependence of the E and G moduli with the magnetization state of the material, consists in measuring the state of vibration at a point on the base of a cylindrical bar of known dimensions and density, when very small torsional and transverse oscillations have been simultaneously provoked on it. In this way, the displacement of the point can be measured in order to calculate the spectrum, from which the value of the E

and G moduli is obtained for each distinct value of the external magnetic field applied.

2 Method

Figure 1 shows a diagram of the experimental arrangement used to determine the natural vibration frequencies of magnetic cylindrical samples. As previously stated, we have also tried to evaluate the variation of the elastic constants in the presence of magnetic fields, that is, to determine ΔE and ΔG simultaneously.

The sample studied is supported by a positioning system to correctly situate it opposite the interferometer and to be able to select the point of detection of the vibration. The ferromagnetic bar is the nucleus of a straight solenoid powered by a DC generator. The solenoid has been designed in such a way as to be able to obtain a field of up to 600 Oe inside it, enough to saturate the samples used. The resultant configuration is such that the field H in the sample is approximately homogenous.

In a single experiment we tried to simultaneously determine E and G , as well as their field variations. G would be obtained from the torsional natural frequencies and E from either the transverse or longitudinal natural frequencies [9]. In order to ensure the free vibration of the bar, it was placed horizontally, supported in the center over a rubber support. The center is a node for the first torsional and longitudinal modes and the second transverse mode. A small quartz sphere measuring 4.40 mm in diameter was used to excite the vibration of the sample, applying a brief impact on the bar. This type of vibration allows the bar to oscillate freely in its natural modes as, once the impact has

finalized, no forces act on the sample.

The torsional and transverse oscillations were originated by means of a grazing horizontal impact delivered perpendicular to the axis of the bar, at a point on the lateral surface close to one end. The percussion is the equivalent of an equipollent percussion applied to the center of the base plus a couple with a moment in the direction of the axis. Thus, transverse and torsional normal modes were simultaneously excited. The resultant vibration was detected by registering the tangential component of the displacements at a point of the opposite base close to the lateral surface. Once the spectrum of the tangential displacement was determined, the transverse modes could be identified, as they ought to follow a determined sequence, that is, given the first transverse frequency, the others are fixed. The lowest frequency to not follow said frequency must be the first torsional frequency. However, a second, optional detection could be carried out at the central point of the base, where only the transverse vibration would register. Comparing the associated spectrums to both registers would allow us to clearly identify the first torsional mode that we could use in a later G calculation. E could be determined from the frequencies of the transverse modes.

The longitudinal vibrations were excited by a perpendicular impact applied at the center of the cylinder base, the out-of-plane component of the displacement was be detected at the center of the opposite base. The frequency associated with the first longitudinal mode could also used to calculate E [9].

A laser interferometer OP-35-I/O from Ultra Optec Inc. was used [10] to measure the sample vibration at a point. With this system, both displacement components - the

perpendicular one (out-of-plane) and the tangential one (in-plane) -can be detected at the same point, although the detection is not simultaneous. Regardless of this, we used the same experimental assembly as it was only necessary to vary the optical paths within the interferometer, a process that is carried out automatically. The in-plane mode would be used to detect the torsional and transverse modes, and so we would use only one experimental configuration to elastically characterize the material [8]. A second, optional detection could be performed for the longitudinal modes by using the out-of-plane mode. The detection principle of the optical system is based on the speckle phenomenon. This phenomenon is observed when a scattering surface is illuminated by coherent light [11],[12]. In this way, the interference from the wavelets diffused by the surface yields an intensity distribution consisting of bright and dark spots. The area size illuminated by the surface is about 20 μm , therefore this system permits point-like detection without contact with the sample. At the same time, the system has a broad bandwidth, from 1 kHz to 35 MHz, allowing the simultaneous detection of various vibration modes, as well as a resolution of approximately 1 nm.

The optical unit is equipped with a 5 mW He-Ne laser, the beam of which is divided in two by a Bragg cell. One beam is identical to the incident and the other has a frequency increased by 40 MHz. In the in-plane configuration, the two beams are focused on the same point of the sample, and the resultant dispersed light is collected by the optical unit and finally sent to a photodetector. A tangential displacement of the detection point δy will originate a variation in the path length of the optical beams. Consequently, the change of the phase is given by the formula $\frac{4\pi \sin \theta \delta y}{\lambda}$, where λ is the wavelength and θ is the angle formed by both incident beams with the normal one at the surface at the detection

point. In the case of the out-of-plane detection, there is only one beam focused on the surface, the other is used as a reference, later interfering with the light dispersed at the surface. An out-of-plane displacement δx produces a phase variation equal to $\frac{4\pi \cos \theta \delta x}{\lambda}$.

A 40 MHz frequency signal, modulated in phase by the displacements, is obtained at the output of the photodiode. The signal is processed by a demodulating unit to yield a signal proportional to the instantaneous displacements of the surface.

Finally, the obtained signal is analyzed using the Fourier transform FFT, with the object of determining the contents in frequencies of the displacements. In our case, a Tektronic TDS-430A digital oscilloscope with the FFT option was used.

For our slender bars, with diameters D much smaller than their length L , the elemental theory of propagation of waves in bars is applicable. The wavelength of the waves studied is great compared with the diameter. The displacements associated with the torsional and longitudinal modes verify a second-order wave equation. Taking into account the free-ends conditions, the vibration normal modes are obtained corresponding to a standing wave, with a wavelength such that a whole number n of half wavelength is contained within the bar length. For torsional waves, the phase velocity v_τ [13] is a function of the shear modulus G and the density ρ , $v_\tau = \sqrt{\frac{G}{\rho}}$. This velocity would vary with the magnetic field and a value would be obtained for each field H on the inside of the solenoid. The natural vibration frequencies are equal to:

$$f_\tau = \frac{n}{2L} \sqrt{\frac{G}{\rho}}, \quad (1)$$

where n is a whole number given to each mode. The shear modulus G as a function of

density ρ , length L and for the case of the first frequency $f_{1\tau}$, would take the value of $G = 4\rho L^2 f_{1\tau}^2$.

The wave equation for the transverse vibrations is a fourth-order differential equation. Imposing the free-ends conditions, natural frequencies are obtained, which, for the present case with a cylindrical bar of diameter D and length L , are equal to:

$$f_t = n^2 \frac{\pi D}{32L^2} \sqrt{\frac{E}{\rho}}, \quad (2)$$

where $n = 3.112$ for the first mode and $5, 7, \dots$ for the following ones.

In our case, to determine E we will use the second transverse mode as it has a node in the center; by which $E = \frac{1.6384\rho L^4}{\pi^2 D^2} f_{2t}^2$.

Equally, the E value would depend on the homogenous field H applied to the bar.

With the E and G values obtained, we will be able to obtain the Poisson's ratio $\mu = (E/2G) - 1$, which varies from 0 to 0.5 in ordinary materials.

3 Experimental results

The samples used were 110 mm long nickel bars with a 10 mm diameter. The metallographic analysis shows that they have a $(99.90 \pm 0.09)\%$ of purity. The density ρ of the material is 8912 kg/m^3 . The mechanical treatment followed was cold drawn and straight length and the heat treatment was annealed at 900°C in continuous furnace.

For the dimensions of these samples, the demagnetizing effect cannot be overlooked. In the presence of a magnetic field, a cylinder creates a demagnetizing magnetic field inside it due to the effect of the poles. This demagnetizing field is a function of the magnetization and the geometrical dimensions [14]. To determine the effective field H_{eff} on the inside of the sample, the following expression would be used:

$$H_{eff} = H_{sol} - H_{des} = H_{sol} - N_d \cdot M, \quad (3)$$

where H_{sol} is the field generated by the solenoid, H_{des} is the demagnetizing field equal to the demagnetizing factor N_d by the magnetization M . The demagnetizing factor N_d is calculated in terms of the ratio r between length and diameter, for this case $r = 11$ and $N_d = 0.2201$. The demagnetizing field in the saturation is equal to 107 Oe, with an intensity of magnetization in saturation $M_s = 0.619$ T.

The results obtained by the method described in the above paragraph are shown in Table 1. The data shown are for samples in demagnetized and technical saturation states. Figure 2 shows an example of the vibration spectrum after the tangential impact of a 4.40 mm diameter quartz ball on the lateral surface of the bar for a 34 Oe effective field. In this spectrum we can see the first torsional frequency ($f_{1\tau} = 13820$ Hz) and the two first transverse frequencies ($f_{1t} = 3555$ Hz and $f_{2t} = 9330$ Hz), also excited by the impact.

As has already been stated, to determine the shear modulus G , the first torsional natural frequency $f_{1\tau}$ was used, for which the central position to which the bar is supported, is a vibration node. To determine the elastic modulus E the second vibration natural frequency f_{2t} was used instead of the first, because the support of the piece corresponds with the maximum amplitude of the first transverse vibration mode, which could have

altered the results.

The table shows the frequency of the second transverse vibration mode f_{2t} and the frequency of the first torsion mode $f_{1\tau}$. With these, and using the equations 2 and 1 the elastic modulus E and the shear modulus G could be obtained - in the table the E and G values are given along with their uncertainties $u(E)$ and $u(G)$ respectively. In order to calculate the uncertainties, we employed the recommendations of the ISO guide for expressing measurement uncertainty [15]. In the uncertainties calculus, the contributions of density, geometrical dimensions and either transverse or torsional natural frequency were taken into account. For density, the data supplied by the manufacturer with an uncertainty $u(\rho)$ equal to 18 kg/m³ was taken. For the dimensional measurements, a digital caliper of 0.01 mm precision was used. The main contribution to the frequency uncertainty $u(f)$ was the resolution obtained in the Fourier analysis (FFT). In this case it was limited by the FFT option of the digital oscilloscope used (Tektronic TDS-430A) at 5Hz for the torsional and transverse measurements.

Figures 3 and 4 show the change in the elastic modulus E and the shear modulus G measured simultaneously in function of the applied magnetic field. The results obtained follow a function of logistic type of the order $p = 3$,

$$f(H_{eff}) = \frac{A_1 - A_2}{1 + \left(\frac{H}{x_0}\right)^p} + A_2, \quad (4)$$

where the coefficients A_1 , A_2 and x_0 are obtained through statistical fit with the experimental data obtained. For example, for Figure 3 the fit $E(H_{eff})$ gives $A_{1E} = 187.67 \pm 0.13$ GPa, $A_{2E} = 194.51 \pm 0.09$ GPa and $x_{oE} = 73.09 \pm 2.57$. In Figure 4, to draw the G curve versus H_{eff} the parameters $A_{1G} = 82.14 \pm 0.06$ GPa, $A_{2G} = 85.25 \pm 0.09$ GPa and

$x_{o_G} = 70.51 \pm 2.34$ were taken.

To obtain the E and G increase between the demagnetized and saturated states (ΔE and ΔG) the parameters of the curves fitted?? were used, with $\Delta E = A_{2_E} - A_{1_E} = 6.84$ GPa and $\Delta G = A_{2_G} - A_{1_G} = 3.10$ GPa, that is to say, the ΔE and ΔG variations were 3.65% and 3.78% respectively.

The results obtained ought to comply with the following relation:

$$\frac{1}{G} = \frac{3}{E} - \frac{1}{3\kappa}, \quad (5)$$

where κ is the compressibility modulus, which ought to be constant in function of the applied field because it is not affected by the change of the domain structure [13]. From equation 5 we can determine the compressibility modulus for each of the E and G values measures simultaneously. In Figure 5, the κ values as a function of the applied magnetic field are shown, and it can be seen that the equation 5 was effectively complied with, as all the values are within the average value $\bar{\kappa} = 88.98$ GPa plus or minus their uncertainty $u(\kappa) = 2.5$ GPa, which was determined from the E and G uncertainties. With the κ results obtained, a practically horizontal line (whose angle is equal to $7.3 \cdot 10^{-3}$ radians) can be fitted??, which shows that it does not depend on the magnetic field H .

With the purpose of comparing the results obtained for ΔE , this value has been also determined from the longitudinal vibrations [9]. In order to do this, E for each magnetic field H was determined by two methods. Firstly, following the procedure previously described, that is, by applying a tangential impact to the cylinder's lateral surface and measuring the in-plane displacement at a point on the opposite side. Secondly, by applying

an axial impact to the center of a base and measuring the out-of-plane displacement vibration on the opposite base.

Figure 6 shows the results obtained in determining ΔE by both methods. It can be seen that the shapes of the curves are analogous and the final ΔE values from the demagnetized state until the saturated state are, in percentages, equal to 3.65% by the torsional excitation method, and 3.19% by the longitudinal excitation method. Also, the ΔE values obtained by each method are included in the uncertainty interval of the other. In this way, the uncertainty interval was $\Delta E = 6.78 \pm 0.64$ GPa for the longitudinal excitation, and $\Delta E = 6.84 \pm 0.83$ GPa for the torsional excitation. Both intervals are co-incidents.

4 Conclusions

In this study, a method was applied to simultaneously determine ΔE and ΔG in bars from the torsional and transverse natural frequencies. The method is based on using a laser speckle interferometer which detects the tangential component of displacements at a point and which also has a broadband frequency response. This technique offers certain advantages over other methods, including speed, high precision, a lack of interaction and the fact that it does not require a complex sample preparation. The experimental results obtained were satisfactory, as the outlined data show. This study has shown the method's usefulness for simultaneously evaluating the variations of elastic moduli with the magnetic field.

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Figure captions

Figure 1. Experimental arrangement showing all the major elements.

Figure 2. Signal spectrum recorded following the torsional impact of a quartz pendulum.

Figure 3. Change in elastic modulus E with the applied magnetic field H .

Figure 4. Change in shear modulus G with the applied magnetic field H .

Figure 5. Compressibility modulus κ versus the applied magnetic field H .

Figure 6. Change in elastic modulus E with the applied magnetic field H by (a) torsional excitation method and (b) longitudinal excitation method.

Tables

Table 1. Results obtained with the torsional excitation method.

Table 1: Results obtained with the torsional excitation method.

State	$f_{2t}(\text{Hz})$	$f_{1\tau}(\text{Hz})$	$E \pm u(E)(\text{GPa})$	$G \pm u(G)(\text{GPa})$	$\Delta E(\text{GPa})$	$\Delta G(\text{GPa})$
Demagnetized	9310	13805	187.74 ± 0.58	82.20 ± 0.17	6.84(3.65%)	3.10(3.78%)
Saturated	9475	14060	194.45 ± 0.60	85.27 ± 0.18		