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34 **Keywords:** *Diesel injection, solenoid-operated injector, fuel injection modelling,*
35 *thermodynamic diagnosis, zero-dimensional model, GTL fuel.*

36

37 **1. INTRODUCTION**

38 Nowadays, the well-known common-rail injection technology achieves better
39 results than the traditional injection system due to higher flexibility for all load ranges. This
40 system makes possible to regulate the injection parameters (such as the start and duration of
41 injections and the injection pressure) with high precision at any operating mode of the
42 engines. Moreover, the study of the rate of fuel injection (RoI) allows controlling the rate of
43 heat release (RoHR) characteristics. In consequence, the fuel injection rate optimization is
44 directly related to the in-cylinder combustion process.

45 Current injection strategies are characterized by short injection times and more than
46 one injection per cylinder and cycle. Several studies have researched the effects of different
47 injection strategies [1, 2]. Normally, these studies require a dedicated and complex facility
48 apart from the engine test bench (where it is difficult to measure the real rate of injection
49 accurately). However, if this research facility is not available, it would be helpful to have a
50 model that is able to provide the rate signal from some known performance parameters
51 (such as injector geometry, the pulse of energization, injection pressure, etc.). The existing
52 literature about models for simulation of the rate is wide, but the premises for the
53 development of the model are not always the same.

54 Most of the researchers working on fuel injection rate simulation focus on the
55 hydraulic one-dimensional modelling (1D) of the injector, which requires a thorough
56 knowledge of the dimensions and operation conditions of the injector. For the modelling
57 the injector components, researchers have used computational tools such as AMESim code
58 [3] or other similar software. Usually, these models are divided according to the injector
59 type used: solenoid [4-8] or piezoelectric [9-11]. Solenoid operated injectors (the most used
60 in last decade) have been studied and reported in several works. Seykens *et al.* [5], using an
61 8-hole injector, modeled the needle displacement caused by injection pressure variation
62 which produced different slopes in the fuel injection rate. Chung *et al.* [6] modeled the rate
63 of injection from the study of fuel flow through the injector and the forces involved during

64 the injection process. In this paper, a single injector outlet is used in the nozzle and, a single
65 rate is simulated. Payri *et al.* [4], modeled a 6-hole solenoid injector and took into account
66 the effects of the discharge coefficient and cavitation on the nozzle. Marcic *et al.* [8] used a
67 6-hole solenoid injector for the development of a needle movement dynamic model to
68 simulate multiple injection rates. The most important disadvantages of these models are the
69 need of accurately knowing the internal dimension of injectors.

70 In order to avoid this issue, Payri *et al.* [12] developed a zero-dimensional model
71 (0D) assuming the injector as a black box, and directly calculating the rate of injection
72 signal as a function of some easy to know geometrical and operating parameters, and the
73 mathematical equations of the fluid-dynamic study at the nozzle outlet. This work is an
74 example the use of 0D models, to reduce calculation time.

75 The use of injection models to simulate the effect of properties of alternative fuels
76 to diesel is not overly usual [5, 7, 9, 11, 13]. This type of models has equations to estimate
77 fuel properties (density, kinematic viscosity or bulk modulus) or experimental data [7, 9]
78 necessary to use the models.

79 Different parameters such as injector type, test conditions or fuel properties may
80 affect the results provided by the model. Although the possible differences (concerning real
81 values) are low, they should be considered when fuel mass rate modelled is used to
82 calculate heat release rate. In this sense, the thermodynamic diagnosis helps to understand
83 the importance of these parameters during combustion [13-16].

84 The present work proposes a zero-dimensional model, which allows obtaining the
85 rate of fuel injection, knowing the injection pressure, the injector energizing time, the total
86 mass injected in each injection, the injection temperature, characteristics of the injector
87 nozzle (diameter and number of holes), the gap between the start of injections (in case of
88 multi-injection) and some fuel properties. Compared to other models, the model presented
89 is able to simulate both single injection and split injection (pre-injection and main
90 injection), in current engine operation modes or other unusual modes. For the development
91 of the model, the experimental results of two solenoid injectors, with a different number of
92 holes, were used. The stated model works with conventional diesel fuel without biodiesel
93 and Gas To Liquid fuel (GTL). Fuel properties, such as density and viscosity, are
94 determined by means of correlations as a function of fuel temperature and pressure. Finally,

95 the effect of the fuel introduction in the combustion chamber has been studied comparing
96 two situations: i) with vs without fuel introduction and ii) modelled and measured injection
97 rates, on the gas properties, in-cylinder mean temperature and heat release law are the main
98 results of the thermodynamic diagnosis presented.

99

100 **2. EXPERIMENTAL SETUP AND TEST PROCEDURE**

101 *2.1. Experimental installation and injectors tested*

102 Figure 1 shows the experimental installation used for determination of the rate of
103 fuel injection. The complete installation is described by Armas *et al.* [17].

104

105 *Figure 1. Simple functional scheme experimental installation.*

106 The experimental fuel injection rates were measured by means of fuel injection rate
107 indicator IAV model EVI2 K-050-49 (IRI).

108 The signal of injector energizing was registered by means of an AC/DC Current
109 Clamp, Hantek® CC 65, 20kHz bandwidth, ($\pm 2\%$ of error in the range used), while the
110 total fuel mass injected (m_f) was measured by a gravimetric balance KERN PFB 3000-2
111 with a precision of $\pm 0,01$ g. Two different solenoid operated injectors have been used in this
112 work: a) a 7-hole injector with 150 μm diameter (Denso model 7H150) and b) an 8-hole
113 injector with 138 μm diameter (Denso model 8H138). Both injectors were used for
114 developing the model approach and also for its validation under different operating modes.
115 The operating parameters of the injectors were for the boost current 20 A and 75 V, and for
116 the hold current 17 A and 13 V. In these injectors, the needle raising conditions cannot be
117 varied, nor can it make boot and ramp shaped main injections.

118 The nozzle geometry was identified by means of an X-ray scanner NIKON model
119 CT-SCAN-XT-H-160. The software of the scanner generates tridimensional (3D) images of
120 the object from two-dimensional images (2D). Figure 2 shows an example of the images
121 used to determine the nozzle diameter.

122

123 *Figure 2. An example of the image from the tip of injector obtained by means X-ray*
124 *analysis.*

125 In this work, it is also used a test bench equipped with the light-duty engine NISSAN
126 YD22, with 7H150 injectors. This installation is described by Payo *et al.* [18]. The main
127 characteristics of the engine are shown in Table 1.

128 *Table 1. Engine characteristics*

129 2.2. Test fuels

130 2.2.1. Fuel properties

131 The development of the model and its application (with both injectors), were done
132 using neat diesel fuel (without biodiesel). The model contemplates the possibility to work
133 with a GTL fuel. This alternative fuel, obtained from natural gas or from renewable sources
134 (biomass gasification) through a Fischer-Tropsch process, presents great potential to be
135 used in diesel engines [19]. The diesel fuel was supplied by REPSOL Co. while the GTL
136 fuel was supplied by SASOL Co. Table 2 shows the main properties of both fuels.

137
138 *Table 2 Fuel properties*

139
140 The atmospheric pressure experimental data for fuel density and kinematic viscosity
141 are presented in Table 3.

142
143 *Table 3. Kinematic viscosity and density of fuels tested at different temperatures with*
144 *atmospheric pressure*

145 146 2.2.2. Fuel density estimation

147 The fuel density (ρ_f) and kinematic viscosity (ν_f) oscillate with temperature and
148 pressure.

149 Equation 1, proposed by Payri *et al.* [20], determines the diesel fuel density as a
150 function of pressure and temperature, the reference temperature (T_0) is 313 K, and the
151 reference pressure (P_0) is 0.1 MPa. The back temperature of injector fuel return is T and
152 injection pressure is P . The ranges of applicability for this expression are 298K to 348K in
153 temperature and 0.1MPa to 180MPa in pressure.

154

155 $\rho(P, T) = 835.698 - 0.628(T - T_0) + 0.491(P - P_0) - 0.00070499(T - T_0)^2 +$
 156 $0.00073739(P - P_0)^2 + 0.00103633(P - P_0)(T - T_0)$ (1)

157 Outcalt *et al.* [21] used the expression (2) to determine the density of GTL fuel as a
 158 function of pressure and temperature. For this expression, is needed a GTL fuel density
 159 correlation, depending on temperature, at the reference pressure (applicability 270K to
 160 470K in temperature and 0.48MPa to 50MPa in pressure).

161

162
$$\rho(P, T) = \frac{\rho(P_0, T)}{1 - 0.082681 \cdot \ln\left(\frac{P+B(T)}{P_0+B(T)}\right)}$$
 (2)

163

164 For calculating $\rho(P_0, T)$, the equation proposed by Rackett [22] and the
 165 experimental data presented in Table 3 was used.

166

167
$$\rho(P_0, T) = 255.5 \cdot 0.519^{-(1+(1-T/578.2)^{0.542})}$$
 (3)

168

169 Where $B(T)$ is Tait equation parameter defined by Dymond and Malhotra [23],
 170 which can be expressed by the equation (4).

171

172
$$B(T) = 325.49 - 298.96 T/273 + 70.73(T/273)^2$$
 (4)

173

174 2.2.3 Fuel dynamic viscosity estimation

175 The fuel density and the fuel dynamic viscosity depends on pressure and
 176 temperature. From the experimental data shown in Table 3, exponential expressions (5) and
 177 (6) were obtained to define the dependence of the dynamic viscosity (μ_f) with the
 178 temperature, at a reference pressure and for each fuel, such as those presented by Payri *et*
 179 *al.* [24].

180

181
$$\mu_{Diesel}(P_0, T) = 5.7694e^{-0.0169T}$$
 (5)

182

183
$$\mu_{GTL}(P_0, T) = 4.7883e^{-0.0189T}$$
 (6)

184

185 In the expressions (5) and (6) the fuel dynamic viscosity is given in cPo while the
186 fuel temperature is given in °C.

187 For calculating the dynamic viscosity variation caused by the fuel pressure,
188 expression (5) and (6) are replaced by the expression (7), proposed by Kousel [25].

189

$$190 \quad \mu_f(P, T) = \mu_f(P_0, T) \exp \left[\frac{P}{10^4} (7.9718 + 37.27967 \mu_f(P_0, T)^{0.278}) \right] \quad (7)$$

191

192 2.3. Test plan

193 In this work, rates of fuel injections from two different test plans were used:

- 194 i) A test plan with an injector denoted as 7H150, previously mounted in an engine
195 coupled to a dynamometer in a test bench.
- 196 ii) A test plan with a similar injector denoted as 8H138, used for determining its
197 discharge coefficient.

198 Originally, the first test plan was used in order to experimentally characterize
199 exhaust pipe heat transfer of the engine [26]. The main advantage of using these two test
200 plans consists of the possibility to test the injectors with multiple injections with short
201 energizing times, characteristic of both urban and extra-urban driving conditions (test plan
202 i), and with long injector energizing time as employed typically of used in a test for spray
203 modelling (test plan ii). Different operating modes from each test plan were used for model
204 approach and its validation.

205 Table 4 shows the characteristic injection parameters of the test plan used for the
206 model approach (modes with asterisks) and its validation (modes without asterisks), using
207 the 7H150 injector and the experimental installation shown in Figure 1. Values of injection
208 pressure and injection timing shown in Table 4 were recorded from tests carried out in the
209 test bench with the light-duty engine NISSAN YD22, equipped with 7H150 injector. Figure
210 3 shows these engine operating modes [26].

211

212 *Table 4 Main characteristic parameters of injection for split injection with Denso 7H150*
213 *injector (9 engine modes).*

214

215 As can be seen in Figure 3, the 9 operating modes are located in the lower-left zone
216 of the engine map (Torque-engine speed), limited by the full load curve (maximum torque
217 at each engine speed). These modes cover the most part of the zone characterized by the
218 New European Driving Cycle (NEDC) [26].

219 The outermost modes (A, C, G, and I) and the central mode (E) were selected for
220 the model approach, while the parameters from modes B, D, F, and H were used for model
221 validation. All modes used have two injections (pre-injection and main injection) and were
222 tested in the installation presented in Figure 1 using two different dwell times (time
223 between injections) at each operating mode.

224

225 *Figure 3. Engine operating modes used for 0D model input data with 7H150 injector.*

226

227 Table 5 shows the injection characteristic parameters of the nine modes for
228 calculating RoI related to the second injector Denso 8H138. Six of them for the model
229 development (denoted with an additional asterisk) and three used for the model validation.

230

231 *Table 5. Main injection characteristic parameters for single injection using Denso 8H138*
232 *injector.*

233

234 2.4. Methodology

235 During all the tests for determining rates of fuel injection (Tables 4 and 5), the fuel
236 temperature at the inlet of the high-pressure pump was constant (around 40°C). The return
237 back pressure (P_{rp}) of injectors was 0.975 bar in all cases and injectors. The methodology
238 presented in this paper was divided into three parts:

- 239 - General methodology for the model approach (Figure 4).
- 240 - Model validation with different fuels (Figure 5).
- 241 - Model application and validation across the thermodynamic diagnosis (Figure 6).

242 Figure 4 shows the methodology used for the model approach. Using geometrical
243 data from both injectors, experimental data from IRI (RoI and energizing signals, and mass
244 injected for each injection) and fuel properties (in this case, density and dynamic viscosity
245 diesel fuel equations) the different characteristic parameters of the model were determined
246 and developed. Experimental data includes values from 11 different operating modes tested

247 with diesel fuel, 5 split injections (pre + main injections) with 7H150 injector and 6 modes
248 only with a main injection with the 8H138 injector. After model approach, modelled and
249 experimental values of the rates of injections were compared.

250

251

Figure 4 Methodology for model approach.

252

253 To validate the model, (as shown in figure 5) two sets of data were used:

254 - Results obtained with diesel fuel (route with arrows and boxes drawn in continuous
255 lines) under stationary modes B, D, F and H of Table 4 with injector 7H150 and
256 modes 3, 6 and 9 of Table 5 with injector 8H138.

257 - Results obtained with GTL fuel (route with arrows and boxes drawn in dashed
258 lines) in order to verify the ability of the model to reproduce rate of injections with
259 different fuels, under operating modes A, C, G, I and E with injector 7H150 which
260 appear in Table 4.

261 All modeled results were compared with the experimental data from the IRI and
262 each fuel.

263

264 *Figure 5 Methodology for model approach and its validation with different fuels.*

265

266 Figure 6 shows the methodology followed to compare modelled and experimental
267 rates of injections applied to a thermodynamic diagnosis model with both Diesel and GTL
268 fuels and 7H150 injector. Experimental and modelled rates of injection were used as input
269 data to the thermodynamic diagnostic model and are based on the estimation of
270 thermodynamic properties of the gas, derived from the experimental pressure signal in the
271 cylinder [27] and improved by Payri *et al.* [14]. It allows obtaining comparative results of
272 the in-cylinder gas mean temperature, RoHR and Heat Release Law (HRL). The
273 comparison of results with the experimental RoI, the modeled RoI and without the RoI,
274 allows the evaluations of the influence of the injection rate on the thermodynamic
275 diagnosis.

276

277

Figure 6 Methodology to compare thermodynamic diagnosis results

278

279 3. MODEL PROPOSAL

280 Figure 7 shows an example of simple injection for an energizing time (ET) of the
281 injector and its corresponding curve of rate of injection versus time. The injection time (t_{inj})
282 is obtained by means of the equation 8 which depends on: i) the energizing time of injector,
283 ii) the time lag (d_1) between the start of the energizing (SoE) and the start of the injection
284 (SoI) and iii) the time lag (d_2) between the end of the energizing (EoE) and the end of the
285 injection (EoI).

286

$$287 t_{inj} = ET - d_1 + d_2 \quad (8)$$

288

289 When the injector works under single injection, the time lags d_1 and d_2 depend on
290 the injector type, the injection pressure and the fuel used [17]. However, when the injector
291 works under split injection, these time lags could be influenced by the time between
292 injections or dwell time (D_T) [28], defined as the time between the end of the previous
293 injection and the beginning of the next injection during the same thermodynamic cycle. In
294 this proposal, the model takes into account this variation. For this reason, the calculated
295 correlations include D_T as a factor in the correlation for determining d_1 .

296

297 Taking into account the data collected from the two injectors under study, the
298 equations 9 and 10 are proposed. The determination of the delay time d_1 is based on a
299 correlation which takes into account: injection pressure and back pressure in return line of
300 the injector (P_{inj} and P_{rp} , respectively), fuel density and dynamic viscosity (μ_f), and D_T . In
301 the specialized literature, most of the studies use the Reynolds number (Re) for estimating
302 both d_1 and d_2 delays. In this work have been taken into account some physical parameters
303 that affect Re determination, but not its value. Correlation includes the effect of D_T on d_1
304 and the effect of ET on d_2 , as it is done in other studies [12].

304

$$305 d_1 \approx a_1 + a_2 \sqrt{P_{inj} - P_{rp}} + a_3 \left(\frac{\sqrt{2\rho_f}}{\mu_f} \right)^2 + a_4 \frac{\sqrt{2\rho_f}}{\mu_f} + a_5 D_T \quad (9)$$

306

307 The d_1 values remain practically constant (around 0.43 – 0.47ms), independently of
308 the injector and the operating parameters. The maximum error of d_1 predicted value is

309 0.05ms, the average absolute error was 0.00193 ms and the R-squared was 99.93 %. Since
310 the P-value, in Table 6, is less than 0.05, there is a statistically significant relationship
311 between the variables with a confidence level of 95.0 %.

312 Related to the d_2 determination, a correlation is proposed (Equation 10). This
313 correlation includes the effect of the following parameters: the number of nozzle orifices
314 (z_0) and its diameter (d_0) of injectors, energizing time, injection pressure and the back
315 pressure downstream of the injector. The maximum error of d_2 predicted value is 0.217ms
316 in the longest energizations (2.5ms). The average absolute error was 0.02848 ms and the R-
317 squared was 99.40 %. Since the P-value in Table 6 is less than 0.05, there is a statistically
318 significant relationship between the variables with a confidence level of 95.0 %.

319

$$320 \quad d_2 \approx b_1 + b_2\sqrt{P_{inj} - P_{back}} + b_3\frac{d_0}{z_0} + b_4ET \quad (10)$$

321

322 Values of the coefficients used in both correlations, are shown in Table 6.

323 *Table 6. Coefficient of d_1 and d_2 correlation.*

324

325 To obtain the modelled fuel injection rate closer to the experimental one and since
326 the rate shape affects the spray penetration [29], it is necessary to take into account that the
327 actual rate signal is composed of three zones (Figure 7): i) first part of the rate with positive
328 slope; ii) second part, quasi-stationary rate with an almost constant value, and iii) third part,
329 rate with negative slope. The positive and negative slopes depend on the injection pressure
330 and the governing mechanism of the needle injector movement [20-25, 27, 30] for the same
331 fuel. Additionally, the use of different fuels could affect the slope and the maximum level
332 that reaches the rate at quasi-stationary zone [15].

333

334 *Figure 7. Experimental actual event of fuel injection process.*

335

336 For reproducing the slopes of the modelled rates of injection curves during their
337 increase (or decrease), data registered during the tests detailed in Tables 4 and 5 were used.
338 In order to simplify the model, a similar correlation was used for up and down slopes (with

339 positive or negative denotation). For reproducing the decrease of the rate of injection
340 curves, was used the same but with a negative sign.

341 The correlations for determining positive and negative slopes are presented in
342 equations 11 and 12, and their coefficients are presented in Table 7. The average absolute
343 error was 0.0922486 and the R-squared was 76.16 %. Since the P-value in Table 7 is less
344 than 0.05, there is a statistically significant relationship between the variables with a
345 confidence level of 95.0 %.

346

$$347 \quad \text{Slope}^+ \approx c_1 + c_2 \sqrt{P_{inj} - P_{back}} + c_3 \frac{d_0}{z_0} \quad (11)$$

348

$$349 \quad \text{Slope}^- \approx -c_1 - c_2 \sqrt{P_{inj} - P_{back}} - c_3 \frac{d_0}{z_0} \quad (12)$$

350

351 *Table 7. Coefficients of correlation for determination of positive and negative slopes.*

352

353 With the combination of the curves generated by both slope correlations (positive
354 and negative), a triangular rate of injection (TRI) is obtained as Figure 8 shows. The
355 maximum value of the experimental rate of injection signal is established by equation 13.

356

$$\dot{m}_f = C_d A_o \sqrt{2 \rho_f (P_{inj} - P_{back})} \quad (13)$$

357

358 Where C_d is the injector nozzle discharge coefficient, A_o is the total injection
359 discharge section area, ρ_f is the fuel density, P_{back} is the back pressure inside the fuel
360 indicator, and P_{inj} is the injection Pressure.

361 The closed area of the TRI may not match with the total mass injected measured
362 under the experimental test. Then, the model uses the total mass injected under the test to
363 achieve the rate of injection shape as follows:

- 364 • When the total mass obtained from TRI is less than the experimental value, the
365 model increases the slopes until matching both modelled and experimental injected
366 total mass. In this case, the modelled RoI is always triangular.

367 • When the total mass obtained from TRI is higher than the experimental value, the
368 model calculates a RoI using the slope correlations. However, the maximum value
369 that the modelled RoI can reach is limited by the maximum delivery for each
370 pressure. The value of maximum delivery for each pressure was calculated using the
371 equation 13. The slopes can be increased (until the mass corresponding to the
372 modelled rate coincides with the experimental mass), to avoid that the modelled RoI
373 never surpasses the maximum value.

374 For refining the coincidence between modelled and measured values under
375 trapezoidal RoI, the model smoothes the transition between the straight lines of the slopes
376 and the rate limit as Figure 8 shows.

377

378 *Figure 8. Simple modelled rates of fuel injection vs experimental rate.*

379

380 In the case of multiple injections, the model is capable of distributing the total mass
381 injected in each cycle, depending on the energization signal to obtain the mass of each
382 injection, before beginning the process of modeling the signal.

383 **4. RESULTS AND DISCUSSION**

384 *4.1. Comparison between modelled and experimental rates of injection.*

385 For a better comprehension of results obtained in this work, this section has been
386 divided into four sub-sections following the same order of the Figures 4, 5 and 6.

387 Figure 9 shows, as an example, the comparison between the modelled and the
388 experimental RoI with injector 7H150 from the four operating modes used for the model
389 validation (B, D, F and H modes). Results show that the model is able to reproduce the
390 shape and the quantity of fuel injected under split injection processes. The maximum shot
391 to shot difference between the modelled and the experimental RoI is 31 % and was
392 observed in F modes. However, the maximum cycle to cycle difference is lower than 1 %
393 and it decreases with the injected mass increase.

394

395 *Figure 9. Modelled and experimental rates of injection. Operating modes B, D, F and H,*
396 *were used for model validation.*

397

398 *4.2. Model application using the injector 8H138.*

399 Figure 10 presents results from 8H138 solenoid-operated injector in order to check
400 the model ability to reproduce rates of injection produced by another injector.

401

402 *Figure 10. Modelled and experimental rates of injection. Operating modes 3, 6 and 9, used*
403 *for model validation of the injector 8H138.*

404

405 In this case, experimental modes used for comparison were 3, 6 and 9 (See Table 5).
406 Modelled and experimental results obtained show that shot to shot differences are greater
407 than in the case of the 7H150 injector. However, cycle to cycle differences are lower than
408 0.5 %.

409 Summarizing, the cycle to cycle differences between modelled and experimental
410 RoI are acceptable while shot to shot differences could be questionable depending on the
411 final use of modelled RoI.

412

413 *4.3. Model application using GTL fuel and the injector 7H150.*

414 In order to demonstrate the capability of the model with different fuel, similar tests
415 to those presented in Figure 9, were carried out with GTL fuel in 5 modes (modes used to
416 approach the model with diesel). Figure 11 shows that the model reproduces the RoI with
417 GTL fuel with similar trends as to when Diesel fuel was tested.

418

419 *Figure 11. Modelled and experimental rates of injection. Operating modes A, E and I, used*
420 *for model approach and GTL fuel.*

421

422 *4.4. Application to thermodynamic diagnosis*

423 As presented in Figure 6, modelled and experimental rates of injection were used as
424 input data in a zero-dimensional thermodynamic model in order to obtain comparative
425 results of the in-cylinder gas mean temperature and both the rate of heat release (RoHR)
426 and the heat release law (HRL). In the thermodynamic model used, the in-cylinder
427 instantaneous mean temperature of the gas (T_i) is calculated using the known gas state
428 equation 14:

429

430
$$\frac{p_i V_i}{m_i R_i} = T_i \quad (14)$$

431

432 Where p_i is the in-cylinder gas pressure, V_i is the in-cylinder calculated volume, m_i is the
 433 instantaneous in-cylinder trapped mass, R_i is the instantaneous specific constant of the In-
 434 cylinder gas.

435 The thermodynamic model works with three species: intake air, evaporated fuel and
 436 stoichiometric burnt products [14, 27]. The introduction of the evaporated fuel as specie,
 437 has two direct effects on the T_i : i) in-cylinder mass will change along the injection process
 438 and ii) R_i will change along the thermodynamic cycle according to the evolution of the
 439 mass fractions (Y_{xi}) of the mentioned species as equation 15 shows.

440

441
$$R_i = R_a Y_{ai} + R_f Y_{fi} + R_b Y_{bi} \quad (15)$$

442

443 Where R_a is the specific air constant, R_f is the specific constant of the evaporated fuel and
 444 both are constant values [27] while R_b (specific constant of burnt products) depends on the
 445 temperature of the products as can be seen in Figure 12.

446 This result was obtained by means of a model which calculates the composition of
 447 the burnt products from a reaction with stoichiometric fuel-air ratio and, in consequence,
 448 the relative fuel-air ratio $Fr = 1$ with air and a diesel fuel with a mean molecular
 449 composition $C_{10.8} H_{18.7}$. The model used was adapted from the proposed model by Way
 450 [31]. The model considers 12 chemical species (including dissociation products) and
 451 establishes 5 atomic balances and 7 characteristic equilibrium reactions.

452

453 *Figure 12. Variation of the specific constant of stoichiometric burnt products [32].*

454

455 Variations of the gas constant of the burnt products with temperature are significant
 456 when it is high enough for dissociation to occur, which leads to the appearance of
 457 monoatomic species, with low molecular weight. However, as the T_i is never high enough
 458 to expect a significant increase in the gas constant, it is acceptable to adopt a fixed value for
 459 this constant, as for air and fuel (equation 16).

460

$$R_f = \frac{R_\mu}{MW_f} = 55.95 \text{ J/kgK}$$

$$R_a = \frac{R_\mu}{MW_a} = 287 \text{ J/kgK} \quad (16)$$

$$R_b = \frac{R_\mu}{MW_b} = 285.4 \text{ J/kgK}$$

461

462 Where MW_f , MW_a , MW_b are molecular weights of fuel, air, and burnt products,
 463 respectively, and R_μ is the universal constant.

464 Figure 13 shows, as an example, the evolution of the air, evaporated fuel and burnt
 465 products mass fractions as a function of crank angle under a combustion engine with the
 466 experimental rate of fuel injection test and under a combustion engine without the RoI. In
 467 this case, the test was carried out with split injection.

468

469 *Figure 13. Evolution of the air, evaporated fuel and stoichiometric burnt products mass*
 470 *fractions a) under a combustion engine without the rate of fuel injection and b) under a*
 471 *combustion engine with a split rate of fuel injection.*

472

473 Figure 14 shows the angle evolution of the R_i calculated with and without taking
 474 into account the introduction of fuel into the combustion chamber of the engine, using the
 475 same data of the test presented in Figure 13.

476

477 *Figure 14 Evolution specific constant of the in-cylinder gas with split injection.*

478

479 As can be seen in Figure 15, the instantaneous R_i reduction does not produce a
 480 decrease of the calculated T_i because of the effect of the increase of the in-cylinder mass is
 481 higher than the reduction of the R_i on the calculated gas temperature.

482

483 *Figure 15 Evolution of the in-cylinder gas mean temperature with split injection.*

484

485 The effect of introducing the rate of injection as input data in the thermodynamic
 486 diagnosis has been evaluated in three possible scenarios (using the experimental RoI, the

487 modelled RoI and without the RoI), in order to evaluate the differences produced on T_i ,
 488 RoHR, and HRL.

489 Figure 16 shows (from top to the bottom) the RoI curves (modelled and
 490 experimental) and the in-cylinder pressure signal (input file for the diagnostic model). Also,
 491 the T_i , the RoHR, and the HRL versus crank angle were calculated with the RoI (modelled
 492 and experimental) and without the RoI, as thermodynamic diagnosis results, from the
 493 operating mode I with diesel fuel as an example. Values of T_i calculated without the RoI
 494 are higher than those obtained considering the RoI (modelled and experimental), as it was
 495 shown in Figure 15.

496 The thermodynamic model used in this work calculates the RoHR by means of the
 497 equation 17 [27]:

498

$$m_i du_i + \left(u_b - \frac{u_f \left(\frac{f}{a}\right)_{st} + u_a}{\left(\frac{f}{a}\right)_{st} + 1} \right) (dm_b + Y_b dm_{bb}) =$$

$$-p_i dV_i - dQ_w + (h_{finj} - u_f) dm_{fev} - R_i T_i dm_{bb} \quad (17)$$

500

501 Where u_i is the internal energy of the in-cylinder trapped mass, u_b is the internal energy of
 502 the stoichiometric burnt products, u_f is the internal energy of evaporated fuel, $\left(\frac{f}{a}\right)_{st}$ is the
 503 stoichiometric fuel to air ratio, u_a is the internal energy of the inlet air, m_b is the mass of
 504 stoichiometric burnt products, Y_b is the stoichiometric mass fraction of the burnt products,
 505 m_{bb} is the mass of blow-by, Q_w is the heat transferred to cylinder walls, h_{finj} is the
 506 enthalpy of fuel under the injection and m_{fev} is the mass of the evaporated fuel.

507 Values of the RoHR were calculated using the equation 17 (left term of the
 508 equation). Curves related to this parameter, and their cumulative values (HRL), were lower
 509 when the RoI is not taken into account since equation 17 is solved without the effect of the
 510 fuel introduction as a specie in the thermodynamic model.

511 Also, Figure 16 shows that the thermodynamic model results with the modelled and
 512 experimental RoI have reasonable accuracy. The shot to shot and cycle to cycle differences

513 between modelled and experimental rates of injection are acceptable for their use in
514 thermodynamic diagnosis.

515

516 *Figure 16. Thermodynamic diagnosis of the combustion process with diesel fuel from*
517 *operating mode I.*

518

519 Results presented confirmed that the use of the proposed model for estimating the
520 rates of injection, as input data in thermodynamic diagnosis under different engine
521 operating modes and fuels, has high accuracy when determining the T_i , the RoHR, and the
522 HRL compared to the thermodynamic diagnosis with the experimental RoI used as input
523 data. These results also confirm the possibility to use this empirical, simple but also a
524 robust zero-dimensional model for calculating instantaneous introduction of fuel in the
525 combustion chamber as a species together with air and burnt products along the
526 thermodynamic cycle.

527

528 **6. CONCLUSIONS**

529 In this work, empirical, simple and robust correlations for determining the delay
530 time between SoE and SoI signals, obtained from the two solenoid-operated diesel
531 injectors, are proposed. The absolute error of the correlations for determining the delay
532 time between SoE and SoI is ± 0.05 ms, this means around 10% of the difference with the
533 mean value of the delay time measured in all tested situations.

534 The model has demonstrated its ability to reproduce the rates of injection, with
535 different solenoid operated injectors, under different situations: i) under single or split
536 injections, ii) under different injection pressure and/or energizing time and/or iii) with
537 different fuel properties within the range established by standards EN590 and EN15940 for
538 diesel and GTL fuels, respectively.

539 Modelled and experimental results obtained show that cycle to cycle differences are
540 lower than 1% independently of the injector model and the fuel tested. These results are
541 acceptable for their use in thermodynamic diagnosis. However, shot to shot differences
542 could be questionable depending on the final use of the modelled RoI, as for example in
543 fuel spray modelling.

544 The use of modelled rates of injections, as input data in thermodynamic diagnosis
545 under different engine operating modes and fuels, has similar accuracy when determining
546 the T_i , the RoHR, and the HRL.

547 Summarizing, this proposed model can be a useful and alternative tool for
548 estimating rates of injection at any operation engine mode and with any calibration thereof,
549 without the need to carry out a test of the rate of injection. Although the differences
550 between shot to shot are sometimes high, do not affect directly the thermodynamic
551 diagnosis. However, the slopes could be calculated more accurately, the model could be
552 improved in this sense in future works.

553

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562

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- 663

664 NOMENCLATURE / ABBREVIATIONS

665 A Amps

666	A_o	Total injection discharge section area
667	$B(T)$	Tait parameter
668	C_d	Injector nozzle discharge coefficient
669	d_1	Time lag between SoE and SoI
670	d_2	Time lag between EoE and EoI
671	D_T	Time lag between injections
672	EoE	End of energizing time
673	EoI	End of injection time
674	ET	injector energizing time
675	Fr	Relative fuel-air ratio
676	$\left(\frac{f}{a}\right)_{st}$	Stoichiometric fuel-air ratio
677	GTL	Gas To Liquid fuel
678	h_{finj}	Enthalpy of injected fuel
679	HRL	Heat release law
680	IRI	Injection rate indicator
681	m	Main injection
682	m_i	Instantaneous In-cylinder trapped mass
683	m_b	Mass of stoichiometrically burnt products
684	m_{bb}	Mass of blow-by
685	m_{fev}	Mass of evaporated fuel
686	\dot{m}_f	Fuel mass flow rate
687	m_f	Total fuel mass injected
688	MW_a	Molecular weights of air
689	MW_b	Molecular weights burnt products
690	MW_f	Molecular weights of fuel
691	$NEDC$	New European Driving Cycle
692	p	Preinjection
693	P_0	Reference pressure
694	P_{rp}	Back pressure of injector fuel return
695	P_{back}	Back pressure inside the fuel indicator
696	p_i	In-cylinder gas pressure
697	P_{inj}	Injection pressure
698	ρ_f	Fuel density

699	Q_w	Heat transferred to cylinder walls
700	Ra	Specific air constant
701	Rb	Specific constant of burnt products
702	R_i	Instantaneous specific constant of the In-cylinder gas
703	R_e	Reynolds number
704	Rf	Specific constant of the evaporated fuel
705	Rμ	Universal constant
706	RoHR	Rate of heat release
707	RoI	Rate of fuel injection
708	SoE	start of injector energizing
709	SoI	Start of injection
710	T0	Reference temperature
711	TRI	Triangulate model of the rate of injection
712	T_i	Instantaneous in-cylinder mean gas temperature
713	t_{iny}	Fuel injection duration
714	u_a	Internal energy of air
715	u_i	Internal energy in-cylinder trapped mass
716	u_b	Internal energy of stoichiometrically burnt products
717	u_f	Internal energy of fuel
718	μ_f	Fuel dynamic viscosity
719	V	Volts
720	V_i	In-cylinder calculated volume
721	ν_f	Fuel kinematic viscosity
722	Y_{xi}	Mass fractions
723	Y_b	Stoichiometrically mass fraction burnt products
724	0D	Zero-dimensional model
725	1D	One dimensional model
726		

Figures

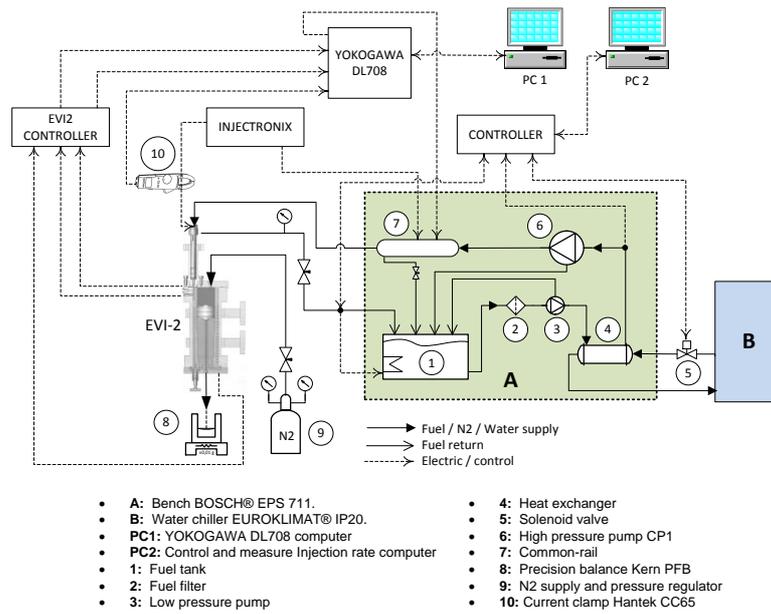


Figure 1. Simple functional scheme experimental installation.



Figure 2. Example of image from the tip of injector obtained by means X-ray analysis.

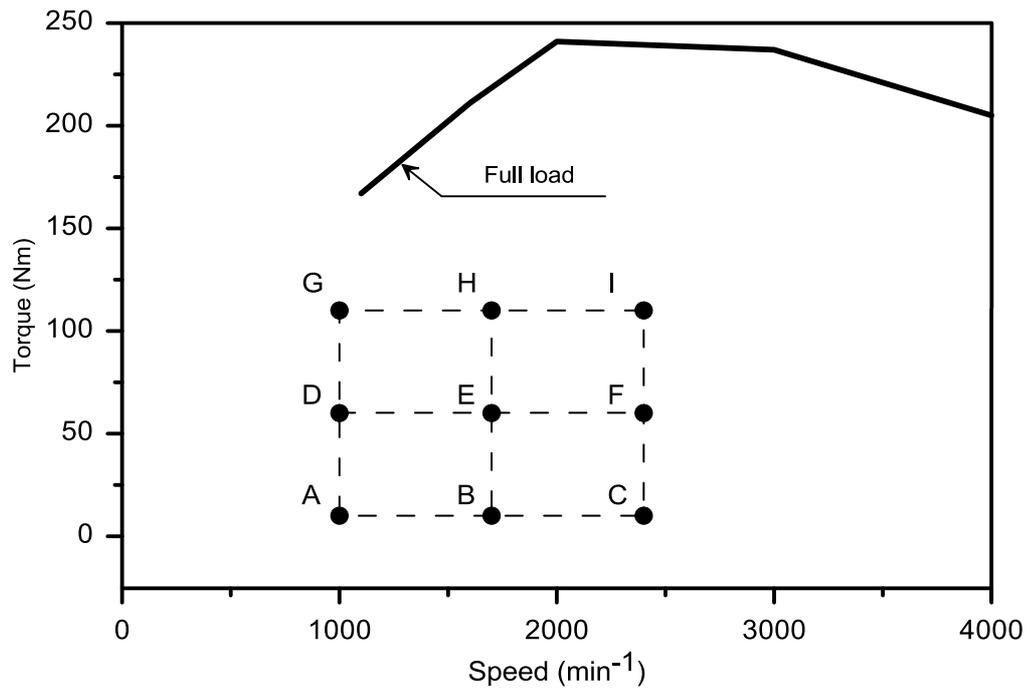


Figure 3. Engine operating modes used for OD model input data with 7H150 injector.

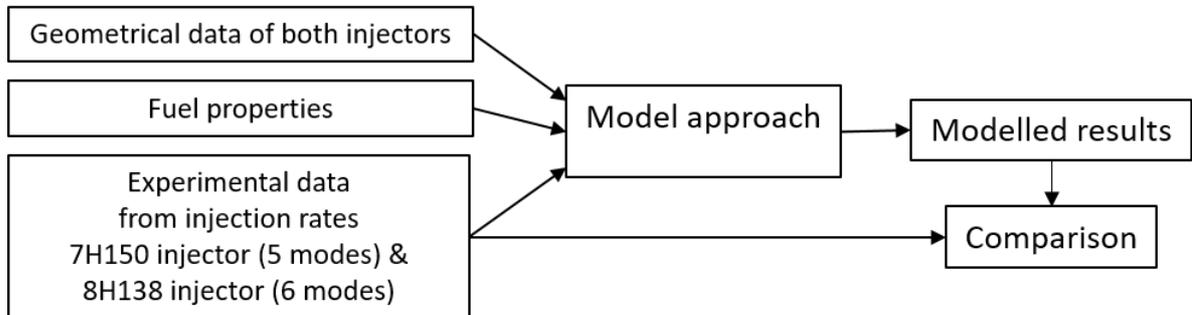


Figure 4 Methodology for model approach.

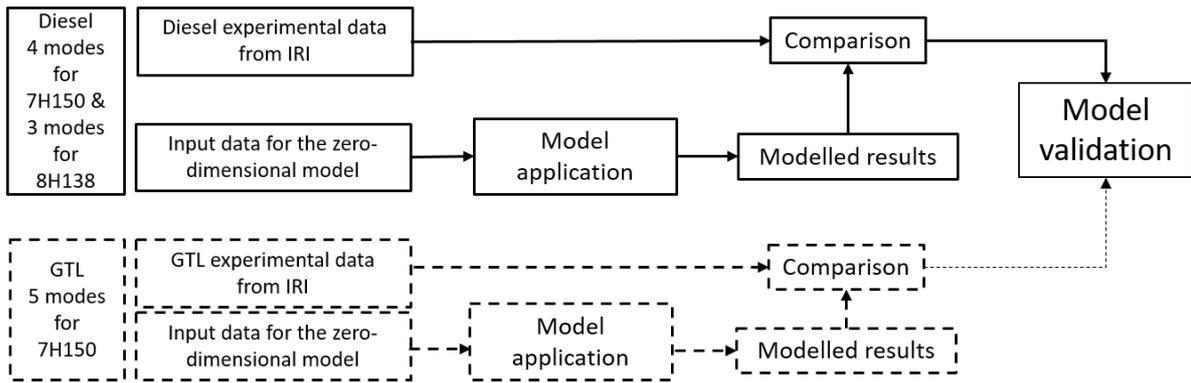


Figure 5 Methodology for model approach and its validation with different fuels.

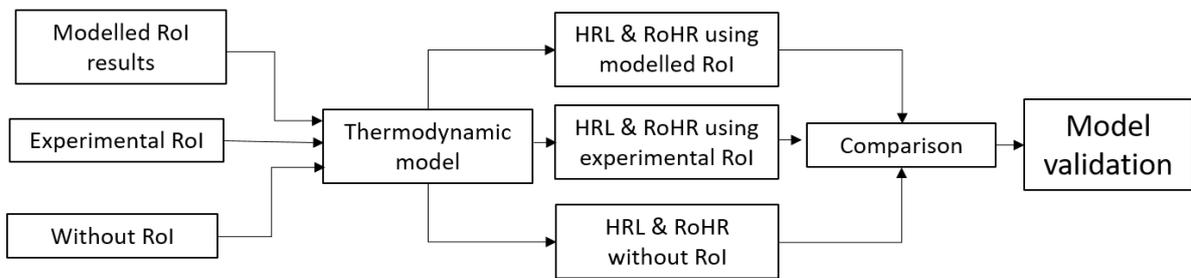


Figure 6 Methodology to compare thermodynamic diagnosis results.

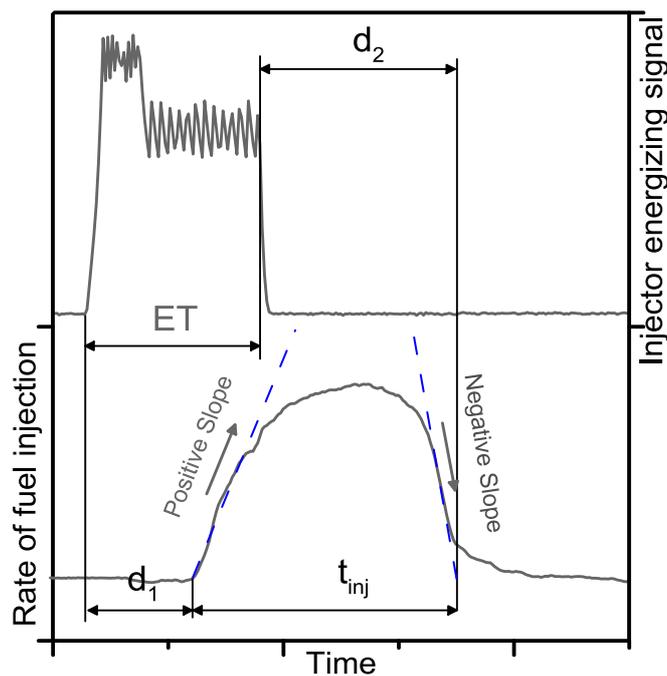


Figure 7. Experimental actual event of fuel injection process.

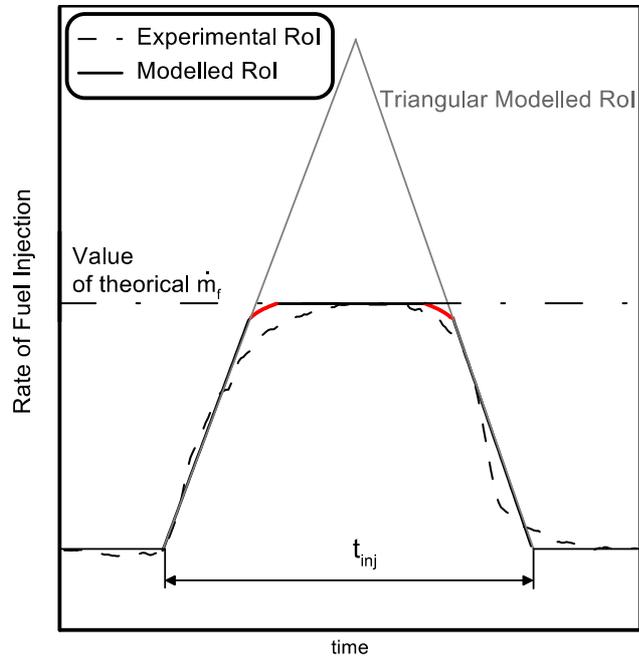


Figure 8. Simple modelled rates of fuel injection vs experimental rate.

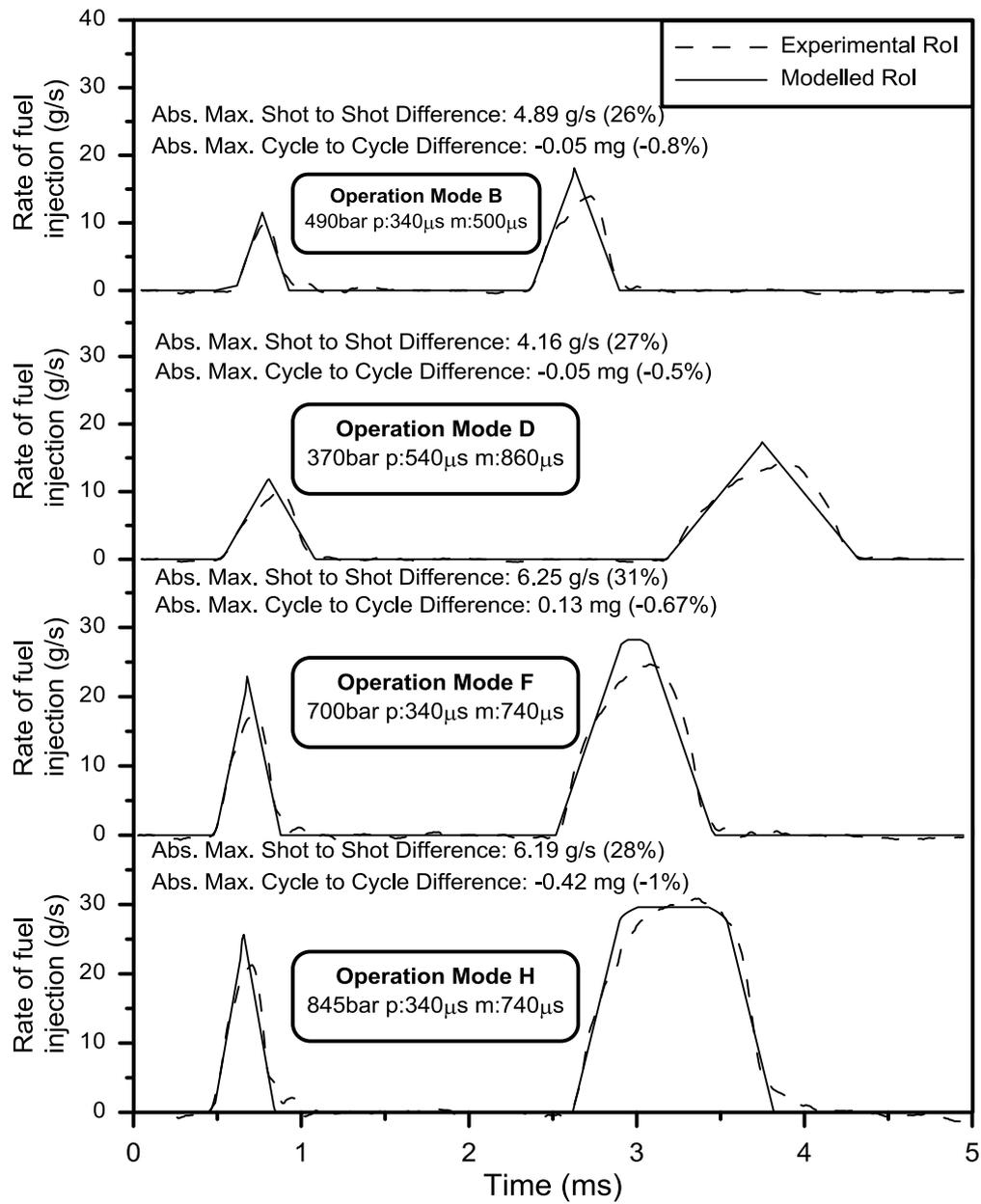


Figure 9. Modelled and experimental rates of injection. Operating modes B, D, F and H, were used for model validation.

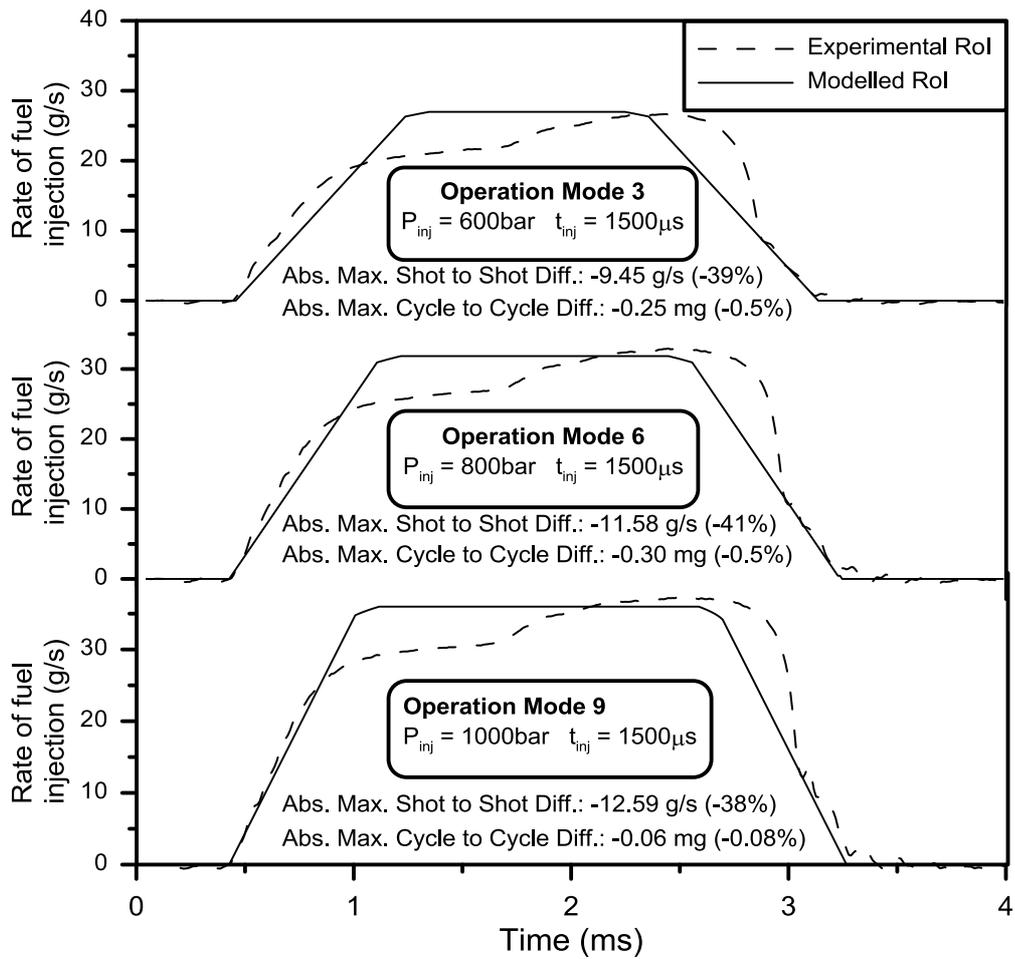


Figure 10. Modelled and experimental rates of injection. Operating modes 3, 6 and 9, used for model validation of the injector 8H138.

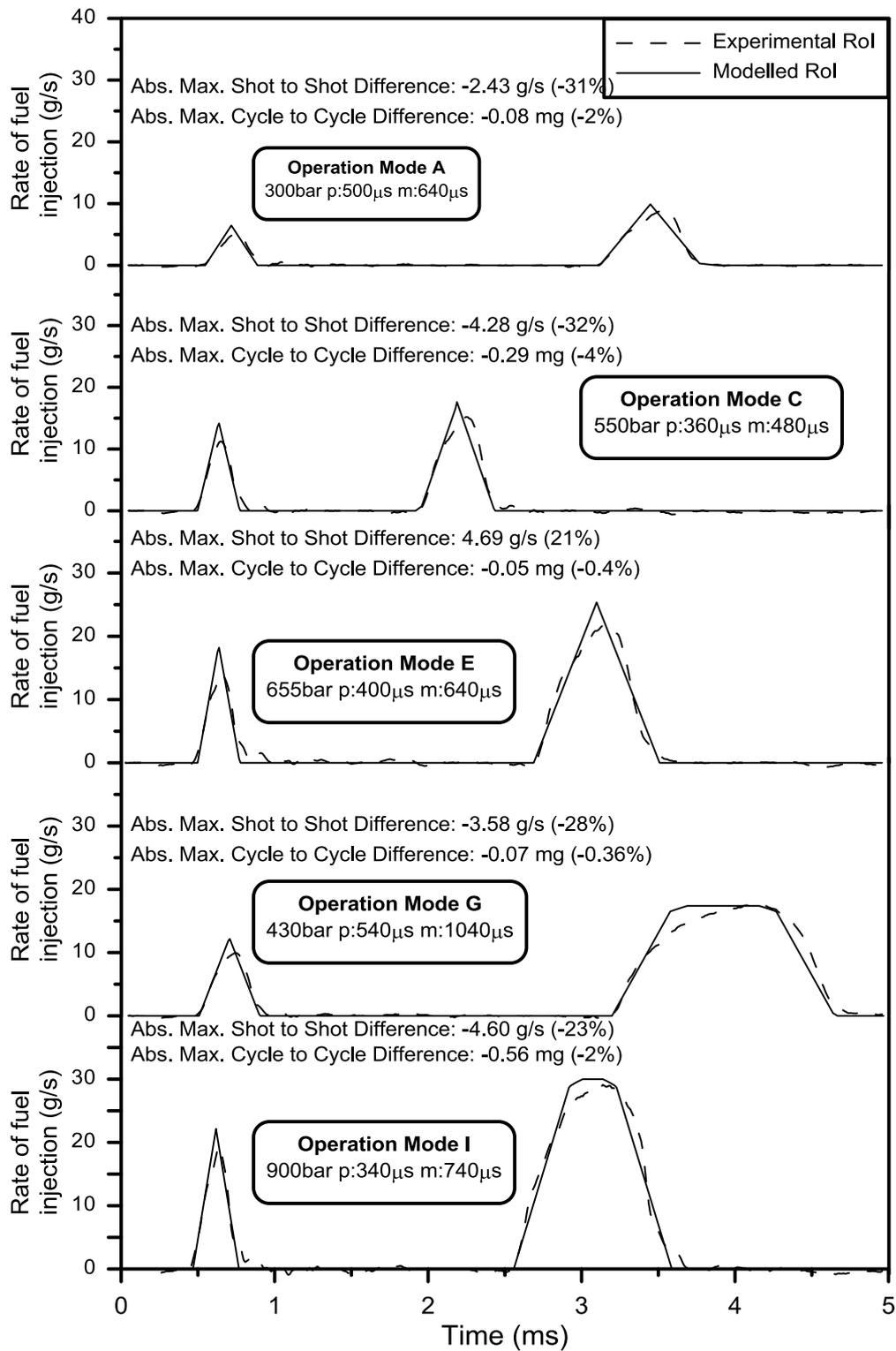


Figure 11. Modelled and experimental rates of injection. Operating modes A, E and I, used for model approach and GTL fuel.

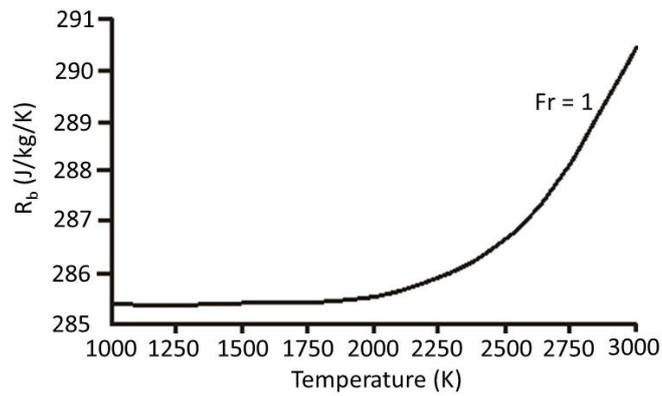


Figure 12. Variation of the specific constant of stoichiometric burnt products [31]

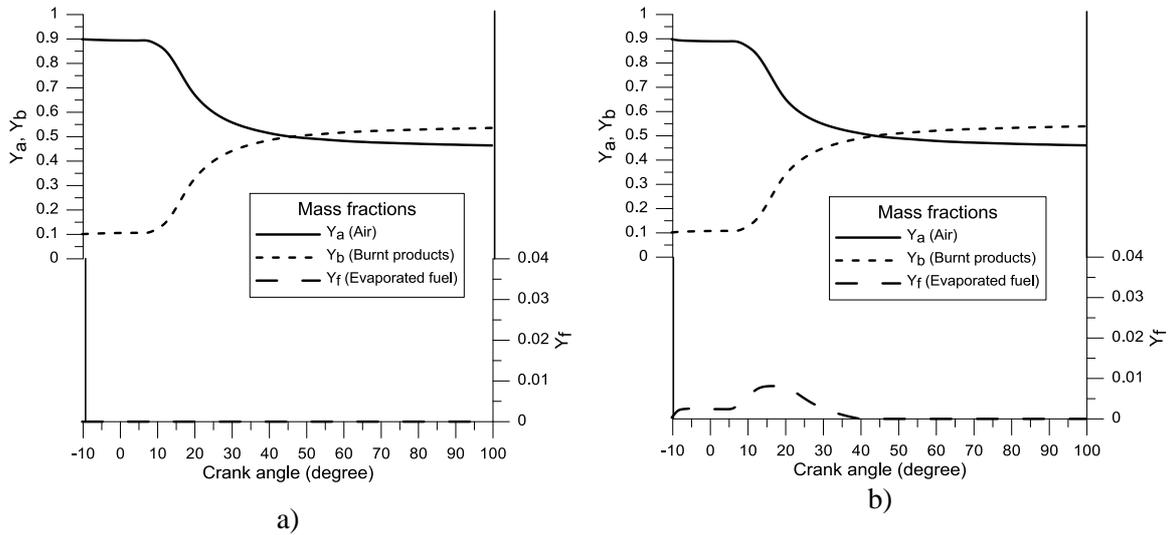


Figure 13. Evolution of the air, evaporated fuel and stoichiometric burnt products mass fractions a) under a combustion engine without rate of fuel injection & b) under a combustion engine with a split rate of fuel injection.

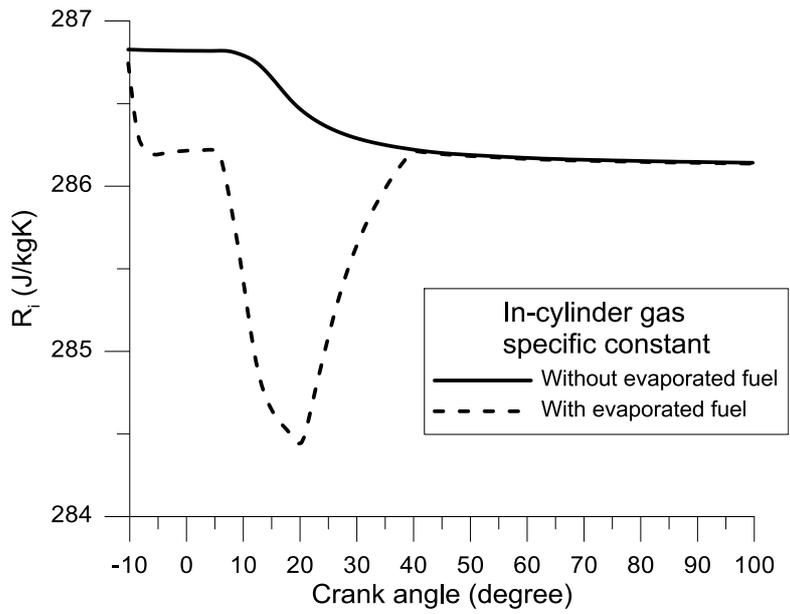


Figure 14 Evolution specific constant of the in-cylinder gas with split injection.

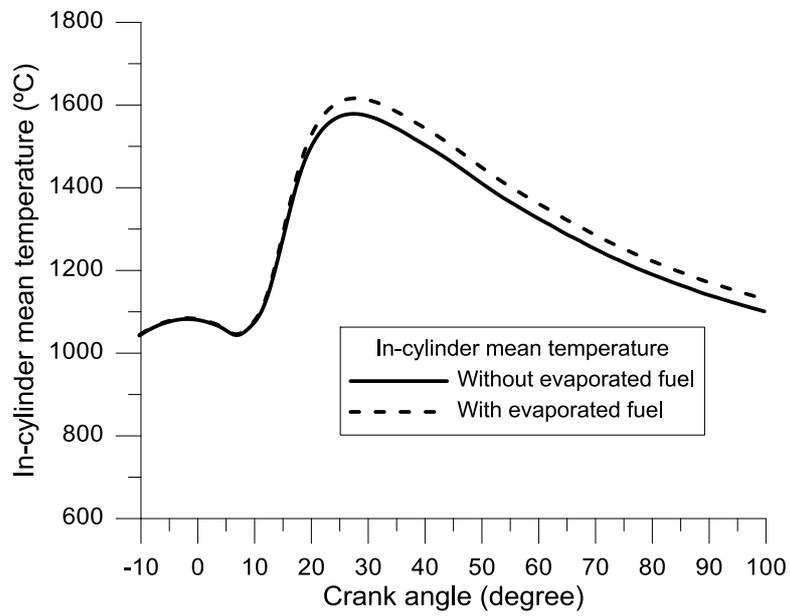


Figure 15 Evolution of mean temperature In-cylinder gas with split injection.

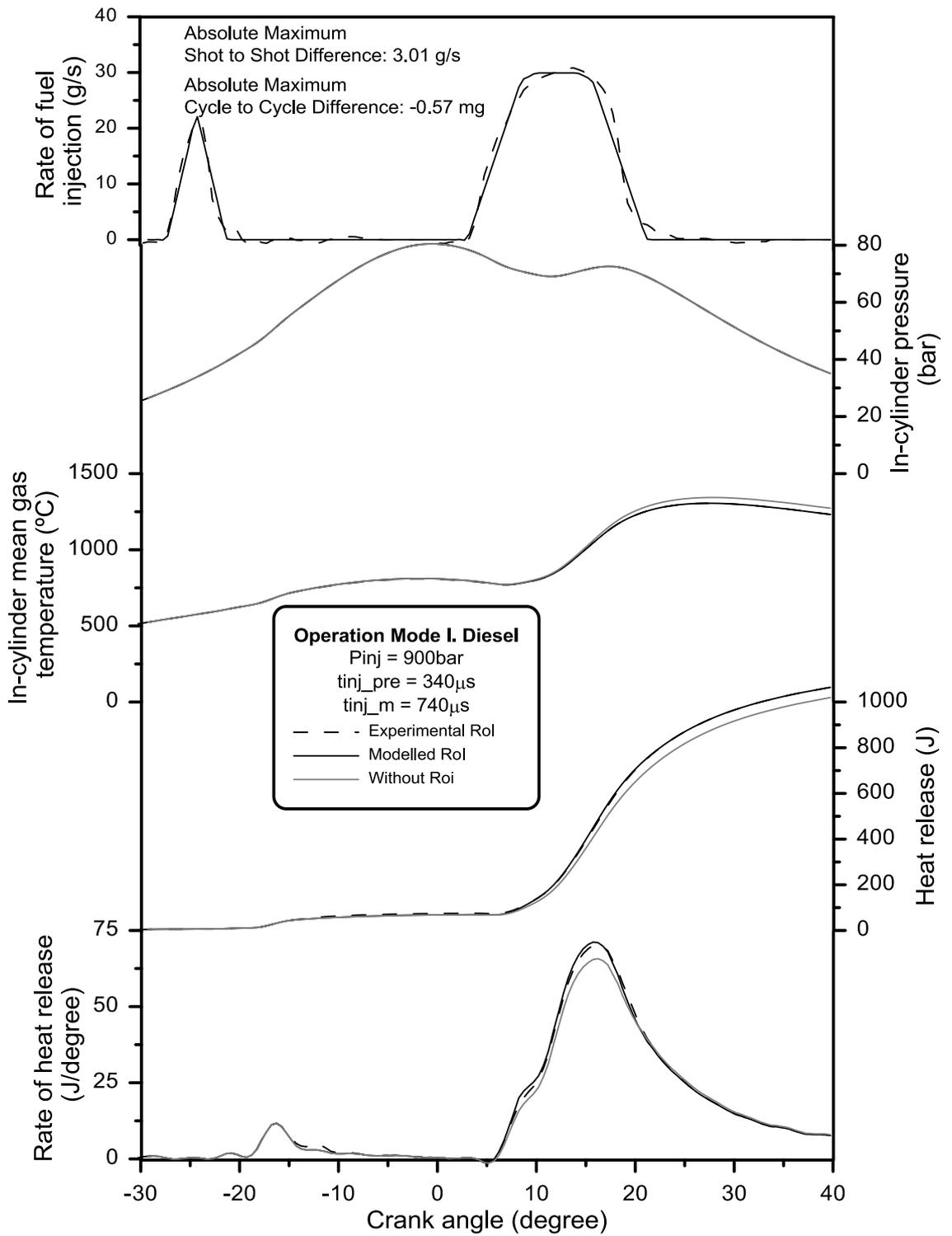


Figure 16. Thermodynamic diagnosis of the combustion process with diesel fuel from operating mode I.

Tables*Table 1. Engine characteristics.*

Type	Nissan YD22, 2.184 l, turbocharged + intercooler
Fuel injection	Common rail, pilot injection
Maximum power	82kW at 4000min ⁻¹
Maximum torque	248Nm at 2000min ⁻¹
Cylinder arrangement	4 cylinders, in line
Bore/stroke	86.5/94mm
Compression ratio	16.7:1

Table 2. Fuel properties.

<i>Properties</i>	<i>diesel</i>	<i>GTL</i>
Molecular Formula	$C_{15.18}H_{29.13}$	$C_{16.89}H_{35.77}$
Molecular weight (g/mol)	211.4	238.6
H/C Ratio	1.92	2.12
Stoichiometric fuel–air ratio	1/14.64	1/14.95
Low heating value (MJ/kg)	42.43	43.86
Cold filter plugging point (°C)	-17	-7
Lubricity (WS1.4) (μm)	333	330
Cetane number	54.2	73

Table 3. Kinematic viscosity and density of fuels tested at different temperatures with atmospheric pressure.

<i>Temperature (°C)</i>	<i>ν_f at Patm (cSt)</i>		<i>μ_f at Patm (cPo)</i>		<i>ρ_f at Patm (kg/m³)</i>	
	<i>diesel</i>	<i>GTL</i>	<i>diesel</i>	<i>GTL</i>	<i>diesel</i>	<i>GTL</i>
15	5.24	4.66	4.43	3.60	845	773.5
25	4.64	3.91	3.90	3.00	840	767
40	3.55	2.97	2.94	2.25	827	757.5

Table 4. Main characteristic parameters of injection for split injection with Denso 7H150 injector (9 engine modes).

Mode	Engine speed (min^{-1}) - Torque (Nm)	Injection pressure (bar)	ET (μs)		Dwell (μs)	P_{back} (bar)
			Pre injection	Main injection		
A*	1000 – 10	300	500	640	2085/2585	60
B	1700 – 10	490	340	500	2130/2670	
C*	2400 – 10	550	360	480	2130/2670	
D	1000 – 60	370	540	860	1425/1765	
E*	1700 – 60	655	400	640	1100/1460	
F	2400 – 60	700	340	740	1855/2255	
G*	1000 – 110	430	540	1040	1655/2015	
H	1700 – 110	845	340	740	2015/2355	
I*	2400 – 110	900	340	740	1815/2155	

* Modes used to model approach

Table 5. Main injection characteristic parameters for single injection using Denso 8H138 injector.

Mode	Injection pressure (bar)	Energizing time (μs)	P_{back} (bar)
1*	600	500	80
2*		1000	
3		1500	
4*	800	500	95
5*		1000	
6		1500	
7*	1000	500	90
8*		1000	
9		1500	

* Modes used to model approach

Table 6. Coefficients of d_1 and d_2 correlations.

<i>Coefficients</i>	<i>Values for d_1</i>	<i>P-value</i>	<i>Coefficients</i>	<i>Values for d_2</i>	<i>P-value</i>
a ₁	1.02565	0.0000	b ₁	0.60913	0.0000
a ₂	-0.00420	0.0000	b ₂	0.01194	0.0000
a ₃	0.00327	0.0000	b ₃	-0.03598	0.0000
a ₄	-0.07541	0.0000	b ₄	0.89404	0.0000
a ₅	0.00632	0.0002	-	-	-

Table 7. Coefficients of correlation for determination of positive and negative slope.

<i>Coefficients</i>	<i>Value Slope</i>	<i>P-value</i>
c ₁	2.2205	0.0000
c ₂	-0.0513	0.0000
c ₃	-0.0894	0.0000