



Journal Paper

“Decision Making via Binary Decision Diagrams: A Real Case Study”

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Chapter 1

Decision Making via Binary Decision Diagrams: A Real Case Study

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Abstract Nowadays logistical and operational systems are being more complex. A quantitative and qualitative analysis in Decision Making (DM) is presented in this paper. Decision Trees (DT) and Binary Decision Diagrams (BDD) are used to find the best solution to the main problem. The BDD method is used to perform the quantification. A real case related to the timeliness on the deliveries is studied in this paper. Importance Measures (IMs) have been considered to rank the basic events of the DT with respect to their contribution to the top event. Thereby, an improvement of the response of a company facing certain problems and the optimization of the company resources is done. Statistical data is used to obtain an approximate measure of the occurrence probability of the events involved.

Keywords Binary Decision Diagrams · Decision Making · Decision Tree · Importance Measures

1.1 Introduction

The aim in a Decision Making (DM) process is to select the most advantageous path among different situations.

According to the DM problem studied in this paper, to establish a logical structure of the problem is essential. DT has been chosen for that purpose [5]. DT leads to complete a logical relation among several single events. These events, alone or by combination of them, are the responsible of the main problem. The interrelation of mentioned events has been implemented by using logical gates.

It is possible to establish which of the events need to be set employing data analysis techniques when the logical structure of the problem, as well as the statistical

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data, is considered. Nowadays the key to optimize the resources is found on these data analysis techniques considering that a wrong approach of the problem is able to make unfruitful and costly decisions.

1.1.1 Decision Making Scenario

DT shows the main problem and it is composed by several events called basic problems. These basic problems are the responsible for the main event to occur. Not all of the basic problems have the same weight and every single event has a different occurrence frequency. The DT has different levels, from the top event to the different roots, which are called basic events. It is indeed on these basic events where it will be necessary to work in order to reduce its occurrence frequency, where a lower occurrence probability of the main problem will be achieved by reducing it.

1.1.2 Binary Decision Diagram

Binary Decision Diagrams (BDDs), as a data structure that represents the Boolean functions, were introduced by Lee [4], and further popularized by Akers [1], Moret [8], and Bryant [2]. The BDD is used in order to analyze the DT. It will allow obtaining an analytical expression depending on the occurrence probability and the logical structure of the tree of every single basic event.

A BDD is a directed acyclic graph (V, N) , with vertex set V and index set N (position of v in the order of variables). Vertex set contains two types of branches. On the one hand, a terminal vertex has as attribute a value: $value(v) \{0, 1\}$, where “1” state corresponds to the occurrence of the main problem, and “0” state corresponds to the non occurrence of the main problem. All the paths that have 1 state provide the cut-sets of the DT. On the other hand, a non-terminal vertex v has as attributes an argument index $index(v) N\{0, 1, \dots, n\}$, and two descendants, $low(v)$ and $high(v)$, that are connected by a branch. Each one has a vertex 0 branch that represents a non-occurrence basic event, or 1 branch that represents an occurrence basic event. For any non-terminal vertex v , if $low(v)$ is also non-terminal, then $index(v) < index(low(v))$, and if $high(v)$ is non-terminal, then $index(v) < index(high(v))$.

BDD has a root vertex v that leads to denote a function f_v defined recursively as follow: Firstly, if v is a terminal vertex and $value(v) = 1$, then $f_v = 1$. In other case, if $value(v) = 0$, then $f_v = 0$. Secondly, if v is a non-terminal vertex with $index(v) = i$, then f_v will be:

$$f_v(x_1, \dots, x_n) = x_i \cdot f_{low(v)}(x_1, \dots, x_n) + x_i \cdot f_{high(v)}(x_1, \dots, x_n). \quad (1.1)$$

The cut sets of a BDD are the pathways to terminal vertices with value of 1. The occurrence probability of the top event can be calculated by the probability of the cut sets of the BDD. It is possible to achieve the occurrence probability of the main problem through the addition of the BDD cut sets. Thus, mentioned cut sets are able to be evaluated by changing the occurrence probability of each event [1, 2, 8].

Fig. 1.1 BDD example

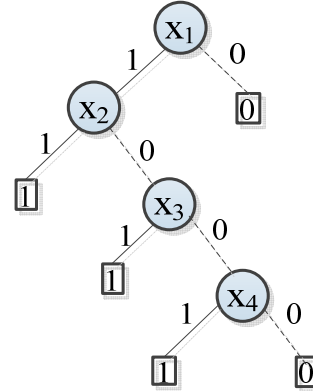


Fig. 1.1 shows a BDD example, which is composed of:

A root vertex: x_1

Two non-terminal vertex: x_2 and x_3

A terminal vertex: x_4

The BDD gives (starting at the x_1 event and finishing in every path to the ones) the cut sets needed, which indeed are:

$$CS_1 = \{x_1, x_2\}$$

$$CS_2 = \{x_1, \bar{x}_2, x_3\}$$

$$CS_3 = \{x_1, \bar{x}_2, \bar{x}_3, x_4\}$$

where \bar{x}_i is the denial of x_i , which means the non occurrence of that event.

1.1.3 Conversion from DT to BDD

The transformation from DT to BDD is achieved by applying certain mathematical algorithms [7]. Hence, it is possible to find an analytical expression of the logical structure desired.

An adequate ranking of the basic events is crucial in order to reduce the size, and thus the computational time to solve the BDD. There are different methods, and any of them will be more adequate to use according to the problem structure, number of variables, etc. In the simulations done in this paper, the AND method have been considered for listing the events [6].

Once the conversion from DT to BDD is done, it is possible to obtain an accurate expression of the main problem occurrence probability by assigning a probability value to each basic event.

$$Q_{sis} = q_{e001} \cdot q_{e002} + q_{e001} \cdot (1 - q_{e002}) \cdot q_{e003} + q_{e001} \cdot (1 - q_{e002}) \cdot (1 - q_{e003}) \cdot q_{e004}, \quad (1.2)$$

where Q_{sis} is the occurrence probability of the main problem and q_{e00i} is the probability of occurrence of the basic event 'i'. Further detailed information about the conversion and variable ordering methods can be found in [4].

1.1.4 Importance Measures

A classification and identification of the events that are influencing the most in the main problem is necessary. IMs reveal some key information such as which of the events are the ones that contribute the most to the main problem to occur.

IMs can be calculated by the Birnbaum, Criticality and Fussell-Vesely heuristic methods. The basic events with higher IM values must be the first to be considered [3]. Focusing on the events that IMs are pointing, will allow the company to reduce the probability of the main problem.

Birnbaum Importance Measure determines that the system is in a certain state that, the occurrence of a certain event causes the main problem. The mathematical expression is:

$$I_i^{Birn} = Q_{sis}(1_i, q(t)) - Q_{sis}(0_i, q(t)) = \frac{\partial Q_{sis}(t)}{\partial q_i(t)}, \quad (1.3)$$

where Q_{sis} is the unavailability of the system, $Q_{sis}(1_i, q(t))$ is the probability of occurrence of the main problem when the basic event "i" is happening, and $Q_{sis}(0_i, q(t))$ is the probability of occurrence of the main problem when the basic event "i" is not happening.

Criticality Importance Measure, unlike Birnbaum, considers the probability of the related basic event:

$$I_i^{Crit} = \left(\frac{q_i}{Q_{sis}} \right) \cdot \left(\frac{\partial Q_{sis}}{\partial q_i} \right) = \left(\frac{q_i}{Q_{sis}} \right) \cdot I_i^{Birn}, \quad (1.4)$$

where Q_{sis} is the occurrence probability of the main problem and q_i is the probability of occurrence of the basic event 'i'.

Fussell-Vesely Importance Measure is that corresponding to the union of those cut sets where such events are presented.

$$I_i^{FV} = \frac{P(E_1^i \cup E_2^i \cup E_3^i \dots E_n^i)}{Q_{sis}(t)}, \quad (1.5)$$

where $(E_1^i \cup E_2^i \cup E_3^i \dots E_n^i)$ is the probability of the union of those cut sets where basic event i is presented and $Q_{sis}(t)$: Main problem occurrence probability.

1.2 Case Study

This paper is focused on the reduction of the occurrence probability of an undesired main event in a decision making scenario. Mentioned event is defined by the orders delay. Firstly, detect which are the events related with the delay in the orders is compulsory (see Fig. 1.3). In order to have an efficient and real decision making, the development of the DT flowchart is crucial. The closest to real decisions the tree is, the better the results will be achieved.

An inner procedure must be in charge of compiling all the information of the basic events. The connections between the events and the logical structure will be obtained by analyzing this information. Mentioned inner procedure need to be done carefully with surveys, questionnaires, meetings and anything needed to create a feedback between employees and the company. The probability associated to each basic event is taken from mentioned feedback and questionnaires.

A repeated event in the DT is possible to be found. This is due to there are basic events which are able to cause the main problem in numerous company areas. For instance in this real case study “Sampling mistakes” ($e006$) is repeated in first and third branches.

Minimal cut sets are obtained by using a mathematical algorithm which converts the DT to BDD as aforementioned in previous sections. Fig. 1.3 shows the basic events interrelation and the probability of occurrence of each basic event. The following calculations have been obtained starting from mentioned probability of occurrence.

1.2.1 System Probability Variation Over the Years

A simulation of the system through ten years has been done. The simulation consists on decreasing the probability of each single event in isolation. With this simulation, a more restrictive control of the influence of each basic event over the system is achieved.

According to the results presented in Fig. 1.2, basic events number ten, from eighteen to twenty-three, twenty-six and twenty-seven, are the ones which affect the most to the system. That means that those events must be modified if the purpose of the company is to reduce efficiently the main event probability. These results will be verified straightaway with the IMs analysis.

Fig. 1.4 shows the importance of each event with the three IM methods. These events can be grouped in a ranking of importance. In this particular case, the event twenty-six has a greater importance than the rest of the events. That means it should

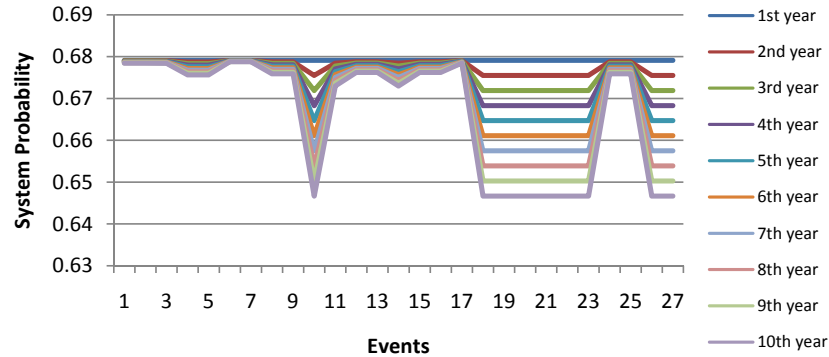


Fig. 1.2 Influence of basic events over the years

be the first event to be considered. However, there is a group of events which has a similar IM value among them (events ten, eighteen to twenty-three and twenty-seven). It is useful to observe the three IMs values to decide how events will be placed in the importance ranking.

The IM value of the rest of the events is so small that they will not have to be considered until the events with the higher IM value had been solved. With this valuable information, it is possible to apply a methodology whereby to obtain a ranking showing the order of the basic events to act.

Looking for the most efficient way to act, it is suggested to start with Criticality IM. In case the basic events have the same IM value, the second step would be to obtain the Birnbaum IM and rank the events. In case the basic events still have the same importance, finally Fussell-Vesely should be obtained and thus have an importance order of the basic events.

1.3 Conclusions

A quantitative analysis in DM problems provides efficient and useful results, e.g. to determinate the basic events that have major influences. Furthermore, to establish a logical structure of the problem is necessary in order to respond as close as possible to the manner the problem is caused.

BDDs are employed to obtain the cut sets that are used to get the analytical expression of the main problem occurrence probability. Thus, the DM offers the chance to simulate dynamically the probability of the main problem when the probabilities of the basic events changeover time.

Decrease the probability of certain events simultaneously entails a lower impact over the main problem probability than to decrease only the probability of the events which the IMs are pointing as more important.

Delay in the orders											
Projects interaction	g01	Projects interaction	g001	Human Resources	g007	Lack of notification	g013	Detailed needs in organization chart	g017	Delay in repair orders	g019
Human Resources	g007	Wrong stock project	g002	Renege on inner procedures.	g008	Unreal warehouse data	g014	Mismatch between needs	g018	Tools shortage	g020
Lack of advice in organization chart	g013	Wrong information exchange, P=0.04	e001	Bad diet	g009	Wrong simulations	g016	Unreal capacity work	e020	Errors in paperwork generation	g022
Delay in repair orders	g019	Wrong stock project	g002	Renege on inner procedures.	g008	Unreal warehouse data	g014	Mismatch between needs	g018	Manufacturing closed late	g023
		Poor production management	g003	Lack of employees formation	g005	Theft employees, P=0.12	e014	Overcharge of capacity, P=0.05	e018	Tools shortage	g020
		Lack of a global planning, P=0.03	e002	Lack of motivation	g010	No coordination between workers	e015	Wrong work planning, P=0.07	e019	Wrong hierarchy of employees, P=0.02	e021
		Poor production management	g003	Lack of employees formation	g005	No coordination between workers	g015			Wrong tools stock	g021
		Forecast mistakes	g004	Employees with low qualification	e004	Poor communication with work orders, P=0.13	e015			Wrong tools stock	g021
		Lack of personal formation	g005	Lack of inner formation courses	e005	Lack of daily notification, P=0.2	e016			Limited tools, P=0.03	e022
		Forecast mistakes	g004	Lack of motivation	g010	Wrong simulations	g016			Limited tools, P=0.03	e022
		Shortage of statistic resources	g006	Limited salary, P=0.07	e009	Lack of simulations, P=0.03	e017			Low tools reliability, P=0.1	e023
		Out-dated analysis techniques, P=0.05	e003	Bad work atmosphere, P=0.08	e010	Forecast mistakes	g004			Paperwork generation errors	g022
		Storage of statistic resources	g006	Bad diet	g009	Forecast mistakes	g004			Employees with limited qualification, P=0.1	e024
		Sampling mistakes, P=0.08	e006	Personal reasons, P=0.06	e008	Out-dated analysis techniques, P=0.05	e003			Lack of resources in some departments, P=0.2	e025
		Parameters selection mistakes, P=0.08	e007	Poor canteen	g011	Shortage of statistic resources	g006			Manufacturing closed late	g023
		Lack of employees training	g005	Poor canteen	g011	Shortage of statistic resources	g006			Limited connection between departments, P=0.3	e026
		Employees with low qualification, P=0.06	e004	Menu variety, P=0.06	e011	Sampling mistakes P=0.08	e006			Manufacturing order not scheduled, P=0.06	e027
		Lack of inner formation courses, P=0.02	e005	Food quality	g012	Parameters selection mistakes	e007				
				Food quality	g012						
				Low kitchen personnel qualification, P=0.05	e012						
				Low ingredients quality, P=0.04	e013						

Fig. 1.3 Delay in the orders DT description

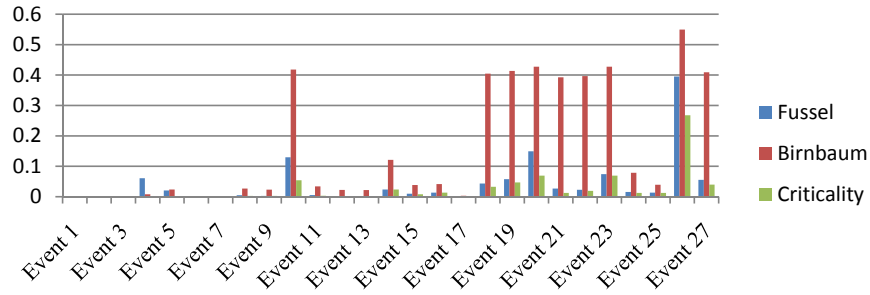


Fig. 1.4 Importance Measures

Applying aforementioned methodology provides the company with a powerful method in the DM process and also an approach to increase the reliability.

As further work the author propose to explore large DM problems, and more complex problems where consider other variables.

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