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B-Spline Approach for Failure Detection and Diagnosis on Railway Point Mechanisms Case Study

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ABSTRACT
In railway transportation the safety and reliability in operations of railway point mechanism¹ must be ensured in order to improve the quality of the service. This paper presents a case study in a railway turnout (US: switch). The case study reports the effect of the operating force data sampled, and monitored during the switching of a railway point mechanism, can be converted into continuous polynomial B-spline functions. These functions are employed to define, and periodically to update, tolerance bands for the purpose of condition monitoring. Data from variously faulty mechanisms were converted similarly, and the profiles found to differ sufficiently not only so as to detect 100% of faults but, from the distinctive shapes of the profiles, also to diagnose (i.e. distinguish correctly between) some 70-80% of them.

KEYWORDS: B-Spline, Failure Detection and Diagnosis, Point Mechanisms, Remote Condition Monitoring.

THE NEED FOR FAILURE DETECTION AND DIAGNOSIS
As part of a guided transportation system, a train can move from one track to another only in certain places, that is, where an appropriate mechanical device has been installed. This device is known as “turnout”. The turnout has moving parts, called switches (US: blades), and which steer the trains in one
of two directions, normal (straight through) or reverse. The switches move from normal to reverse or reverse to normal direction. The basic operation of points has remained the same since their development at the beginning of the railway era. To ensure correct and safe operations of points, position detection and locking devices for the switch blades have been developed. The former are electromechanical devices, while the latter are generally mechanical components which prevent point movement due to vibration, etc. The detection and the point locking systems increase the inversion cost and then the maintenance costs.

There are many cases with catastrophic consequences of point failures, such as at Eschede (1998), Potters Bar (2002) and Grayrigg (2007). It has impulse a large number of research studies, as automatic detection (Fararooy et al. 1996, García Márquez and Chacón 2009), failure analysis and diagnosis (García Márquez et al. 2010), or wear assessment (García Márquez 2011), where is used the condition monitoring (CM) equipment, as illustrated in Figure 2 (García Márquez et al. 2007).

![Figure 1: Point mechanism example, Beijing Underground (China).](image-url)
A robust failure detection and diagnosis (FDD) method is required. This paper presents a new approach in section 4. Previous work employing digital filters (Hou and Andrews 1978) for this purpose includes the development of a basic model in the State Space framework to solve the linear discrete data filtering problem employing the Kalman filter. In this paper, the authors demonstrate that a simple digital filter can be employed with the objective of reducing the complexity of previous models and their computational cost (García Márquez and Pedregal 2007). A recent work has involved forecasting using a vector auto-regressive moving-average model to predict the signal and then comparing with the operation when a fault free is present (García Márquez et al. 2003). By contrast, this paper presents an FDD approach that meets these requirements rather differently, i.e. by employing the well-known B-spline approach (Papaelias et al. 2008) for the representation, processing and interpretation of the force signals during switching. Further information and a review of profile monitoring applications can be found in Noorossana et al. (2011) and Woodall (2007).

**B-SPLINE FOR SIGNAL PROCESSING**

The appropriateness of B-splines in this context arises principally from the sampling of the force signals being necessarily discrete. Values are only precisely “known” at the moments at which the signal is sampled and must be somehow inferred at others. Straightforward linear interpolation might be thought to suffice but would involve excessive reliance upon individual samples, any of which may in fact be noise. Another alternative is the fitting of continuous functions to the data (using, for example, linear,
logarithmic, exponential or Gaussian methods), but the resulting accuracy then depends upon the user selecting an appropriate method based on the shape of the signal. Fitting the various data instead to an appropriate number of polynomial segments (Park 2011), e.g. the global B-spline least-square curve approximation (Piegl and Tiller 1997), these difficulties can be avoided.

Splines are a good choice not only for signal processing (Park 2011, Unser 1999, Unser et al. 1993a), but also in Computer-Aided Design (Flöry and Hofer 2008, Pedregal et al. 2009), computer graphics (Wang et al. 2006), curve reconstruction in reverse engineering (Ueng et al. 2007), CAD/CAM model generation (Donoso et al. 2010) and image processing (Lee 1989, Unser et al. 1993b). The advantages of using B-spline curves are:

- can be fitted, even with polynomials of low degree, to a large variety of data regardless of the shape of the input signal;
- B-spline polynomials of low degree provide a smooth approximation with less undulation than polynomial approximations of high degree (e.g. Lagrange polynomial);
- allow different signals to be compared more readily than otherwise (see section 3);
- can be augmented with smoothing equations so as to achieve a smooth approximation of noisy data;
- provide a good trade-off between accuracy and computational cost compared with other fitting methods because implementations are relatively simple.

**B-spline function formulation**
A B-spline function \( b(r) \) is the union of polynomial function segments defined by several control points \( d_j \) and represented in the Bernstein basis (Figure 3). The joining points of these polynomial segments are called knots, \( r_j \), that define the basic functions, \( N_j^n(r) \), of the Bernstein basis (Chacon et al. 2013). The knots define the length of the polynomial segments. A uniform B-spline has equally spaced knots. A B-spline function of degree \( n \) with \( L+1 \) control points is defined by equation (1):

\[
b(r) = \sum_{j=0}^{L} d_j N_j^n(r),
\]

where the basic functions are defined recursively by equation (2):
\[
N_j^0(r) = \begin{cases} 
1 & r \in [r_j, r_{j+1}] \\
0 & r \notin [r_j, r_{j+1}] 
\end{cases},
\]

\[
N_j^n(r) = \frac{r - r_j}{r_{j+n} - r_j} \ast N_j^{n-1}(r) + \frac{r_{j+n} - r}{r_{j+n} - r_{j+1}} \ast N_{j+1}^{n-1}(r), \quad j = 0, \ldots, L.
\]

The polygon connecting the control points \(d_j\) is called control polygon.

![Control Points, Control Polygon, Knots](image)

**Figure 3:** B-spline curve of degree \(n = 3\).

**B-spline function approach**

The proposed method computes a global approximation B-spline function \(b(r)\) of degree \(n\) with \(k+1\) knots, \(r_0, \ldots, r_k\), that represents an approach to a set points \(p_i\) associate with the parameter \(w_i\), where \(i = 0, \ldots, P\). This approximation minimizes the mean squared errors given by the expression (Farin 2002):

\[
\sum_{i=0}^{P} \left\| p_i - b(w_i) \right\|^2 = \sum_{i=0}^{P} \left\| p_i - d_j N_j^n(w_i) \right\|^2,
\]

The minimum of the sum (3) is found by setting to zero the first derivative. The result of minimizing the distance (3) is the lineal system:

\[
\sum_{j=0}^{L} \sum_{i=0}^{P} d_j N_j^n(w_i) N_k^n(w_i) = \sum_{i=0}^{P} p_i N_k^n(w_i), \quad k = 0, \ldots, L,
\]

where the control points \(d_j\) are the unknown parameters. The evaluated basis functions \(N_j^n(w_i)\) of the sum (4) define the matrix \(B\) with elements \(b_{ij}\), \((i = 0, \ldots, P\) and \(j = 0, \ldots, L\):
\[
B = \begin{pmatrix}
N^a_0(w_0) & \cdots & N^a_L(w_0) \\
\vdots & \ddots & \vdots \\
N^a_0(w_p) & \cdots & N^a_L(w_p)
\end{pmatrix}.
\]

The system (4) can be rewritten in terms of the column vectors of control points, \( \mathbf{D} \), and data points, \( \mathbf{P} \), by equation (5):

\[
B' \times \mathbf{D} = B' \times \mathbf{P}.
\]

The parameters \( w_i \) are defined as (Lee 1989):

\[
w_i = \frac{\| p_i - p_{i-1} \|_f}{\sum_{j=0}^{P} \| p_j - p_{j-1} \|_f}, \quad i = 1, \ldots, P,
\]

where the initial value \( w_0 = 0 \). Equation (6) generates three different parameterizations regarding to the values of \( f \): uniform \( (f = 0) \), centripetal \( (f = 0.5) \) and chord length \( (f = 1) \).

**Smoothing B-spline approach**

Smoothing equations have been introduced into the B-spline approximation in order to achieve a smooth approximation. These smoothing equations are given by the following equations (Farin 2002):

\[
\begin{align*}
0 & = 2d_1 + d_2 \\
& \vdots \\
0 & = 2d_L + d_{L+1}
\end{align*}
\]

Equation (7) can be rewritten, in terms of the smoothing matrix \( \mathbf{B}_{\text{smooth}} \), as:

\[
\mathbf{B}_{\text{smooth}} \times \mathbf{D} = 0.
\]

Equation (8) is introduced into (5) to obtain the following equation in terms of the extended matrices \( \mathbf{B}_1 \) and \( \mathbf{P}_1 \):

\[
B_1' \times B_1 \times \mathbf{D} = B_1' \times \mathbf{P}_1, \quad B_1 = \begin{pmatrix} \mathbf{B} \\ \mathbf{B}_{\text{smooth}} \end{pmatrix}, \quad \mathbf{P}_1 = \begin{pmatrix} \mathbf{P} \\ \mathbf{0} \end{pmatrix}.
\]
Equation (9) does not generate enough control over the smoothing conditions. The weighting with a $\alpha$ parameter of the two components $B_1$ and $P_1$ provides better control of the smoothing conditions (Farin 2002), therefore equation (9) is defined as:

$$B_1 = \begin{pmatrix} 1 & \cdots & 0 \\ B_{\text{smooth}} \end{pmatrix}, \quad P_1 = \begin{pmatrix} 1 & \cdots & 0 \\ 0 \end{pmatrix}, \quad \alpha \in [0,1].$$

The value of $\alpha$ depends on the noise level of signal. In this paper is assumed $\alpha = 0.5$.

**End conditions of a B-spline function**

Equation (9) computes a local B-spline approximation $b(u)$ for a set points $p_i$. This curve is joined to the previous approximation. This union is not continuous and it leads to appear gaps. In this paper end conditions are considered in equation (9), thereby obtaining a continuous approach at the joints. However, this approximation presents inflexion points. In order to minimize these irregularities are also considered prescriptions of the endpoints derivatives from equation (9).

The B-spline function interpolates the endpoints to achieve $C^0$ continuity at the joints, i.e. the curves are joined:

$$d_0 = p_0, \quad d_L = p_p. \quad (10)$$

The end matrices $B_{\text{end}}$ and $P_{\text{end}}$ are defined as:

$$B_{\text{end}} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix}, \quad P_{\text{end}} = \begin{pmatrix} p_0 \\ p_p \end{pmatrix}. $$

Consequently, equation (10) can be written as equation (11):

$$B_{\text{end}} \cdot D = P_{\text{end}}. \quad (11)$$

Equation (11) is introduced into (9) to obtain the following equation as a function of the matrices $B_2$ and $P_2$:

$$B_2^T \cdot B_2 \cdot D = B_2^T \cdot P_2, \quad B_2 = \begin{pmatrix} B_1 \\ B_{\text{end}} \end{pmatrix}, \quad P_2 = \begin{pmatrix} P_1 \\ P_{\text{end}} \end{pmatrix}. \quad (12)$$

As the previous smoothing approximation, equation (12) does not generate enough control over the end conditions. It is solved giving a weight to the components of the matrix $B_2$ and $P_2$ by the $\beta$ parameter:
\[
B_2 = \begin{pmatrix} 1 \\ B_{\text{end}} \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 \\ P_{\text{end}} \end{pmatrix}, \quad \varepsilon \in [0,1].
\]

In this paper is assumed \( \beta = 0.9 \) to obtain a continuous approach. Analogously, in order to obtain a \( C^1 \) B-spline approximation, prescription of two derivatives, \( w_0 \) and \( w_L \) have been considered.

**COMMISSIONING CURVES AND CONSTRUCTION OF TOLERANCE BANDS**

To investigate the feasibility of B-splines to the condition monitoring of point mechanisms, operating current (consumed by the motor) and force (exerted by the system) data were collected at a sampling frequency of 10Hz during a total of 476 point moves (or attempted point moves) (García Márquez et al. 2010) as part of a confidential study for Balfour Beatty Rail (UK). The data were classified in terms of direction of movement (reverse to normal direction or vice-versa) and faults were detected with “current (A) vs. time (s)” curves and “force (N) vs. time (s)” curves. It was observed that the latter were the better choice for detecting faults, therefore B-splines were then constructed from the force profiles, assuming cubic B-spline (polynomial of degree 3) as suggested in (Farin 2002, Lee 1989) with five uniform polynomial curve segments per sample set (García Márquez and Schmid 2007) and centripetal (rather than uniform) parameterization (Chacón et al. 2013, Papaelias et al. 2008). Centripetal parameterization by Lee (1989) is based on the physical analogy of someone driving on a curved highway. Smoothing equations were introduced into the approximation, and end conditions implemented into the algorithm so as to yield smooth curves with \( C^1 \) continuity at the joints. All computations have been carried out in the software package Mathematica®9, on a PC Intel® Core™2 Quad CPU at 2.5 GHz.

![Figure 4: Representation of fault-free sample data (“as commissioned”) using B-splines.](image-url)
Figure 4 shows a case “fault free” mechanism, i.e. “as commissioned”, where is demonstrated how the noise in the signals can be smoothed satisfactorily. Each sample set was found to be slightly different and hence there was some variability between the B-splines; furthermore even greater variability might be expected in service as discussed above in the introduction. Any tolerance bands must be therefore be defined and redefined periodically rather than once at the time of commissioning.

The scheme illustrated in Figures 5 and 6 is proposed for this purpose. Referring to the first “as commissioned” B-spline curve as $d_1$, and having also then obtained a second curve $d_2$, the initial tolerance band is defined so as to encompass both as shown in Figure 5, i.e. upper and lower limit curves $c^+$ and $c^-$, parallel to $d_1$ but respectively displaced up and down by whatever are the greatest positive and negative excesses (if any) of $d_2$ over $d_1$. A third (still “fault free”) curve $d_3$ is then obtained and, if this lies outside the tolerance band, refine it as shown in Figure 6, i.e. raising $c^+$ and lowering $c^-$ by the greatest positive and negative excesses of $d_3$ over these two respectively. The procedure to compute the tolerance band is the same if $d_2$ does not cross to $d_1$. For example, if $d_2$ is above $d_1$, then $c^-$ coincides with $d_1$, and if $d_2$ is below $d_1$, then $c^+$ coincides with $d_1$. The tolerance band thus formed can then be updated periodically in a similar fashion with further curves $d_4$, $d_5$, etc. (as long as the mechanism remains “fault free”) so as to allow for wear and other variations; the result might be similar to Figure 7 in which the band is divided into five sections, each corresponding to one segment of the underlying B-spline. The computation time increases with the number of sections. In this case study, the 5 divisions of tolerance band is the minimum number that identifies all the faults considered with a minimum computational cost. It has been demonstrated that variations of this number of sections do not provide better results.
Figure 5: Initial definition of tolerance bands (c⁺, c⁻) from first two “as commissioned” (fault-free) data sets d₁, d₂.

Figure 6: Updating of initial tolerance band (c⁺, c⁻) with third “as commissioned” (fault-free) data set d₃. Note that subsequent periodic updating would be upon a similar basis.
DIAGNOSIS OF FAILURES AND INCIPIENT FAULTS

The tolerance bands defined above become crucial in FDD. In the present work further force profiles were obtained again at 10Hz for the eight typical types of faults listed in Table 1. It was found that these differed markedly from the “as commissioned” curves, and also sufficiently from each other to suggest that the various shapes alone, might be enough for fault identification and diagnosis. The signal is analyzed online, and the lined up on the horizontal axe is automatically done by the condition monitoring system when the mechanism starts to work, i.e. the signals are not lined up on the horizontal axe before they are analyzed.
B-splines were thus constructed from each sample set (using three degrees of freedom and five segments). Figure 8 shows the result for one particular fault type, “obstruction reverse side at toe” (ORS), when switching from normal to reverse. This is compared in Figures 9 and 10 with what might be expected if the mechanism were “fault free”, i.e. with particular reference to the sections of the tolerance band obtained above. The mechanism can be seen initially to stop pushing a little less slowly than when “fault free” and, after 1.5 seconds or so, is still exerting 144N in excess of what is necessary; thereafter it tends to start pulling much too quickly, and by the end of switching the force is 981N greater than intended. This latter deviation was in fact the maximum observed as listed in Table 2, wherein the second and third columns indicate that it arose below the fifth section of the tolerance band. Note how these characteristics differ when switching from reverse to normal as listed in Table 3.

In fact ORS emerges in Table 2 as rather different from the other faults that were investigated: the maximum deviation during N-R switching is not only, by some margin, the greatest, but also the only one to occur in the fifth section. This alone is enough to distinguish it from the others and, at the bottom of Table 3, three further faults stand out when switching R-N: the maximum deviation for “back drive slack off toe end RHS” (BDS-R) will occur uniquely below the second section, “drive rod stretcher bar loose RHS” (DRSB-R) above the second, and “backdrive tight at heel end RHS” (BT-R) above the first.

The remaining four – “tight lock reverse side” (TLRS), “diode snubbing block disconnected” (DSBD), “backdrive tight at heel end LHS” (BT-L) and “back drive slack off toe end LHS (BDS-L) – are not so

<table>
<thead>
<tr>
<th>Fault</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORS</td>
<td>12mm obstruction reverse side at toe</td>
</tr>
<tr>
<td>TLRS</td>
<td>Tight lock reverse side (sand on bearers both sides)</td>
</tr>
<tr>
<td>DSBD</td>
<td>Diode snubbing block disconnected</td>
</tr>
<tr>
<td>BT-L</td>
<td>Back drive tight at heel end LHS</td>
</tr>
<tr>
<td>BDS-L</td>
<td>Back drive slack off toe end LHS</td>
</tr>
<tr>
<td>BDS-R</td>
<td>Back drive slack off toe end RHS</td>
</tr>
<tr>
<td>DRSB-R</td>
<td>Drive rod stretcher bar loose RHS</td>
</tr>
<tr>
<td>BT-R</td>
<td>Back drive tight at heel end RHS</td>
</tr>
</tbody>
</table>

Table 1: Selected typical faults on point mechanisms
easy to tell apart, and all of them give rise to maximum deviations above the first section of the tolerance band when switching N-R and below the fifth when in reverse, and therefore to distinguish correctly between them thus requires knowledge of the magnitudes of the deviations. With the benefit of hindsight it should thus be no surprise that it was amongst these tests that there were some failures to diagnose correctly.

Figure 8: B-spline representation of sample data from mechanism switching from normal to reverse but having ORS.
Figure 9: Comparison of faulty (ORS) force profile with “fault free” tolerance bands (N-R).

Figure 10: Force relative to when “fault free” – mechanism with fault ORS switching N-R.
### Table 2: Maximum deviations from tolerance when faulty and switching N-R.

<table>
<thead>
<tr>
<th>Failure</th>
<th>Tolerance Band</th>
<th>Force (N)</th>
<th>Tested</th>
<th>Detected</th>
<th>Correctly Diagnosed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sign max (D+,D-)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ORS</td>
<td>5</td>
<td>-</td>
<td>981</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>TLRS</td>
<td>1</td>
<td>+</td>
<td>222</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DSBD</td>
<td>1</td>
<td>+</td>
<td>187</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BT-L</td>
<td>1</td>
<td>+</td>
<td>145</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BDS-L</td>
<td>1</td>
<td>+</td>
<td>47</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>BDS-R</td>
<td>1</td>
<td>-</td>
<td>236</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DRSB-R</td>
<td>1</td>
<td>-</td>
<td>204</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BT-R</td>
<td>1</td>
<td>+</td>
<td>491</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>23</td>
<td>100%</td>
<td>70%</td>
</tr>
</tbody>
</table>

### Table 3: Maximum deviations from tolerance when faulty and switching R-N.

<table>
<thead>
<tr>
<th>Failure</th>
<th>Tolerance Band</th>
<th>Force (N)</th>
<th>Tested</th>
<th>Detected</th>
<th>Correctly Diagnosed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sign max (D+,D-)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ORS</td>
<td>5</td>
<td>+</td>
<td>712</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>TLRS</td>
<td>5</td>
<td>+</td>
<td>144</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DSBD</td>
<td>5</td>
<td>+</td>
<td>102</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BT-L</td>
<td>5</td>
<td>+</td>
<td>86</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BDS-L</td>
<td>5</td>
<td>+</td>
<td>105</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>BDS-R</td>
<td>2</td>
<td>-</td>
<td>217</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DRSB-R</td>
<td>2</td>
<td>+</td>
<td>244</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>BT-R</td>
<td>1</td>
<td>+</td>
<td>330</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>23</td>
<td>100%</td>
<td>83%</td>
</tr>
</tbody>
</table>
CONCLUSIONS
Remote Condition Monitoring is usually used to collect signals from a mechanism. These systems often contain some kind of alarm system based around thresholding techniques, the limits of which are set by local maintenance personnel. The methodology presented above was found to detect every fault that was presented, and to diagnose 70% of them correctly when switching from normal to reverse and some 83% for reverse to normal, rates which might well have been higher but for the force profiles of some fault types being too similar and perhaps for having divided the tolerance band into only five sections. The case study considers only a switch in order to show the procedure clearly. However, the system can monitor simultaneously various switches, it will depend of the condition monitoring system, using the same method presented in this paper for all the switches, where the confident band and the patterns for fault identification will be set for each switch.

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