Connection Science
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/ccos20

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Published online: 14 Apr 2014.

To cite this article: Sergio Miguel-Tomé & Antonio Fernández-Caballero (2014): On the identification and establishment of topological spatial relations by autonomous systems, Connection Science, DOI: 10.1080/09540091.2014.906389

To link to this article: http://dx.doi.org/10.1080/09540091.2014.906389

Connection Science, 2014
http://dx.doi.org/10.1080/09540091.2014.906389

On the identification and establishment of topological spatial relations by autonomous systems

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(*Received 19 July 2013; accepted 28 February 2014*)

Human beings use spatial relations to describe many daily tasks in their language. For a mobile robot to be useful in daily life, it is necessary to have navigation algorithms capable of identifying and establishing spatial relations. To date in robotics, the navigation problem has been thoroughly researched as a task of guiding a robot from one spatial coordinate to another. Therefore, there is a difference in degree of abstraction between the language of human beings and the algorithms used in robot navigation. This article introduces a piece of research performed on the use of topological relations for the formalisation of spatial relations and navigation. So far, topological relations have been applied widely in geographical information systems and also in spatial logics. There are some proposals in robot navigation which use them for planning but there is no research about making decision in robot navigation. Our research focuses on decision-making methods to establish spatial relations. The main result is a new heuristic, called the Heuristic of Topological Qualitative Semantics (HTQS), which allows the identification and establishment of spatial relations decision-making from a set of actions. To demonstrate its effectiveness, HTQS has been implemented in the form of agents that can move in a two-dimensional virtual environment. HTQS opens a new door to designing algorithms for navigation based on the identification and establishment of spatial relations.

Keywords: qualitative navigation; spatial relations; Heuristic of Topological Qualitative Semantics

1. Introduction

Robots have proven to be useful tools for police officers, surgeons and cleaning staff. However, in each of these cases, the robots have been designed for specific tasks. Currently, the construction of multifunctional robots is being studied: robots able to navigate in environments where flexible behaviour is required such as offices or homes, and which are able to work directly with humans in various activities. Up to now, many successes in robotics have been linked to the reactive paradigm (Brooks, 1986). However, it seems difficult for the reactive paradigm to enable the development of multifunctional robots, as their manner to interact in the environment will constantly change, depending on the task at hand. It has thus been proposed that adequate decision-making in heterogenous environments can only be made possible by means of collecting information from the environment (Miguel-Tomé, 2011). It should also be kept in mind that the natural way of communicating spatial tasks by humans is by making use of spatial relations. A multifunctional robot would continually receive tasks such as:

Take the package that is on the table and leave it in the closet.

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Given this, it would seem that it is necessary for a multifunctional robot capable of functioning in human environments to possess a high degree of spatial reasoning, and more specifically, about qualitative spatial relations. The ability of a robot to move around and interact in an urban environment depends on the capacity to perform tasks such as capturing the environment, self-localisation, navigation, planning and natural language processing. The processes involved in these tasks can communicate with each other through the robot's spatial representation. Data collected by the sensors, e.g. Fernández-Caballero, Castillo, Martínez-Cantos, and Martínez-Tomás (2010) and Pavón, Gómez-Sanz, Fernández-Caballero, and Valencia-Jiménez (2007), are used to form a representation of the space, and the self-localisation method calculates the position of the robot in the representation, the navigation and planning methods that calculate the movements from the representation, and natural language processing converts the voice signal into a sentence on the representation.

Depending on the complexity of the environment and the objective assigned to a robot, the architecture for decision-making may be divided into three levels: execution, navigation[1] (or local navigation) and planning (or global navigation). In each of these levels, the methods can use either quantitative or qualitative spatial representations (Shi, Wang, & Yang, 2010; Wolter & Lee, 2010). The possible combinations of levels and types of methods give rise to different types of architectures, as shown in Figure 1. The most investigated architecture in robotics is I (shown in the diagram). This architecture is useful for precision industrial applications, but is not appropriate in the case of interaction with humans. As all spatial representations of this architecture are quantitative, it is impossible for the robots to understand commands as shown in the example above. This problem when coupled with the need to manage uncertainty, gave birth to the research into decision-making methods for qualitative navigation (Levitt & Lawton, 1990; McDermott & Davis, 1984). The development of qualitative methods generates the emergence of alternative architectures. After the first years of research into qualitative methods, the difficulty of creating qualitative methods for navigation gave rise to 'the poverty conjecture' (Forbus, Nielsen, & Faltings, 1991). If this conjecture is true, it implies that creating type III architectures is impossible. Although, according to some authors, the recent research has weakened the poverty conjecture (Cohn & Renz, 2008). Even so, until now, only type II architecture has been successfully developed. Therefore, to create multifunctional robots, new qualitative methods of navigation are required and the capacity to reason about space has to be increased. It is therefore necessary to create new methods to make use of spatial relations.

The formalisation of spatial relations has been investigated in other disciplines as well as robotics, such as logic and geography. In geography, one of the most frequent ways to formalise spatial relations has been by means of topological relations. Topological relations have allowed geographic information systems (GIS) to recognise and analyse the spatial relations that exist in stored geographic information and, in turn, spatial relations allow complex spatial modelling and

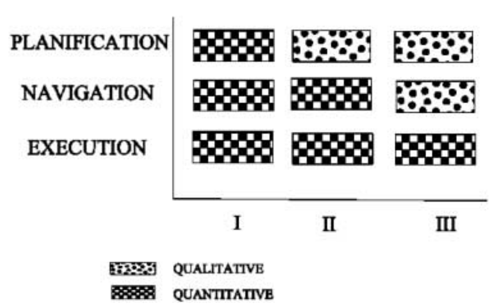

Figure 1. A robot must make decisions at various levels. Depending on the type of method used by the robot, a type of architecture for decision-making is obtained.

analysis. Furthermore, psychological research indicates that topological relations are an important part of human spatial knowledge (Renz, 2002).

In the fields of logic and symbolic paradigm of artificial intelligence (AI), research in depth into the problems of satisfiability of spatial configurations and deduction of spatial relations has been carried out (Li & Li, 2004; Randell, Cui & Cohn, 1992; Renz, 2002; Renz & Nebel, 1999; Schockaert, De Cock, & Kerre, 2009), satisfiability of spatial configurations of temporal intervals and deduction of temporal relations (Allen, 1983; Freksa, 1992). In the very early symbolic paradigm, planning algorithms attempting to satisfy spatial configurations has been researched (Fikes & Nilsson, 1971). However, only recently decision-making algorithms for local navigation have been begun to be research (Dylla et al., 2008).

1.1. *A new research programme: multifunctional robots on topological notions*

Based on the above, we believe that within the symbolic paradigm, it is possible to develop decision-making algorithms founded on topological relations and apply these to navigation and other tasks. We thus propose the development of a research programme which studies the usefulness of topological notions for navigation and leads to the creation of multifunctional robots. The programme will be called, Multifunctional Robots On Topological Notions (MROTN). The hypothesis behind the programme is that topological relations are a useful and effective tool in creating a multifunctional robot. The attainment of that goal only can be achieved by attaining numerous subgoals. At the beginning of the research programme, issues such as uncertainty, multiagents or processing of natural language cannot be addressed because the initiation of a new line of research implies the necessity of studying methods for more simple environments. It is initially necessary to successfully develop and understand some concepts in simple environments before confronting the problems posed by more complex environments. The first goal in our research programme is thus to achieve algorithms which can perform navigation based on identifying and establishing spatial relations in simple environments as happened historically in the case of AI and robotics in other lines of research. The reader should thus note that although these methods can only be used in simple environments, this is not because the research is inadequate but because there is no existing background for the new line of research and further research is thus needed in order to confront the creation of the basis of the new line of research. Given the above, simple environments are needed at first.

In this article, we further develop the first results about identifying and establishing spatial relations through topological notions which were presented in IWINAC 2013 (Miguel-Tomé, 2013) and earlier in Miguel-Tomé (2008). In Section 2, spatial representation methods used in AI and their application are examined. Then, Section 3 deals with decision-making to establish a topological relation and introduces the Heuristic of Topological Qualitative Semantics (HTQS). Later, Section 4 describes a method for identifying topological relations. Then, Section 5 shows the tests performed by agents using heuristics for the navigation of agents in a virtual world. Finally, in Section 6, our results and future projects are outlined.

2. Representation of space for navigation

As previously mentioned, spatial representations can be divided into two groups: quantitative and qualitative (Wolter & Lee, 2010). In turn, methods of robot decision-making can also be divided into quantitative and qualitative, depending on the type of spatial representation used. In this section, we examine the representation methods used in AI and their application.

2.1. *Quantitative methods*

In quantitative methods, space is always represented as a metric space. The quantitative problem of mobile robot navigation is broadly defined as follows:

> Given a starting point A reach destination point(s) B $(B_1, B_2, \ldots)$ by using [the robot's] knowledge and the [robot's] sensory information received.

This problem has been the most extensively investigated problem in robotics. The algorithms developed to solve it make a robot move from one to another numeric spatial coordinates (Choset et al., 1995). The great advantage of quantitative navigation methods is that with only one level in the spatial representation different tasks can be performed. For example, the information captured by the sensors can be used directly to build a representation of space, the method of local and global navigation can be directly used for the representation of space, and the localisation method directly calculates the position of the robot in the representation (e.g. odometry, global positioning system, inertial navigation, lights, landmarks). Within quantitative navigation, the methods of spatial representation can be divided into two classes: one robot one point (OROP), and, one robot many points (ORMP).

The OROP representation approach is used to navigate to a point in space with robots without articulations and whose shape remains constant. In this approach, the whole robot is represented as a single point in space. In the case that a detailed map of the environment is available, algorithms of potential function (Newman & Hogan, 1987), force (Borenstein & Koren, 1991), harmonic functions (Masoud, 2010) and evolutionary approaches (Kala, Shukla, & Tiwari, 2010, 2012), among others, are covered by this approach. In cases where there is no map to navigate, reactive algorithms such as Bug (Kamon, Rimon, & Rivlin, 1998; Lumelsky & Stepanov, 1987) are often used. Bug algorithms are based on a reaction to the environment that allows the robot to avoid obstacles when navigating towards a given destination. The great advantage of this method of representation is that the algorithm only has to run the calculations for a given point in space. Obviously, a mobile robot does not occupy a point in space. Therefore, it is usual to push the boundaries of obstacles to prevent or avoid collisions, or every point in space is equivalent to the surface of the robot.

The ORMP representation approach is usually chosen because the robot used has articulations and the complexity of the environment requires planning of articulation trajectories (Chestnutt et al., 2005; Kuffner, Nishiwaki, Kagami, Inaba, & Inoue, 2001). Each mobile part of the robot is thus represented with one or more points in space (Barraquand & Latombe, 1993; Girard, 1987). If the robot has to be represented with more than a few points, the calculation of the motion of this set of points in space has a large computational cost. One of the proposals which has been made to address these cases is the configurations space, where each point in the space represents a configuration. What is done is to find a path from the point of starting configuration to the point of final configuration space (Lozano-Perez, 1983). However, building space configurations also has a very high computational cost, and therefore most algorithms that use space configurations have an implicit representation which is explored to find the trajectory (Lavalle, 2006).

2.2. *Qualitative methods*

The fundamental difference between qualitative and quantitative methods is in the manner in which space is represented. In qualitative methods, space is not represented by a metric space. However, the specific issue of the best way to represent space in qualitative methods has not yet been resolved. When the first years of research on general methods of qualitative spatial reasoning proved to be unsuccessful, 'the poverty conjecture' was set out (Forbus et al., 1991). This conjecture states that: 'There is no purely qualitative, general-purpose representation of spatial properties'.

However, recent research seems to refute or at least weaken 'the poverty conjecture' (Cohn & Renz, 2008).

Among the objects of a space, there are three types of spatial information: topological, orientational and positional. Proposals for building qualitative spatial representations are based on at least one of the above types of spatial information in a qualitative basis. Some of the proposals made a space representation using topological information are RT_0 (Asher & Vieu, 1995; Hahmann, Winter, & Gruninger, 2009), region connection calculus (RCC) (Randell et al., 1992) and the topological map (Remolina & Kuipers, 2004). For positional information, panorama (Schlieder, 1993; Wagner, Huebner, & Visser, 2003), oriented regions (Levitt & Lawton, 1990) and cardinal direction calculus (Liu, Zhang, Li, & Ying, 2010) have been described. And, for distance information, linguistic variables (Kim, Seong, Hyung, & Kim, 1998; Lea, Hoblit, & Jani, 1993) and distances systems (Clementini, Di Felice, & Hernández, 1997) have been proposed. Among the various proposals to use topological information, RCC is the most widespread and widely studied framework to represent spatial relations between objects. Among other issues, satisfiability, decidability, complexity of different variants, composition of spatial relations and their mixture with fuzzy techniques for handling uncertainty (Li & Li, 2004; Li & Wang, 2006; Li & Ying, 2004; Renz, 2002, 1999; Renz & Nebel, 1999; Schockaert et al., 2009; Stell, 2000) have been studied. However, RCC only allows static spatial descriptions and its calculus to infer relations or the satisfiability of a spacial configuration, while a robot is confronted with a dynamic space.

For the last decade, some proposals have been made to produce dynamic descriptions such as spatio-temporal constraint calculus (Gerevini & Nebel, 2002), as well as reasoning about movement such as qualitative trajectory calculus (Delafontaine, Weghe, Bogaert, & De Maeyer, 2008; Li & Ying, 2004; Van de Weghe, Cohn, De Tre, & De Maeyer, 2006; Van de Weghe, Cohn, De Maeyer, & Witlox, 2005; Van de Weghe et al., 2007). However, these proposals are used to represent objects in space and time, and the problem investigated by these proposals is the satisfiability of these descriptions.

Since the emergence of AI, the problem of generating a sequence of actions to achieve a qualitative spatial configuration between objects starting from another qualitative spatial configuration (blocks world problem, Towers of Hanoi problem and so on) has been researched. The problem can be stated generally as follows.

> Given the starting relation(s) $S(S_1, S_2, \ldots)$ among objects $[O, O']$ $([O_1, O'_1], [O_2, O'_2], \ldots)$ establish the spatial relation(s) $G(G_1, G_2, \ldots)$ by using operators $A_1, A_2, A_3, \ldots$ and knowledge base B.

This type of problem was a focus of research in the period between the 1950s until the 1980s. The world is described by means of a formal language, in most cases using first-order logic. Some of the most important proposals to solve the above type of problems are Stanford Research Institute Problem Solver (Fikes Nilsson, 1971), situation calculus (McCarthy & Hayes, 1987) and event calculus (Kwalski & Sergot, 1986). Situation calculus has been implemented in GOLOG (Levesque, Reiter, Lespérance, Lin, & Scherl, 1997) for the control of automatic systems. Also, variants of GOLOG have been created to control robots in the ROBOCUP (Ferrein, Fritz, & Lakemeyer, 2005). However, these languages are focused on soccer and their primitives manage only the spatial relation of one object inside a region (Dylla et al., 2008). There are thus are no general methods to translate the information captured by the robot sensors into a high-level representation with a full group of spatial relations.

On the other hand, some experts in the field of robotics think that qualitative topological information is appropriate for planning, but too abstract to allow the realisation of navigation (Schlieder, 1993). Thus, in robotics, two lines of research can be found for the use of qualitative methods: hierarchies of spatial description languages and languages less abstract than topological information.

- In the hierarchy of spatial language, it is proposed that the movement of a robot to deal with an environment has to be split into different problems and that each problem be resolved at a particular hierarchical level (Kuipers, 2000; Levitt & Lawton, 1990; McDermott & Davis, 1984; Sacerdott, 1973; Sobek & Chatila, 1988). In these hierarchies, the qualitative topological information is used to solve scheduling problems.
- Among the methods with languages less abstract than the topological information, there is 'panorama', which uses order information (Schlieder, 1993). This method has been used in the RoboCup (Fogliaroni, Wallgrün, Clementini, Tarquini, & Wolter, 2009; Wagner & Huebner, 2005; Wagner et al., 2003).

2.3. *Navigation based on spatial relations*

Following this summary, we can see that up to now spatial relations have not been applied in robot navigation. However, as shown in the example of a task set in natural language, as described above, spatial relations are of great significance in achieving multifunctional robots. Therefore, in order to create multifunctional robots, it is necessary to investigate the use of spatial relations for navigation. Regarding spatial relations and navigation, the first two problems to be resolved are as follows:

- How to construct a description of spatial relations from a quantitative description.
- How to decide which action to take with a description of spatial relations.

Based on these two questions, one can define a new type of navigation problem, which will be called 'navigation problem based on spatial relations', which is expressed as follows.

> Given a starting point A establish the spatial relation(s) $G = (G_1, G_2, \ldots)$ among objects $[O, O'] = ([O_x, O_{x'}], [O_y, O_{y'}], \ldots)$ by using [the robot's] knowledge and sensory information received.

The navigation problem based on spatial relations allows specifying what we mean by qualitative navigation methods and which are the elements distinguishing the architecture of type III. Also, it is a key element in our research, since one of the objectives is the study of topological notions to resolve the navigation problem based on spatial relations. The proposal presented in this article contains our first results about the usefulness of topological notions that, although limited, have been positive. The main discovery is a heuristic, called the HTQS, which allows decision-making to establish topological relations obtained from a representation of the physical space with a non-dense metric topological space.

3. Representation, reasoning and making decisions through topological spaces

This section introduces the HTQS, which allows the establishment of spatial relations. Notice that the establishment of a spatial relation by an agent implies its navigation. Spatial relations are formalised as topological relations in HTQS. Topological relations have allowed a GIS to recognise and analyse spatial relations that exist in the stored geographic information and, in turn, spatial relations allow complex spatial modelling and analysis. Egenhofer (1989), Egenhofer and Franzosa (1991), and Egenhofer and Herring (1990) have shown that spatial relations can be defined from sets associated with the space occupied by an object in a topological space. The starting hypothesis of the research was that topological relations are a useful and effective tool to achieve navigation based on identifying and establishing spatial relations. In order to achieve this, the concept of topological relation is briefly explained as the research which has been carried out is based on this concept. To illustrate other general concepts of topology used in the research, a small appendix (Appendix 2) has been added. More details can be found in any standard text on topology (Kelley, 1975).

3.1. *Topological relation*

A topological relation (Kelley, 1975) is a relation between two sets in the same topological space. To characterise a topological relation r_x, it is necessary to use invariants. Invariants permit the recognition that a certain relation exists between two sets, A and B, of a topological space.

Egenhofer (1989) developed a method to characterise topological relations between two sets of a topological space by means of intersections. In order to apply the intersections method, the only requirement is that the topological space be connected (Egenhofer & Franzosa, 1991). One version of their method is the '9-intersection model' (Egenhofer, Sharma, & Mark, 1993), which has been applied in the development of commercial tools in the field of GIS. The '9-intersection model' is based on the invariant empty/nonempty produced by nine intersection sets between the sets associated with the sets A and B. The associated sets used to perform the intersection are the inside, the boundary and the exterior of each one of the two objects (see appendix). The result of each intersection is valued as empty or not empty because properties empty and not empty are invariant under homeomorphisms. The nine intersections of the '9-intersection model' are denoted by I, where I corresponds to:

$$I = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}.$$

This method enables to distinguish $2^9 = 512$ configurations but many are not possible. In $\mathbb{R}^2$, the topological relations between two regions of the space without holes that can be defined by the '9-intersection model' are only 8, and they can be seen graphically in Figure 2. These eight relations are jointly exhaustive and pairwise disjoint on $\mathbb{R}^2$, and we call them R_8.

3.2. *Formalisation using topological spaces*

This section presents the formal definitions proposed for the navigation problem based on spatial relations and their solution. The proposed definitions are made using topological notions.

Two levels exist to formalise the space through topological spaces. The first level specifically describes the coordinates of space which are occupied by the objects. This first level is important

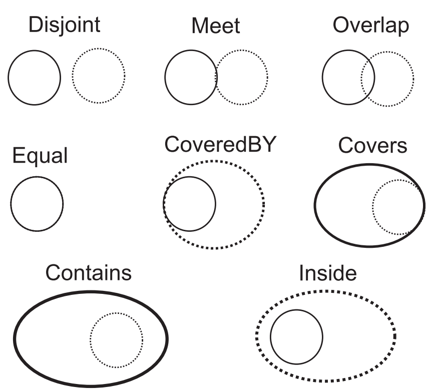

Figure 2. Definable topological relations between two regions of the space without holes on $\mathbb{R}^2$ by the '9-intersection model' method (Egenhofer et al., 1993).

because it permits the linking of the capture mechanisms of a robot. The sensors use a numerical language and it is therefore possible to create a representation employing the notion of metric topological space, using sensory information as a starting point. The second level, which is more abstract, specifies the topological relations between objects. This second level allows the formalisation of spatial relations in the problem set. In the problem, the initial setup and the one received by the robot through its sensors will be described by the language of the first level, while the configuration sought is described by that of the second level. The robot must be able to recognise that when it reaches a first level configuration, the task has been accomplished . Thus, it is necessary to establish methods of equivalence between both levels. A set of definitions for the formalisation of the problem at both levels and how they relate is here presented:

DEFINITION 3.1 (Quantitative topological state) *Given a topological space* $(X, \mathcal{T})$ *and a set of objects* $O = \{o_1, \ldots, o_m\}, e = \{(o_1, A_1), \ldots (o_m, A_m)\}$ *is named a quantitative topological state, where* $A_1, \ldots, A_m \subset X$.

Before defining the description of the configuration space through the second level, it is necessary to define topological relations between objects. This definition will be the link between the two levels of the topological language.

DEFINITION 3.2 (Topological relation between objects) *Given a topological space* $(X, \mathcal{T})$ *and a quantitative topological state* $e = \{(o_1, A_1), \ldots, (o_m, A_m)\}$, r_x *is a topological binary relation between objects* o_j *and* o_k *in state* e *if* $A_j r_x A_k$ *is fulfilled.*

Qualitative topological state is defined by the concept of topological relation between objects, as follows:

DEFINITION 3.3 (Qualitative topological state) *Given a set of topological relations* $R = \{r_1, \ldots, r_n\}$ *and a set of objects* $O = \{o_1, \ldots, o_m\}$, *set* e *is named a qualitative topological state, being* $e = \{(o_1 r_x o_2), \ldots, (o_1 r_v o_m), (o_2 r_{x'} o_1), (o_2 r_u o_3), \ldots, (o_m r_v o_{m-1})\}$.

It should be noted that an object o_j always has the topological relation equal to itself, and therefore it is not necessary to make this information explicit. Also, it must be understood that if every $r_x \in R$ is a reciprocal relation, the topological relation fulfills:

$$o_j r_x o_k \Leftrightarrow o_k r_x o_j.$$

So, one of them does not have to be mentioned.

Once quantitative and qualitative topological states have been defined, we can formalise the concept of navigation problem based on spatial relations.

DEFINITION 3.4 (The establishing topological relations problem) *Given a topological space* $(X, \mathcal{T})$, *a set of objects* O, *and a set of topological functions* $F = \{f_1, \ldots, f_s\}$, *where* $f_i : X \to X,\ i \in \{1, \ldots, s\}$, *the tuple* $((X, \mathcal{T}), O, e_1, F, R, \Omega, \Lambda)$ *is named establishing topological relations problem*, *where*

- e_1 *is a quantitative topological state*; e_1 *is the initial state*;
- R *is a set of topological relations*; $R = \{r_1, \ldots, r_n\}$;
- $\Omega = \{(o_j r_x o_k), \ldots\}$ *is the set of topological relations between objects that have to be fulfilled*;
- Λ *is a function that assigns to each object a set of functions representing the actions capable of being executed by the object*;

$$\Lambda : O \to \mathcal{P}(F).$$

The set of functions that Λ *associates to each object is always finite.*

The capture information mechanisms of robots gather discreet information about their environment (a finite number of sensors in the sonar or laser, a finite number of pixels in the camera and so on). Henceforth, of the numerous problems associated with the establishing topological relations problem, research has focused on those that have $\mathbb{Z}$ as its topological space with order topology (see Appendix 2). $\mathbb{Z}$ is a non-dense space which attributes discreet values to positions, but also $\mathbb{Z}$ has neither lowest nor highest values, allowing it to be taken as a generalisation of all the cases of non-dense finite spaces. $A_1, \ldots, A_m$ are restricted to elements of the base that define the order topology, $\mathcal{B}_<$ (see Appendix 2). Moreover, analysing the problem of navigating in a one-dimensional space has the advantage of less complexity than analysing the problem over a space of several dimensions, and therefore allows a better analysis of the problem.

Once a formal representation of a navigation problem based on the notion of topological space has been defined, the next step is to define what constitutes a solution and its formal representation.

Definition 3.5 (Qualitative topological solution) *Given a establishing topological relations problem* $((X, \mathcal{T}), O, e_1, F, R, \Omega, \Lambda)$ *a sequence* $\langle((o_1,f_{\mu_1(1)}), \ldots, (o_m,f_{\mu_m(1)})), \ldots, ((o_1,f_{\mu_1(z)}), \ldots, (o_m,f_{\mu_m(z)}))\rangle$ *is named a qualitative topological solution, such that*:

- $\forall (o_j r_x o_k) \in \Omega \quad f_{\mu_j(z)}(, \ldots, (f_{\mu_j(1)}(A_j))) \quad r_x \quad f_{\mu_k(z)}(, \ldots, (f_{\mu_k(1)}(A_k))),$

where $(o_j, A_j), (o_k, A_k) \in e_1$.

Function $\mu_k(x)$ determines which one of all the possible functions assigned to object o_k by function Λ is used at position x of the sequence. It is understood that the functions have been ordered under any criterion that enables naming them because they are a finite number. Using the above formal definitions, the main purpose of the investigation can now be stated. We must find an automatic reasoning method for getting the topological solution to the problem of establishing topological relations. That is, if $\overrightarrow{p}$ is a establishing topological relations problem, we seek an algorithm h to generate a qualitative topological solution $\overrightarrow{s}$, so that $h(\overrightarrow{p}) = \overrightarrow{s}$. The reader should be aware that the method, which is being looking for, does not take into account physical restrictions. However, physical restrictions are really important, e.g. one robot can be inside of a swimming pool but it can be inside of a block of concrete. The physical restrictions must be considered when the topological relation target is chosen. So, the architecture must have a knowledge base to propose targets relations although it is out of the scope of this article.

3.3. *Problems with reasoning by means of topological spaces*

Freksa (1990, 1992) originally proposed representing the relations between relations and he called it a 'conceptual neighbourhood': conceptual neighbourhood. The idea was research about reasoning methods with incomplete or coarse knowledge about temporal relations and spatial relations (Freksa, 1991b). Independently, Egenhofer and Al-Taha (1992) proposed a similar idea though they formalised spatial relations with topological relations. Our research is derived from the work of Egenhofer and Al-Taha (1992). They introduced a way to relate topological relations by defining a distance between them and their representation in a graph. In the graph, each one of the topological relations is represented by a node and each node has an arc within that node with a minimal distance to their respective topological relations. This type of graph is called the Closest Topological Relationship Graph (CTRG). The same authors also created a revised version that includes arcs between nodes when there is a transformation that enables passing two objects from one topological relation to another. A graphic representation of the revised CTRG can be seen from Figure 3.

In their work, Egenhofer and Al-Taha studied the possible use of CTRG for inference and prediction. The transformation in the topological relations between two objects is represented in

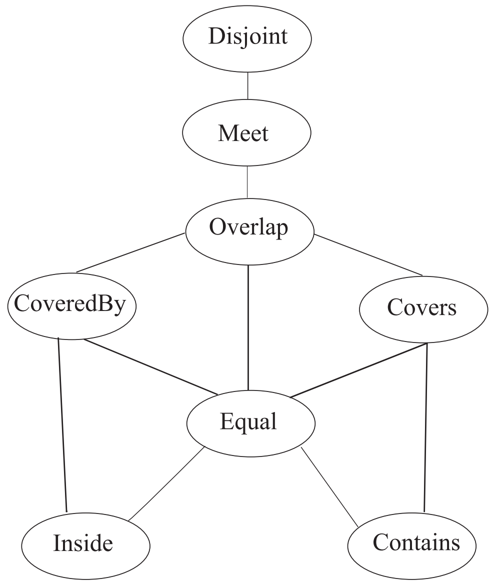

Figure 3. Revised CTRG, adapted from Egenhofer et al. (1993).

a CTRG with a line which makes reference to a way in the graph, called a *deformation diagram*. An example is shown in Figure 4.

Their conclusions were that the CTRG could only be used to infer and predict in only in a few cases. The problem discovered by Egenhofer and Al-Taha is that the CTRG produces more than one possible deformation diagram in several cases.

3.4. *Topological reasoning linear graphs*

The results of Egenhofer and Al-ha do not determine that an algorithm to make decisions based on topological relations cannot be developed, although a CTRG does not serve as a method for inference and prediction. Thanks to Ernst's (1969) research, we know that an analysis of means and ends will find a solution if one can define a complete order relation on the differences. Linear graphs can be obtained from the CTRG considering conditions of isoshape and isosize. However, the linear graphs from the CTRG are not enough to make decisions because they cannot distinguish some cases as is shown in Section A.1 in Appendix 1.

To solve the problem shown in Section A.1 in Appendix 1, it is necessary to construct a linear graph which fulfills the function of generating a single path in the graph for all the situations described with the same topological relation. To this end, the new graph must have more discriminatory power. The way to increase the discriminatory power is by increasing the number of nodes, which is equivalent to having more topological relations. The problem of topological relations defined with the '9-intersection model' is that one cannot tell when an object is on either side of the object in the topological relation. Mathematically, this is reflected in the fact that topological relations are reciprocal.

Thus, for each of the topological relations that cause the appearance of multiple paths, two relations are defined. One covers the situation when an object is on one side of the reference object, and the other covers the situation when an object is on the other side. In order to distinguish between the set of relations to be defined and R_8, the new set of relations is called *relative topological relations* and is denoted by S_{13}. The new set is formed by the following 13 relations: Disjoint-0, Meet-0, Overlap-0, CoveredBy-0, Covers-0, Inside, Equal, Contains, Disjoint-1, Meet-1, Overlapping-1,

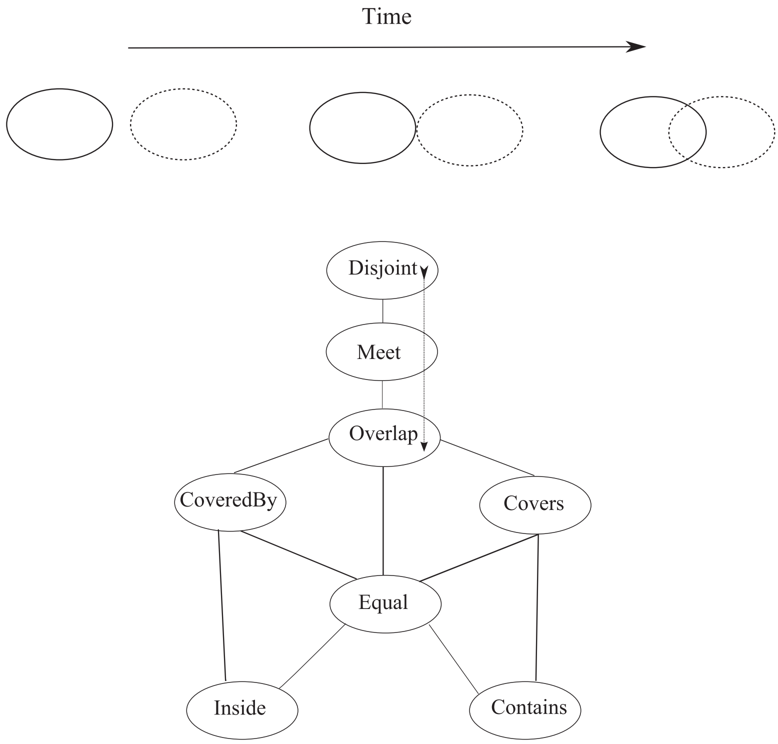

Figure 4. The topological changes of two objects in A are represented in the CTRG by a pathway between nodes in B, adapted from Egenhofer et al. (1993).

CoveredBy-1 and Covers-1. The relative topological relations are jointly exhaustive and pairwise disjoint. Their formal definition will be explained later on.

The heuristics introduced in this article is to seek a solution for a problem $((\mathbb{Z}, \mathcal{T}), O, e_1, F, R, \Theta, \Lambda)$, where $\mathcal{T}$ is the order topology and the following restrictions are fulfilled:

(1) $\forall (o_j, A_j) \in e_1 \quad A_j \in \mathcal{B}_{\mathbb{Z}} = \{(x_1, x_2) : x_1, x_2 \in \mathbb{Z}\} \quad j \in \{1, \dots, m\}$,
(2) $\forall o_k \quad \Lambda(o_k) = \{i\} \quad k \in \{2, \dots, m\} \quad \forall A \in \mathcal{B}_{\mathbb{Z}} \quad i(A) = A$.

The first constraint ensures that objects have no holes. This is important because the set of topological relations are defined for environments with objects without holes. The choice of the topological space $\mathbb{Z}$ is due to the mechanisms of perception that discreetly capture the environment, and one of the objectives is that the methods developed are the most directly applicable. The second constraint says that all the objects except one are static since they cannot change positions. That is to say, only the robot moves in the environment.

The criteria for assigning a TRLG to make decisions in connex topological spaces, fulfilling isosize and isoshape, is the ratio size between the agent and the object. However, we focus on non-dense sets, which implies that the size ratio is not sufficient to assign a graph, since in each case subcases arise by the difference between the dense and non-dense sets. Table 1 presents all subcases for non-dense topological spaces, with their corresponding graphs, assuming isosize and isoshape in the objects. Each of these graphs is called a *topological reasoning linear graph* (TRLG).

Some of the graphs of Table 1 are represented in Figures 5–7. The reader must remember that these graphs represent changes between two objects in their relative topological relation.

Table 1. The conditions between two objects, A and B, to assign a TRLG and the TRLG assigned in a topological space generated with a set no dense.

Case	Subcase	Linear topological reasoning graph	Name
$\lvert A\rvert > \lvert B\rvert$	$\lvert A\rvert = (\lvert B\rvert + 1)$	$\langle s_1, s_2, s_3, s_5, s_{10}, s_{11}, s_{12}, s_{13}\rangle$	GT1
	$\lvert A\rvert > (\lvert B\rvert + 1)$	$\langle s_1, s_2, s_3, s_5, s_8, s_{10}, s_{11}, s_{12}, s_{13}\rangle$	GT2
$\lvert A\rvert < \lvert B\rvert$	$(\lvert A\rvert + 1) = \lvert B\rvert$	$\langle s_1, s_2, s_3, s_4, s_9, s_{11}, s_{12}, s_{13}\rangle$	LT1
	$(\lvert A\rvert + 1) < \lvert B\rvert$	$\langle s_1, s_2, s_3, s_4, s_6, s_9, s_{11}, s_{12}, s_{13}\rangle$	LT2
$\lvert A\rvert = \lvert B\rvert$	$\lvert A\rvert = \lvert B\rvert = 1$	$\langle s_1, s_7, s_{13}\rangle$	ET1
	$\lvert A\rvert = \lvert B\rvert > 1$	$\langle s_1, s_2, s_3, s_7, s_{11}, s_{12}, s_{13}\rangle$	ET2

Notes: The notation is s_1 = Disjoint-0, s_2 = Meet-0, s_3 = Overlap-0, s_4 = CoveredBy-0, s_5 = Covers-0, s_6 = Inside, s_7 = Equal, s_8 = Contains, s_9 = CoveredBy-1, s_{10} = Covers-1, s_{11} = Overlap-1, s_{12} = Meet-1, s_{13} = Disjoint-1.

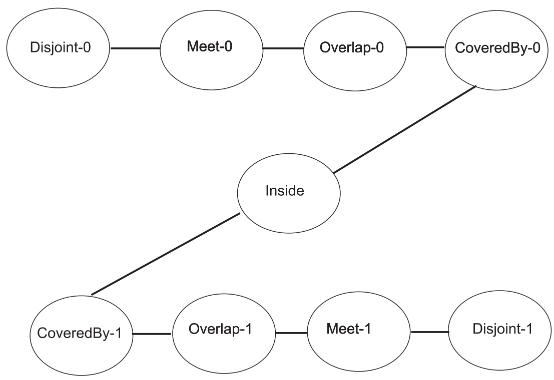

Figure 5. Diagram for $LT2$.

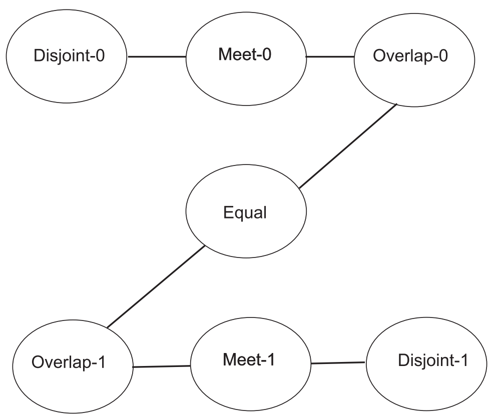

Figure 6. Diagram for the case $ET2$.

TRLGs are key to the heuristics because in these graphs, the problem of multiple paths disappears, as is shown in Figure 8.

3.5. *Heuristic of topological qualitative semantics*

In previous sections, we discussed the reasoning of spatial relations, but the fact that an agent establishes a spatial relationship involves making decisions. The reasoning methods only address

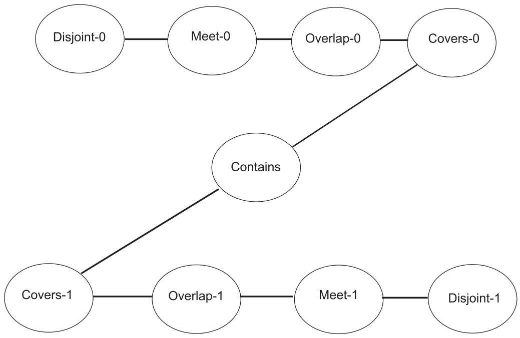

Figure 7. Diagram for the case *GT*2.

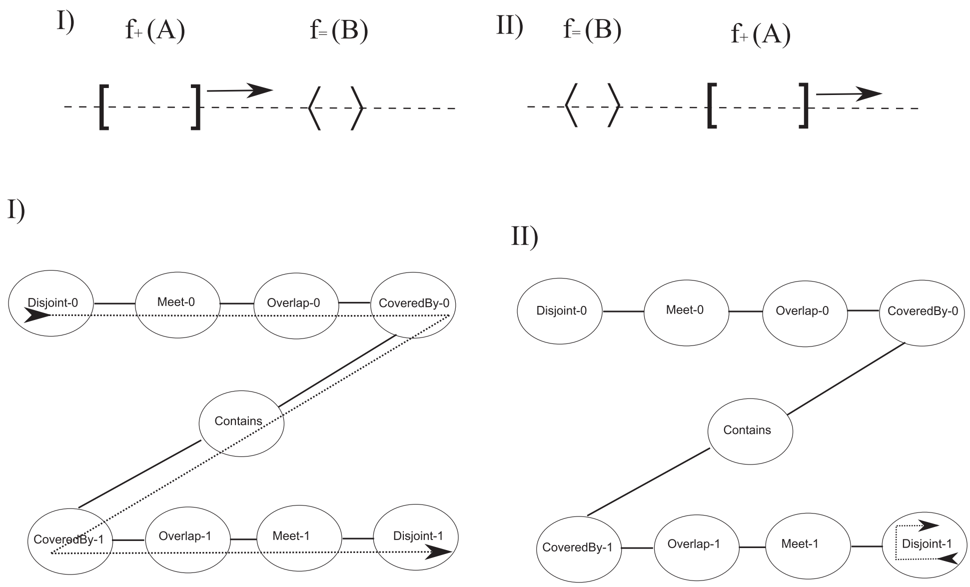

Figure 8. In TRLG there is no problem of multiple paths. The functions are $\mathbf{f}_+|_x(x) = x + 1$ and $\mathbf{f}_=|_x(x) = x$.

the satisfiability problems or changes that must occur to achieve a state. So, a decision-making method is thus required. The decision-making method must indicate which action must be performed to establish a spatial relation. The method that has been developed is a heuristic. The heuristic finds a solution to the problem of establishing topological relations. The method is denominated, a HTQS. First, the problem that exists for the creation of heuristics is explained. In problems concerned with implementing the means-ends analysis (Newell, Shaw, & Simon, 1959), the goal sought is that an object changes its value into others, by applying some operators acting on values of the object. In the means–ends analysis, there are three elements:

- Substitution: The pair (x, y) consisting of two values, x and y. Value y can be the value that substitutes x when applying an operator f, or the value to be established, where $x, y, f(x) \in X$.
- Type of substitution: The relation given in a substitution; $(x, y), (x, f(x)) \in r$.
- Meta-type of type of substitution: The domain of a set of types of substitutions, $r, s, \ldots \subset X \times X$.

In problems related to the means–ends analysis, the meta-type of the type of change caused by an operator and the meta-type of the type of change that one wants to produce on the object are the same. This allows that the action which leads to the destination can automatically be selected, since by using the target and current value of the object, it is possible to calculate the type of change needed, and with that type of change calculated, it is only necessary to select the operator that causes such change. But this approach is not valid for a establishing topological relations problem as the meta-type of type of the change of a function and the meta-type of the type of change that cause a modification in the topological relation are different. The types of change of the functions describe how the functions change the coordinates of the object, and the types of change that appear to consider the objectives of the problem describe the change between the current and the destination topological relation. Thus, to create a heuristic for the establishing topological relations problem, an element that transforms one type of substitution in the topological relation into a type of substitution of coordinates is required. It has been observed that the element which allows that translation is a TRLG.

A generic example is provided to illustrate the operational nature of HTQS. Imagine one has two objects: an object that has the capacity to act, o_A, and a reference object, o_R, in respect of which one wants that o_A be able to fix a concrete relative topological relation. Thus, suppose that o_A can work with three actions:

$$\Lambda(o_A) = \{f_1|_x(x) = x + 1,\ f_2|_x(x) = x,\ f_3|_x(x) = x - 1\}$$

and o_R cannot move and remains static:

$$\Lambda(o_R) = \{f_4|_x(x) = x\}.$$

The first data structure used is a table that is constructed by applying a means–ends analysis on the actions. Thus, the functions are labelled with the way in which the positions in space are changed, assigning an order relation to be satisfied when the action applies. If isosize and isoshape conditions are met, there are only three possible cases:

- $\forall x \in X, x < f(x)$
- $\forall x \in X, x = f(x)$
- $\forall x \in X, x > f(x)$

Thus, the functions of o_A are labelled creating the results contained in Table 2.

The second data structure used is the TRLG. The nodes of each TRLG are labelled with an incremental enumeration. In the example shown, it is considered that o_A and o_R are equal in size and bigger than 1. Since we are in $\mathbb{Z}$ the form of an object is necessarily the same if the size is the same. Therefore, the graph used is in Figure 9.

Table 2. Function labelling table.

Function	Label
f_1	$<$
f_2	$=$
f_3	$>$

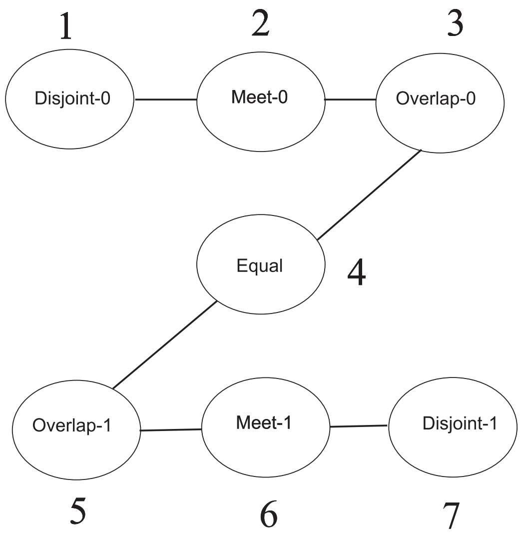

Figure 9. TRLG with the incremental enumeration.

Once the above data structures have been created, the algorithm to select the action requires the following steps:

(1) The relative topological relation between the objects o_A and o_R is calculated from their current positions, s_c, and the number associated to the node of s_c , n_{s_c}, is stored.
(2) The number associated to the node of the relative topological relation which is the target, s_t, between the objects o_A and o_R, n_{s_t}, is obtained.
(3) It is checked what order relation holds between n_{s_c} and n_{s_t} from the following:
 - $n_{s_c} < n_{s_t}$,
 - $n_{s_c} = n_{s_t}$,
 - $n_{s_c} > n_{s_t}$.
(4) It is examined at the means–ends table using the order relation that holds between n_{s_c} and n_{s_t} in order to select the action that is labelled with the same order relation. This is the action selected by heuristics.

4. Calculation of relative topological relations

The previous section described the heuristic in order to solve a establishing topological relations problem. The first step of the heuristic states that the relative topological relation is calculated using the positions of the two objects. However, so far, we have not given an explanation as to how the calculation should be performed. This is important because it connects the mobile robot's perception of the environment with the navigation algorithm. Thus, this section introduces a method to calculate the relative topological relation of two regions of space.

4.1. *Problems to calculate*

Initially, the use of the '9-intersection model' might be considered as a valid calculatory model. Unfortunately, there are two impediments that prevent implementation. The first obstacle is that the '9-intersection model' was proposed for dense topological spaces, such as $\mathbb{R}^n$, and fails for non-dense spaces. The '9-intersection model' is based on topological concepts of interior, frontier and exterior. However, when the mechanism of the '9-intersection model' is applied to non-dense spaces, such as $\mathbb{N}^n$ or $\mathbb{Z}^n$, the method cannot characterise the eight topological relations as the definition of frontier:

$$\partial A = \bar{A} \cap (\overline{X - A})$$

and, as

$$X - A = A^{-}$$

then

$$\partial A = \bar{A} \cap (\overline{A^{-}}).$$

Thus, in non-dense independent spaces of A, it is always the case that:

$$\partial A = \emptyset.$$

Therefore, the '9-intersection model' does not permit the characterisation of the eight topological relations in non dense topological spaces. In order to solve this problem, the use of interior and border definitions different from those contained in the topology have been proposed (Egenhofer Herring, 1990). Unfortunately, these new definitions from the field of digital topology (Rosenfeld, 1979) loose the simplicity that makes the '9-intersection model' so interesting. The second and most important impediment is that the number of relative topological relations is 13, compared with the eight relations that the '9-intersection model' can define for regions in $\mathbb{R}^2$. Therefore, the '9-intersection model' reconstructed with the definitions of digital topology is incapable to define the 13 relative topological relations that have to be defined. Due to the two obstacles just mentioned, we have created a variant of an old formalism. This old formalism is the method for finding relations between time intervals used in the Allen and algebra (1983). Freska noted that the 13 relationships defined by Allen's formalism could be interpreted as spatial in a spatial context (Freksa, 1991a). It is true that Allen applies his method to convex intervals in $\mathbb{R}$, and these do not correspond with opens sets of the topological connected space over $\mathbb{R}$. Nevertheless, the convex intervals over $\mathbb{Z}$ coincide with the open sets of the order topology on $\mathbb{Z}$. Thus, instead of using the Allen method to calculate the relations between two time intervals, it is used to calculate a topological relation. Indeed, the interpretation given to $\mathbb{Z}$ is that of a spatial dimension. Allen's method also avoids the second impediment cited for the '9-intersection model' as it can characterise the relative topological relations. Although it is needed to do some variations because Allen was using $\mathbb{R}$, a dense set, like a model of time. However, $\mathbb{Z}$ is a not dense set. Allen's formalism is the next set of conditions

$$\begin{pmatrix} \min(X) < \min(Y) & \min(X) < \max(Y) \\ \max(X) < \min(Y) & \max(X) < \max(Y) \\ \min(X) = \min(Y) & \min(X) = \max(Y) \\ \max(X) = \min(Y) & \max(X) = \max(Y) \end{pmatrix}.$$

Allen used the next two conditions to define meets.

- $\min(X) = \max(Y)$,
- $\max(X) = \min(Y)$.

He did this because he used his formalism on a dense set-like model of time. Now, we want to use the formalism on a no-dense set-like model of space. For this reason, we introduce the following changes:

Table 3. Relative topological relations defined by means of the order propositions matrix.

Case	Subcase	Linear topological reasoning graph
Disjoint-0 $P^{<} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	Meet-0 $P^{<} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$	Overlap-0 $P^{<} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
CoveredBy-0 $P^{<} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$	Covers-0 $P^{<} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$	Inside $P^{<} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
Equal $P^{<} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$	Contains $P^{<} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	Covers-1 $P^{<} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$
CoveredBy-1 $P^{<} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$	Overlap-1 $P^{<} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	Meet-1 $P^{<} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$
Disjoint-1 $P^{<} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$		

- $\min(X) - 1 = \max(Y)$,
- $\max(X) + 1 = \min(Y)$.

This changes are in our opinion a better definition of the relation meets for space because two objects do not share points of space, but they possess adjacent points. By incorporating the last variations to Allen's method, all relative topological relations can be characterised for no-dense sets. The new formalism, called *order propositions matrix* and denoted by $P^{\leqq}(X, Y)$ consists of the following matrix:

$$P^{\leqq}(X, Y) = \begin{pmatrix} \min(X) < \min(Y) & \min(X) < \max(Y) \\ \max(X) < \min(Y) & \max(X) < \max(Y) \\ \min(X) = \min(Y) & \min(X) - 1 = \max(Y) \\ \max(X) + 1 = \min(Y) & \max(X) = \max(Y) \end{pmatrix}.$$

Each of the elements of the matrix is a proposition that takes a value of $\mathbb{B} = \{0, 1\}$, depending on whether the proposition is false(0) or true(1). Table 3 provides the characterisation of each of the relative topological relations through the order propositions matrix.

Thus, the values taken by matrix $P^{\leqq}(X, Y)$ directly show the relative topological relations between two sets, and therefore, the node of the TRLG associated with them. The reader should realise that Allen's method enables characterising 13 binary relations (see Table 4). These 13 relations are composed of 6 binary relations, their corresponding 6 inverse binary relations, and the binary relation 'equal'. The inverse relation of 'equal' is itself. These relations defined for the temporal dimension can also be used in spatial dimensions.

If we recall the definition of inverse

$$R^{-} = \{(y, x) : (x, y) \in R\},$$

Table 4. The 13 binary relations enabled by Allen's method.

Allen's relations	Symbol	Relative topological relations (S_{13})	Symbol
Before	r_1	Disjoint-0	s_1
Meets	r_2	Meet-0	s_2
Overlaps	r_3	Overlap-0	s_3
Starts	r_4	CoveredBy-0	s_4
Finished-by	r_6^-	Covers-0	s_5
During	r_5	Inside	s_6
Equal	r_7	Equal	s_7
Includes	r_5^-	Contains	s_8
Finishes	r_6	CoveredBy-1	s_9
Started-by	r_4^-	Covers-1	s_{10}
Overlapped-by	r_3^-	Overlapping-1	s_{11}
Meet-by	r_2^-	Meet-1	s_{12}
After	r_1^-	Disjoint-1	s_{13}

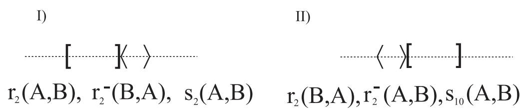

Figure 10. The graph shows two situations that exemplify the problem. Each of the situations have two objects: object o_A represented by [ ] and o_B represented by $\langle\ \rangle$, both objects are situated in a one-dimensional space represented by the dotted line. In each situation, two Allen's relations are true.

one realises that independent of the spatial locations of two objects, they satisfy a relation and its inverse. When describing the temporal relationship between two events, it is irrelevant whether a relation or its inverse is used, since the observer is describing from the outside, and both relations contain the same information. However, when the observer is not describing from the outside because the observer is one of the objects and the observer must make a decision to move; so it matters if the observer uses a relation or its inverse, since actions are labelled for a specific relation. Thus, for HTQS only one of two relations is useful; the other leads to a misunderstanding of the algorithm.

So, what is the relation that must be used in the HTQS? The answer is that it is chosen according to what object is applying HTQS to make decisions. In $P^{\leqq}(X,Y)$, X must always be the agent making the decision, and Y is the object respect of which the agent must make a decision. Figure 10 shows these two states. Note that in the two states, two Allen relations are satisfied in each of them.

In these two cases, it is the object o_A the one that is applying HTQS to act, and, therefore, in situation (I) the relative topological relation is s_2 and in situation (II) it is s_{10}.

4.2. *Generalisation to n dimensions*

The heuristic presented in the previous section allows finding solutions to problems of type $((\mathbb{Z},\mathcal{T}),O,e_1,F,R,\Omega,\Lambda)$, where $\mathcal{T}$ is the order topology and R is the set of relative topological relations. Unfortunately, an algorithm to establish relations in one-dimensional space is not useful in the real world, which is made up of three dimensions. So the next point is the generalisation of the heuristic to dimension n. Therefore, this section will discuss the application of the TRLG to problems with the topological space $\mathbb{Z}^n$, although it is clear that the real interest arises for $n = 2$ or $n = 3$.

The first issue concerning the generalisation is about the creation of a topological n-dimensional space. To do this, several results from topology can be used to build the topological space $\mathbb{Z}^n$. It is known that, given a topology on X and another on Y, there is a canonical way to create a topology on the cartesian product $X \times Y$, the so-called product topology (see Appendix 2). Thus, from the base $\mathcal{B} = \{(x_1, x_2) : x_1 < x_2 \; x_1, x_2 \in \mathbb{Z}\}$, which generates the order topology on $\mathbb{Z}$, we can construct the product topology on $\mathbb{Z} \times \mathbb{Z}$. The definition of the product topology on $X \times Y$ can be applied equally to $X_1 \times X_2 \times \cdots \times X_n$. Therefore, there is no problem in building a generalisation to dimension n of the topological space.

The next topic is the topological relations within a space $X_1 \times X_2 \times \cdots \times X_n$. Since space is constructed by the Cartesian product, the n-dimensional topological relation between two objects is defined by a tuple of n components, the component k being the topological relation that occurs in the dimension k between the two objects. Thus:

$$\forall A, B \in X_1 \times \cdots \times X_n \quad A r_x B \Leftrightarrow A \langle r_{x'}, r_{x''}, \ldots, r_{x^n} \rangle B,$$

where r_{x^k} is the relation between A and B in dimension k.

Therefore, the generalisation to n dimensions of the HTQS consists in applying it successively in each dimension. Nevertheless, this generalisation needs some restrictions in the real world.

One problem that appears when changing to $\mathbb{Z}^n$ is related to the contour of the objects. In $\mathbb{Z}$ objects have a contour that does not vary with respect to the reference points taken to calculate the relative topological relation, since the contour is only a point at each end of the interval. In $\mathbb{Z}^n$ any contour is possible, so that the points of the interval may have values that do not correspond to the values of other points of the contour. Thus, for the generalisation, it is necessary that the sides of each object have right angles to each other, what it will be called, orthogonal concave shapes. This way, two-dimensional objects are rectangles, three-dimensional objects are cubes and so on. This ensures that the corresponding topological relation in one dimension can be changed without affecting the previous one. That is to say, it satisfies the principle of optimality, and the combination of a solution for every dimension is the solution to the complete problem.

Therefore, we need the following condition for the existence of the principle of optimality for problems $((\mathbb{Z}^n, \mathcal{T}), O, e_1, R, \Theta, \Lambda)$, where $\mathcal{T}$ is the product topology, and to allow the HTQS to be applied to each dimension to obtain the solution.

$$\forall (o_j, A_i) \quad A_i \in \mathcal{B}_{\mathbb{Z} \times \cdots \times \mathbb{Z}} = \{((x_1', x_2'), \ldots, (x_1^n, x_2^n)) : x_1, x_2, \ldots, x_1^n, x_2^n \in \mathbb{Z}\}.$$

When the above condition is met in space $\mathbb{Z}^2$, the objects have the form $((x_1, x_2), (y_1, y_2))$ as shown in Figure 11.

Another issue applying the HTQS in a space of dimension $n > 1$, built with the topological product, is related with the actions because the actions can also act on several dimensions. Therefore, for the principle of optimality to be satisfied, the number of dimensions on which the actions act also have to be taken into account. If the actions act on the values of more than one dimension, then navigation cannot be treated for each dimension as an independent problem. Thus, for the HTQS to be applied to each dimension, in addition to the condition that the sides of each object are at right angles to each other, each function f_i must change only the values of one dimension.

These restrictions can be strong for the case of a general environment but there are some concrete environments where these restrictions do not pose a problem. In the case of orthogonal convex shape, a warehouse, a loading dock, a road and a port are places where the shape of objects can be approximated by orthogonal convex shapes. Also the displacements in ports and warehouses are normally in only one-dimensional due to the spatial organisation.

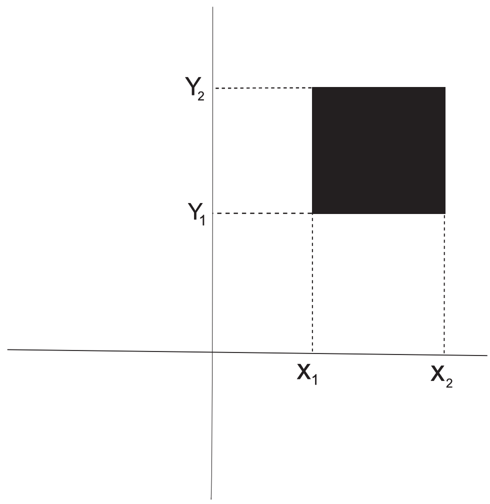

Figure 11. An element $((x_1, x_2), (y_1, y_2))$ of the base $\mathcal{B}_X \times \mathcal{B}_Y$ is graphically represented.

5. Implementation of HTQS

The final step is to test the heuristics in a virtual environment. The use of a virtual environment allows the testing of heuristics avoiding the problems inherent to computer vision when detecting objects (e.g. López-Valles et al., 2005; Schlieder, 1993).

5.1. *Description of the environment*

The designed virtual environment is two-dimensional and consists of two types of entities: agents and objects. Space entities are constituted by one or more parts. Each part occupies a point in the space of the environment. The agents are equipped with a remote sensing system that enables the longitudinal position to be captured and the depth of the objects with respect to the agent that perceives them. Some of the features of the virtual environment are as follows:

- Agents can perform actions that are able to modify their position in space.
- Objects are elements that have no ability to modify their position.
- Agents and objects cannot occupy the same point in space.
- Agents have the ability to move forward, backward, left, right and make 90° turns.

Each agent perceives its environment and returns a list of objects captured, including information about its lateral position and the depth encountered with respect to the agent. In this environment, the agent's goal is to provide a one-dimensional relative topological relation between the agent and a given object. To implement HTQS, a new data type whose elements are the 13 relative topological relations is first defined. These types of data serve both to describe the problem that arises for an agent and to build the TRLGs. Each TRLG is an array and each array element is a record that contains the following:

- A field that stores a relative topological relation.
- A function which, when passing two intervals, returns true if the relative topological relation of the field that contains the position of the array is fulfilled, and false if it is not fulfilled.

Table 5. Actions of the labelled agent.

Action	Dimension 1	Dimension 2
function1	<	–
function2	>	–
function3	–	<
function4	–	>

Note: Symbol – indicates that it does not affect values of that dimension.

For actions, there is a table that indicates how the actions change the spatial position of the objects surrounding the agent for each dimension, based on an egocentric reference frame, as given in Table 5. It is important to highlight that the labelling of the actions is carried out taking into account an egocentric reference frame, and not an allocentric one. If an allocentric frame had been chosen, then an action of an agent should have a different label, depending on the orientation of the agent. In the implementation, we have opted for a single labelling, and the values are maintained in reference to the perception mechanism of the robot.

The final structure that corresponds to the establishing topological relations problem is a record with two fields bearing the following information:

- The first field is an identifier of the object for which a relative topological relation is looked for.
- The second field is an array of the same size as the number of dimensions that the space in the virtual environment possesses attained. Each position in the array stores the topological relation to be achieved in the corresponding dimension with the object specified by the first field of the record.

The algorithm implemented in each agent HTQS is basically Algorithm 1.

5.2. *Data and results*

Once implementation of the HTQS has been programmed in an agent, the algorithm has been tested with several establishing topological relations problems. Each problem is formalised in a configuration of the virtual world. We fix the numerical coordinates of the agent, the numerical coordinates of the target and the topological target relation in one of the dimensions. The sizes of the agent and the target are fixed as constants. The data for the position of the agent, which are given to the programme, are AXC (agent's X coordinate) and AYC (agent's Y coordinate). These two data allow the automatic generation of the rest of the agent's coordinates. The data for the position of the target are TXC (target's X coordinate) and TYC (target's X coordinate). The latest data provided are TRTRX (Target Relative Topological Relation in X), which determines the spatial relation that the agent should achieve in the X dimension with the target.

To generate a solution for the problem, the agent has the following actions in the X dimension: $1 : x = x + 1$; $2 : x = x - 1$. Each action modifies all the coordinates of the agent.

Different runs have shown how HTQS enables that agents to navigate and fulfill their goals. The results generated by HTQS in each scenario of the virtual environment are given in Table 6.

5.3. *The establishing topological relations problem*

The implementation of the HTQS solves the establishing topological relations problem with the following structure $((X, \mathcal{T}), O, e_1, F, R, \Omega, \Lambda)$, where R is the set of relative topological relations

Algorithm 1 n-dimensional HTQS algorithm

```
Require: dimMaximum: INTEGER; number of dimensions of the space
    A, R: ARRAY[1.. dimMaximum] of INTERVAL;
    Target: ARRAY[1..dimMaximum] of relativeTopologicalRelations;
Ensure: sequenceSolution.
1. dim := 1;
2. IF size(A,dim) > size(R,dim)    THEN IF size(A,dim) = size(R,dim) + 1
           THEN relationSize := greaterThan1;
           ELSE relationSize := greaterThan2;
     ELSE IF size(A,dim) < size(R,dim)
         THEN IF size(A,dim) + 1 = size(R,dim)          THEN relationSize := lessThan1 ;
           ELSE relationSize := lessThan2 ;
         ELSE IF size(A,dim) = 1          THEN relationSize := equalTo1 ;
           THEN relationSize := equalTo2 ;
3. CASE relationSize
   • greaterThan1 THEN TRLG^x := TRLG^GT1;
   • greaterThan2 THEN TRLG^x := TRLG^GT2;
   • lessThan1 THEN TRLG^x := TRPG^LT1;
   • lessThan2 THEN TRLG^x := TRLG^LT2;
   • equalTo1 THEN TRLG^x := TRLG^ET1;
   • equalTo2 THEN TRLG^x := TRLG^ET2;
4. relativeTopologicalRelationCurrent := Calculate-relative-topological-relation(A,R, dim);
5. nodeRelationCurrent := Get(TRLG^x, relativeTopologicalRelationCurrent);
6. nodeRelationTarget := Get(TRLG^x, Target(dim));
7. IF (nodeRelationCurrent < nodeRelationTarget)
   THEN heuristics = INCREMENT
   ELSE IF (positionRelationCurrent > positionRelationTarget)
     THEN heuristics := DECREMENT
     ELSE heuristics := RIGHT
8. IF heuristics = RIGHT
     THEN
       BEGIN
       dim := dim +1;
       IF (dim > dimMaximum)
         THEN Return(sequenceSolution);
     END
     ELSE
       BEGIN
       action := LookTableActions(heuristics, dim);
       Launch(action);
       sequenceSolution := sequenceSolution + action;
       GOTO step 3;
       END
```

Table 6. Different runs of the HTQS.

Problem	AXC	AYC	s-a	TXC	TYC	s-t	TRTRX	Solution
1	1	1	7	10	10	3	Contains	$\langle 1, 1, 1, 1, 1, 1\rangle$
2	14	1	7	10	10	3	Contains	$\langle 2, 2, 2, 2, 2\rangle$
3	1	1	7	1	10	3	Meet-0	$\langle 1, 1\rangle$
4	30	30	7	40	40	3	Contains	$\langle 1, 1, 1, 1, 1, 1\rangle$

Notes: AXC: agent's X coordinate; AYC: agent's Y coordinate; s-a: size of the agent; TXC: target's X coordinate; TYC: target's Y coordinate; s-t: size of the target; TRTRX: Target Relative Topological Relation in X. Actions: $1 : x = x + 1$; $2 : x = x - 1$.

S_{13}. It should be noted that HTQS can also be used for the establishing topological relations problem, where R is the set of topological relations R_8. This is possible because each one of the relative topological relations is contained only in one topological relation and the topological relations R_8 are disjoint.

$$\forall s \in S_{13} \quad \exists! r \in R_8, \quad s \subseteq r,$$
$$\forall r, r' \in R_8, \quad r \neq r' \rightarrow r \cap r' = \varnothing.$$

The establishing topological relations problems can be resolved with HTQS if each topological relation target is substituted by a relative topological relation which fulfills that is contained in the topological relation. The selection of the relative topology relation among the possible ones in a topological relation can be carried out in different ways (e.g choosing randomly, in a predeterminate way or calculated by any criteria such as distance).

6. Discussion and future research

Language and human thought make use of spatial relations for the description of many daily tasks. Thus far, the approaches most commonly used for representing space in navigation algorithms have been through numeric spatial coordinates. The creation of algorithms to perform navigation tasks, identifying and establishing spatial relations, is a logical step in the objective of creating multifunctional robots that can be integrated into human society. The application of topological relations for the representation of spatial relations has been used in cartography and geography for the construction of commercial GIS. However, topological relations to date have not been investigated in robot navigation, despite the importance of spatial relations for human navigation and communication. Therefore, we have decided to start a research to study the usefulness of topological notions in order to create multifunctional robots. This article presents the first results of the programme research MROTN into making decisions to establish spatial relations.

The starting point for our research is the study of the '9-intersection model' for prediction and inference of topological changes between objects used in GIS (Egenhofer & Al-Taha, 1992). In their work, the authors define a data structure, the CTRG, for prediction and inference. The findings for CTRG were not encouraging, as it was found that, considering solely the initial and final positions of the objects, inferring what exactly happens is impossible. The reason why making inferences with certainty is impossible is that there are multiple paths between two nodes in the CTRG. Thanks to the work described in Ernst (1969), the importance of linearity for the analysis of means and ends is known; and CTRG is not linear. Therefore, the next step in the research is to attain linear structures that represent gradual changes in topological relations between objects. It has been concluded that, if conditions of isosize and isoshape in the transformations are met, one can create a new set of graphs for prediction and inference, the LCTRGs, which are themselves linear. The LCTRGs can make inferences and prediction of the gradual changes in topological relations that occur, given the initial and final situations of the objects. This result does not invalidate the conclusions of Egenhofer and Al-Taha, as the isoshape and isosize assumptions are not met in the field of GIS. Fortunately, it can be assumed to be met in most of the urban environments where robots can navigate.

In the course of our research, we also found that the ability to infer and predict changes that are achieved by linearisation of the CTRG is not sufficient for decision-making to establish relations. The problem lies in the low expressive power of the set of topological relations defined by the '9-intersection model'. Therefore, we have defined a new set of relations, called relative topological relations, which are also jointly exhaustive and pairwise disjoint. The set of relative topological relations has 13 relations. To define and identify the new relations, a variation of Allen's (1983) method to define temporal relations between time intervals has been created. With the relative topological relations new linear structures, called TRLG, have been defined. The TRLG's are the fundamental elements of a new heuristic, called HTQS, that enables the construction of a solution to the establishing topological relations problem. The heuristic has been developed for one-dimensional topological spaces. Moreover, it has also been generalised to higher dimensional spaces, although it is required to observe certain restrictions on the shapes of the objects for the generalisation to be effective. The conditions are that objects should not have holes and that the sides should form convex right angles to each other. Given these constraints, we have developed a two-dimensional virtual environment with agents and objects, where HTQS is the algorithm used by agents to establish spatial relations in the environment. The implementation of the programme has shown that HTQS allows identifying and establishing spatial relations.

The reader can notice that the environments used here are simple in relation to much of the current research. However, we are beginning a new line of research and we are not seeking to improve or advance an existing line of research. While the quantitative methods to make decisions, or local navigation, in simple environments were done in the 1970s and 1980s we lack an equivalent for qualitative methods. The proposed research line should not be underestimated based on the complexity of the environments tackled in this article as the comparison is not fair. The reader should be aware of the difference between the final goal of the research programme and the steps which have to be carried out in order to attain the final goal, this article being one of those steps.

6.1. *TRLGs and topological maps*

The reader should note that when we talk about qualitative navigation with topological notions, we are not talking about using Voronoi diagrams or other kinds of topological maps. Voronoi diagrams are used for global navigation, they simplify an environment and permit a robot to have a representation of the ways to travel from one place to another place. The idea behind these kinds of diagrams is to group in a node the equivalent positions in the environment, and the links between nodes represent the paths between sites. TRLGs cannot be used as elements for global navigation because they do not indicate pathways in an environment though they can be used for local navigation. If the environment is unknown, then the agents require an exploratory phase in order to make a topological map of the environment. However, the TRLGs do not depend on the environment; the reason being that the sequence of relations cannot happen physically in a different order if the agents move in the same direction towards the object. For that reason, TRLGs are used for local navigation. Henceforth, the difference between algorithms which use topological maps and HTQS is that HTQS is focused on local navigation, while the topological maps are focused on planning of routes. However, if topological maps are given in a general definition (Remolina & Kuipers, 2004), then the TRLGs can be considered as the topological maps of the environment in the cases where the local environment is also the global environment. Usually, the nodes of a topological map contain quantitative information in each node but there is no restriction about the information they can contain, so they can also contain qualitative information, and then if the environment has only one object with the agent and the information stored in each node is about relative topological relations in the topological maps, then the topological map for that situation is a TRLG. If the human environments have more than one object, the topological map will not be a TRLG.

6.2. *Restrictions of HTQS and prospective goals*

The reader should note that the environments used here are relatively simple when compared with much current research. However, we are initiating a new line of research rather than merely improving or moving forward an existing line of research. While quantitative decision-making methods in simple environments were created in the 1970s and 1980s, we lack an equivalent for methods which use topological notions. So while the environments which are currently addressed by quantitative lines of research are much more complex than those which we seek to address, the complexity of the environments which we seek to address in this article should not be underestimated. A simple comparison is unfair. The reader should note the difference between the final goal of our programme of research and the steps which have to be carried out in order to reach that final goal, this article being just one of those steps.

(i) *Isoshape*
The isoshape was a necessary feature in the reasoning involved in developing the TRLG. However, it does not limit the HTQS. Let us suppose that the shape of the object respect to the agent which is establishing the spatial relation changes. In this case, the algorithm requires only a small change to avoid problems of isosize. The algorithm can be modified changing the sentence GOTO stage 3 of stage 8 by GOTO stage 2. This change permits the agent to take decisions even if the shape is changing. This modification permits the system to make decisions in real time.

(ii) *Isosize*
The isosize was a feature required in the reasoning process which led to the development of TRLG. Change of size can be seen as a special case of change of shape. This is the reason that the same modification in the HTQS code in the case of the no-isoshape for the HTQS can also be used in the case where no-isosize occurs in the environment.

(iii) *Static objects*
The feature that the object of the environment must be static has been taken into account in the reasoning done to develop the TRLG. However, this feature is not required due to the fact that it implies making the right decision for the agent but if the object is moving then the situation becomes a hunter-prey situation. The agent is going to pursue, or avoid, the object trying to establish the fixed spatial relation. Finally, a moving object can be considered as a special case of change of size so the code can be modified as in the last cases.

(iv) *Penetrable objects*
This is not a restriction of HTQS but a restriction of the knowledge which is needed to fix the spatial relation with an object. This restriction avoids the necessity of a knowledge system to fix the spatial relation goal because all spatial relations can be achieved physically. Hereinafter, the creation of a knowledge system which can be mixed with HTQS is a very important step in furnishing autonomy to an agent which uses HTQS.

(v) *Flawless map of the environment*
Another feature of the development of the TRLG is that the information gathered from the environment is precise. This requirement should not be a necessity in the use of topological notions. First, because the use of topological notions introduces the capacity to deal with

uncertainty because they are also qualitative notions. Second, it could also facilitate research into the creation of fuzzy versions of HTQS.

(vi) *Actions parallel to the reference system*
This is the second most important feature to be investigated, as this restriction limits the use of HTQS to a particular kind of robot or to a subset of the robots' actions. The research into this restriction is to determine which kind of robots can use either HTQS or some other version of it. The execution of non-parallel movements can be important in environments where it is essential to save time or energy, but there are environments where this requirement is not important because the environment can do necessary kinds of movements such as in libraries or stores. One line of research which may permit us to overcome the limitation of performing only parallel actions may be the use of transformation of coordinates.

(vii) *Orthogonal convex shapes*
This requirement is also important because it fixes the kind of environments where the algorithm can be used. For some environments, the requirement fits correctly, e.g. a warehouse, a loading dock, a road or a port. However, in order to extend the use of HTQS to environments where the restriction is not satisfied, it is necessary to research and elaborate new ways to calculate the relative topological relations (Rosman & Ramamoorthy, 2011).

(viii) *No holes in the objects*
This requirement is needed because the possibility of cavities in the objects adds too much complexity to the number of spatial relations which appear in the environment and as has been mentioned, at this time it is required in order to focus on a simple problem, thus establishing a firm basis for this line of research. Until further research has been carried out in more complex environments, this requirement will not be satisfied. One of the lines of research may be the addition of new relations to S_{13}. Independent of the issue, there are environments where HTQS can be useful because the objects have not holes, i.e. packing environments.

After analysis of the requirements, we think that the next step in research to create a multifunctional robot must be the creation of an architecture which uses HTQS to make decisions. The research must focus on implementing a knowledge base capable of storing information in order to choose its targets and avoid the physical restrictions of the environment. That architecture will be of type III. Once the architecture is ready, the time will have come to address more complex environments. One of the most important issues will be the creation of a mathematical method to identify the relative topological relation between two objects. The current method imposes important restrictions in the generalisation to a n-dimensional space. A deep analysis would allow the creation of another method to identify the relative topological relations which allows more environments where HTQS is effective. One option could be the use of learning spatial relations Rosman and Ramamoorthy (2011). Related to this is the technique of computer vision used because it is linked to spatial representation. An analysis of different methods will be needed in order to fully perceive the advantages and disadvantages of each one with HTQS.

6.3. *Conclusions*

In summary, our research has established a method for making decisions based on identifying and establishing spatial relations. We have linked concepts of different fields, namely the work of Egenhofer and colleagues that belongs to the realm of GIS, the research of Ernst on heuristic analysis of means and ends, and Allen's algebras for temporal reasoning. Three results, which apparently were not related, are the foundation that has allowed the development of the HTQS heuristics. The new theoretical results presented here are the basis on which we are developing new navigation algorithms. The work presented here thus offers the first results in the agenda of our research programme to develop a navigation architecture based on topological notions for the creation of robots which use spatial relations in their interactions with humans and in their decision-making processes.

Note

1. In this article, the term navigation only refers to local navigation. The term planning will be used to refer to global navigation.

References

Allen, J. F. (1983). Maintaining knowledge about temporal intervals. *Communications of the ACM, 26*(11), 832–843.

Asher, N., & Vieu, L. (1995). Toward a geometry of common sense: A semantics and a complete axiomatization of mereotopology. In C. S. Mellish (Ed.), *Proceedings of the 14th international joint conference on artificial intelligence* (pp. 846–852). San Francisco, CA: Morgan Kaufmann.

Barraquand, J., & Latombe, J. C. (1993). Nonholonomic multibody mobile robots: Controllability and motion planning in the presence of obstacles. *Algorithmica*, 10(2–4), 121–155.

Borenstein, J., & Koren, Y. (1991, June). The vector field histogram-fast obstacle avoidance for mobile robots. *IEEE Transactions on Robotics and Automation, 7*(3), 278–288.

Brooks, R. (1986, March). A robust layered control system for a mobile robot. *IEEE Journal of Robotics and Automation, 2*(1), 14–23.

Chestnutt, J., Lau, M., Cheung, G., Kuffner, J. J., Hodgins, J., & Kanade, T. (2005). Footstep planning for the Honda ASIMO humanoid. *Proceedings of the 2005 IEEE international conference on robotics and automation* (pp. 629–634). Piscataway, NJ: IEEE.

Choset, H., Lynch, K. M., Hutchinson, S., Kantor, G., Burgard, W., Kavraki, L. E., & Thrun, S. (1995). *Principles of robot motion: Theory, algorithms, and implementations*. Cambridge, MA: The MIT Press.

Clementini, E., Di Felice, P., & Hernández, D. (1997). Qualitative representation of positional information. *Artificial Intelligence, 95*(2), 317–356.

Cohn, A. G., & Renz, J. (2008). Qualitative spatial reasoning. In F. Van Harmelen, V. Lifschitz, & B. Porter (Eds.), *Handbook of knowledge representation* (pp. 581–584). London: Elsevier Science.

Delafontaine, M., Weghe, N. Van de, Bogaert, P., & De Maeyer, P. (2008). Qualitative relations between moving objects in a network changing its topological relations. *Information Sciences, 178*(8), 1997–2006.

Dylla, F., Ferrein, A., Lakemeyer, G., Murray, J., Obst, O., Röfer, T., ... Wagner, T. (2008). Approaching a formal soccer theory from behaviour specifications in robotic soccer. In P. Dabnichki & A. Baca (Eds.), *Computers in sports* (pp. 161–181). London: WIT Press.

Egenhofer, M. J. (1989). A formal definition of binary topological relationships. *Lectures Notes in Computer Science, 367*, 457–462.

Egenhofer, M. J., & Al-Taha, K. K. (1992). Reasoning about gradual changes of topological relationships. In A. U. Frank, I. Campari, & U. Formentini (Eds.), *Proceedings of the international conference GIS* (pp. 196–219). London: Springer-Verlag.

Egenhofer, M. J., & Franzosa, R. (1991). Point-set topological spatial relations. *International Journal of Geographical Information Systems, 5*(2), 161–174.

Egenhofer, M. J., & Herring, J. (1990). A mathematical framework for the definition of topological relationships. In K. Brassel & H. Kishimoto (Eds.), *Proceedings of the fourth international symposium on spatial data handling* (pp. 803–813). Columbus, OH: The Ohio State University.

Egenhofer, M. J., Sharma, J., & Mark, D. (1993). A critical comparison of the 4-intersection and 9-intersection models for spatial relations: Formal analysis. *Auto-Carto, 11*, 1–11.

Ernst's, G. W. (1969). Sufficient conditions for the success of GPS. *Journal of the ACM, 16*(4), 517–533.

Fernández-Caballero, A., Castillo, J. C., Martínez-Cantos, J., & Martínez-Tomás, R. (2010, December). Optical flow or image subtraction in human detection from infrared camera on mobile robot. *Robotics and Autonomous Systems, 58*(12), 1273–1281.

Ferrein, A., Fritz, C., & Lakemeyer, G. (2005). Using Golog for deliberation and team coordination in robotic soccer. *Künstliche Intelligenz, 19*, 24–30.

Fikes, R. E., & Nilsson, N. J. (1971). Strips: A new approach to the application of theorem proving to problem solving. *Artificial Intelligence, 2*(3), 189–208.

Fogliaroni, P., Wallgrün, J. O., Clementini, E., Tarquini, F., & Wolter, D. (2009). A qualitative approach to localization and navigation based on visibility information. In K. S. Hornsby, C. Claramunt, M. Denis, & G. Ligozat (Eds.), *Proceedings of the 9th international conference on spatial information theory* (pp. 312–329). Heidelberg: Springer.

Forbus, K. D., Nielsen, P., & Faltings, B. (1991). Qualitative spatial reasoning: The clock project. *Artificial Intelligence, 51*(1–3), 417–471.

Freksa, C. (1990). *Temporal reasoning based on semi-intervals* (Tech. Rep. No. ICSI TR-90-016). Berkeley: University of California.

Freksa, C. (1991a). *Cognitive and linguistic aspects of geographic space* (pp. 361–372). Dordrecht: Kluwer Academic Publishers.

Freksa, C. (1991b). Conceptual neighborhood and its role in temporal and spatial reasoning. In M. G. Singh & L. Travé-Massuyès (Eds.), *Proceedings of the IMACS workshop on decision support systems and qualitative reasoning* (pp. 181–187). San Francisco, CA: Elsevier.

Freksa, C. (1992). Temporal reasoning based on semi-intervals. *Artificial Intelligence, 54*(1–2), 199–227.

Gerevini, A., & Nebel, B. (2002). Qualitative spatio-temporal reasoning with rcc-8 and allen's interval calculus: Computational complexity. In F. Van Harmelen (Ed.), *Proceedings of the 15th European conference on artificial intelligence* (pp. 312–316). Amsterdam: IOS Press.

Girard, M. (1987, June). Interactive design of 3d computer-animated legged animal motion. *IEEE Computer Graphics and Applications, 7*(6), 39–51.

Hahmann, T., Winter, M., & Gruninger, M. (2009). Stonian p-ortholattices: A new approach to the mereotopology RT0. *Artificial Intelligence, 173*(15), 1424–1440.

Kala, R., Shukla, A., & Tiwari, R. (2010). Dynamic environment robot path planning using hierarchical evolutionary algorithms. *Cybernetics and Systems, 41*(6), 435–454.

Kala, R., Shukla, A., & Tiwari, R. (2012, August). Robotic path planning using hybrid genetic algorithm particle swarm optimisation. *International Journal of Information and Communication Technology, 4*(2/3/4), 89–105.

Kamon, I., Rimon, E., & Rivlin, E. (1998). Tangentbug: A range-sensor based navigation algorithm. *International Journal of Robotics Research, 2*, 934–953.

Kelley, J. L. (1975). *General topology*. Basel: Birkhäuser.

Kim, C., Seong, K. A., Hyung, L. K., & Kim, J. O. (1998, May). Design and implementation of a fuzzy elevator group control system. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, 28*(3), 277–287.

Kuffner, J. J., Nishiwaki, K., Kagami, S., Inaba, M., & Inoue, H. (2001). Footstep planning among obstacles for biped robots. *Proceedings of the 2001 IEEE/RSJ international conference on intelligent robots and systems* (vol. 1, pp. 500–505). Piscataway, NJ: IEEE.

Kuipers, B. (2000). The spatial semantic hierarchy. *Artificial Intelligence, 119*(1–2), 191–233.

Kwalski, R., & Sergot, M. (1986). A logic-based calculus of events. *New Generation Computing, 4*(1), 67–95.

Lavalle, S. M. (2006). *Planning algorithms*. Cambridge: Cambridge University Press.

Lea, R. N., Hoblit, J., & Jani, Y. (1993). Fuzzy logic based robotic arm control. *Second IEEE international conference on fuzzy systems* (vol. 1, pp. 128–133). Piscataway, NJ: IEEE.

Levesque, H. J., Reiter, R., Lespérance, Y., Lin, F., & Scherl, R. B. (1997). Golog: A logic programming language for dynamic domains. *The Journal of Logic Programming, 31*(1–3), 59–83.

Levitt, T. S., & Lawton, D. T. (1990). Qualitative navigation for mobile robots. *Artificial Intelligence, 44*(3), 305–360.

Li, S., & Wang, H. (2006). Rcc8 binary constraint network can be consistently extended. *Artificial Intelligence, 170*(1), 1–18.

Li, S., & Ying, M. (2004). Generalized region connection calculus. *Artificial Intelligence, 160*(1), 1–34.

Li, Y., & Li, S. (2004). A fuzzy sets theoretic approach to approximate spatial reasoning. *IEEE Transactions on Fuzzy Systems, 12*(6), 745–754.

Liu, W., Zhang, X., Li, S., & Ying, M. (2010). Reasoning about cardinal directions between extended objects. *Artificial Intelligence, 174*(12–13), 951–983.

López-Valles, J. M., Fernández, M. A., Fernández-Caballero, A., López, M. T., Mira, J., & Delgado, A. E. (2005). Motion-based stereovision model with potential utility in robot navigation. *Lecture Notes in Artificial Intelligence, 3533*, 16–25.

Lozano-Perez, T. (1983, February). Spatial planning: A configuration space approach. *IEEE Transactions on Computers, C-32*(2), 108–120.

Lumelsky, J. V., & Stepanov, E. A. (1987). Path planning strategies for a point mobile automaton moving amidst unknown obstacles of arbitrary shape. *Algorithmica, 17*(9), 403–430.

Masoud, A. (2010, March). Kinodynamic motion planning. *IEEE Robotics Automation Magazine, 17*(1), 85–99.

McCarthy, J., & Hayes, P. J. (1987). Some philosophical problems from the standpoint of artificial intelligence. In M. L. Ginsberg (Ed.), *Readings in nonmonotonic reasoning* (pp. 26–45). San Francisco, CA: Morgan Kaufmann.

McDermott, D., & Davis, E. (1984). Planning routes through uncertain territory. *Artificial Intelligence, 22*(1), 107–156.

Miguel-Tomé, S. (2008). *Estudio de algoritmos de semántica cualitativa aplicados a la navegación* (Unpublished master's thesis). Universidad de Castilla la Mancha, Ingeniería Informática.

Miguel-Tomé, S. (2011). General theory of exobehaviours: A new proposal to unify behaviors. *Lecture Notes in Computer Science, 6686*, 20–29.

Miguel-Tomé, S. (2013). On the identification and establishment of topological spatial relations. In J. M. Ferrández Vicente, J. R. Álvarez Sánchez, F. de la Paz López, & F. J. Toledo Moreo (Eds.), *Natural and artificial computation in engineering and medical applications* (vol. 7931, pp. 461–470). Lecture Notes in Computer Science. Berlin, Heidelberg: Springer.

Newell, A., Shaw, J. C., & Simon, H. A. (1959). Report on a general problem-solving program. *Proceedings of the international conference on information processing* (pp. 256–264). London: Butterworths.

Newman, W., & Hogan, N. (1987, March). High speed robot control and obstacle avoidance using dynamic potential functions. *Proceedings of the 1987 IEEE international conference on robotics and automation* (vol. 4, pp. 14–24). Piscataway, NJ: IEEE.

Pavón, J., Gómez-Sanz, J. J., Fernández-Caballero, A., & Valencia-Jiménez, J. J. (2007, December). Development of intelligent multi-sensor surveillance systems with agents. *Robotics and Autonomous Systems, 55*(12), 892–903.

Randell, D. A., Cui, Z., & Cohn, A. G. (1992). A spatial logic based on regions and connection. *Proceedings of the 3rd international conference on knowledge representation and reasoning* (pp. 165–176). San Francisco, CA: Morgan Kaufmann.

Remolina, E., & Kuipers, B. (2004). Towards a general theory of topological maps. *Artificial Intelligence, 152*(1), 47–104.

Renz, J. (1999). Maximal tractable fragments of the region connection calculus: A complete analysis. In T. Dean (Ed.), *Proceedings of the 16th international joint conference on artificial intelligence* (pp. 448–454). San Francisco, CA: Morgan Kaufmann.

Renz, J. (2002). *Qualitative spatial reasoning with topological information*. Heidelberg: Springer.

Renz, J., & Nebel, B. (1999). On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the region connection calculus. *Artificial Intelligence, 108*(1–2), 69–123.

Rosenfeld, A. (1979). Digital topology. *The American Mathematical Monthly, 86*, 621–630.

Rosman, B., & Ramamoorthy, S. (2011). Learning spatial relationships between objects. *The International Journal of Robotics Research, 30*(11), 1328–1342.

Sacerdott, E. D. (1973). Planning in a hierarchy of abstraction spaces. In N. J. Nilsson (Ed.), *Proceedings of the 3rd international joint conference on artificial intelligence* (pp. 412–422). San Francisco, CA: Morgan Kaufmann.

Schlieder, C. (1993). Representing visible locations for qualitative navigation. In N. P. Carreté & M. G. Sing (Eds.), *Qualitative reasoning and decision technologies* (pp. 523–532). Barcelona: Universitat Politècnica de Catalunya.

Schockaert, S., De Cock, M., & Kerre, E. E. (2009). Spatial reasoning in a fuzzy region connection calculus. *Artificial Intelligence, 173*(2), 258–298.

Shi, C., Wang, Y., & Yang, J. (2010). Online topological map building and qualitative localization in large-scale environment. *Robotics and Autonomous Systems, 58*(5), 488–496.

Sobek, R. P., & Chatila, R. G. (1988). Integrated planning and execution control for an autonomous mobile robot. *Artificial Intelligence in Engineering, 3*(2), 103–113.

Stell, J. G. (2000). Boolean connection algebras: A new approach to the region-connection calculus. *Artificial Intelligence, 122*(1–2), 111–136.

Van de Weghe, N., Bogaert, P., Cohn, A. G., Delafontaine, M., De Temmerman, L., Neutens, T., ... Witlox, F. (2007). How to handle incomplete knowledge concerning moving objects. In B. Gottfried (Ed.), *Proceedings of the international workshop on behaviour monitoring and interpretation* (pp. 91–101). Retrieved from ftp://SunSITE.Informatik.RWTH-Aachen.DE/pub/publications/CEUR-WS/Vol-296.zip

Van de Weghe, N., Cohn, A. G., De Maeyer, P., & Witlox, F. (2005, November). Representing moving objects in computer-based expert systems: The overtake event example. *Expert Systems with Applications, 29*(4), 977–983.

Van de Weghe, N., Cohn, A. G., De Tre, G., & De Maeyer, P. (2006). A qualitative trajectory calculus as a basis for representing moving objects in geographical information systems. *Control and Cybernetics, 35*(1), 97–119.

Wagner, T., & Huebner, K. (2005). An egocentric qualitative spatial knowledge representation based on ordering information for physical robot navigation. *Lecture Notes in Artificial Intelligence, 3276*, 134–149.

Wagner, T., Huebner, K., & Visser, U. (2003). An extended panorama: Efficient qualitative spatial knowledge representation for highly dynamic environments. In U. Visser & P. Doherty (Eds.), *Workshop on issues in designing physical agents for dynamic real-time environments* (pp. 109–116). Heidelberg: Springer.

Wolter, D., & Lee, J. H. (2010). Qualitative reasoning with directional relations. *Artificial Intelligence, 174*, 1498–1507.

Appendix 1. CTRG linearisation

The results of Egenhofer and Al-ha do not determine that one cannot develop an algorithm to make decisions based on topological relations. So, if we want to develop an algorithm to make decisions based on topological relations, first we must understand the cause because CTRG fails and so we could avoid it.

Thanks to Ernst's (1969) investigations, we know that an analysis of means and ends will find a solution if one can define a complete order relation on the differences. A graph is a binary relation on a set. Therefore, as the CTRG is a graph with cycles, it is impossible to apply an analysis of means and ends from it. So, the next step is to obtain a linear binary relation on the topological relations. The reason for the CTRG to be non-linear is the wide range of changes that the objects may suffer. Clearly, GIS takes into account changes in the size of an object, as rivers, lakes, sea or forest stands may increase or decrease surface drastically. However, in a common navigation environment, such as a building, this does not happen to objects (i.e. to robots). Therefore, we must take into account the following two conditions: temporal isosize and temporal isoshape.

Temporal isosize: the size of an object does not change over time.
Temporal isoshape: the shape of an object does not change over time.

The consequence of meeting the above two conditions is that all topological relations that the CTRG contains between two objects cannot appear. Possible topological relations between two objects when they meet isosize and isoshape depend on the existing size relation between the two objects. Thus, between any two objects on a dense domain, three possible sets of topological relations are possible, and the resulting set is a function of the size relation between the two objects. The possible cases are as follows:

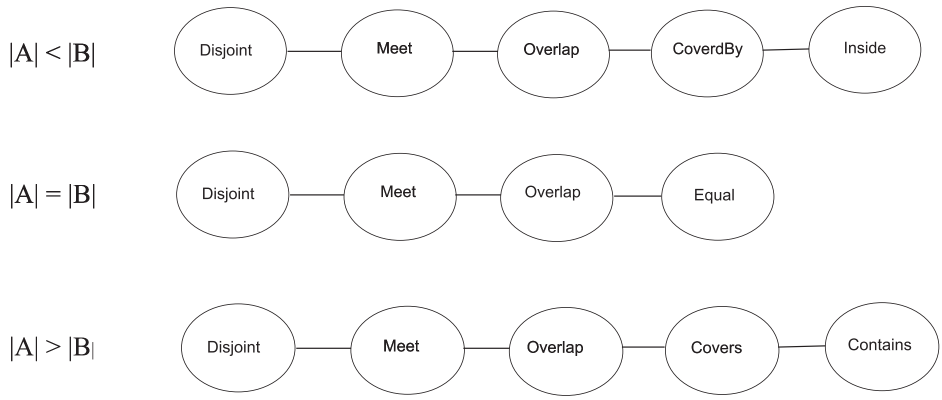

Figure A1. If two objects cannot change size and shape, then the transformations that can undergo topological relations between these two objects are described by one of the three linear graphs shown. The graph that describes the changes depends on the size ratio between the two objects.

- size(A) $<$ size(B)
  If the size of A is smaller than the size of B, the graph that represents the possible topological relations is the one shown in Figure A1.
- size(A) $=$ size(B)
  If the size of A is equal to the size of B, the graph that represents the possible topological relations is the one shown in Figure A1.
- size(A) $>$ size(B)
  If the size of A is greater than the size of B, the graph that represents the possible topological relations is the one shown in Figure A1.

Each graph which can be built with the set of topological relations that can occur in each of the above cases will be called Linear Closest Topological Relationship Graph (LCTRG).

A.1. *LCTRGs do not permit navigation*

In the previous section, we have seen that LCTRGs are obtained in an environment where isosize and isoshape conditions are met. LCTRGs do not have the problem of the existence of more than one possible path between two nodes of the graph (this problem exists in the approach by Egenhofer and Al-Taha). But, is this linearisation sufficient to achieve an algorithm that allows establishing spatial relations? During the investigation, it has been seen that the answer is no. This can be explained by the following example: imagine that in the topological space $\mathbb{R}$ there are two objects, o_A and o_B, which occupy space areas A and B. It is intended to pass from relation *disjoint* to relation *meet*. As shown in Figure A2, there are two situations in which there is one set of actions that serve as a solution, if applied in one of the two situations but not in the other. Indeed, the graph shows two situations that exemplify the problem. Each of the situations have two objects: object o_A represented by [ ] and o_B represented by $\langle\ \rangle$, both objects are situated in a one-dimensional space represented by the dotted line. Depending on the specific location of objects in space, different ways are drawn in LCTRG, although they start from the same topological relation and the same functions are applied in both cases. The functions are $\mathbf{f}_+|_x(x) = x + 1$ and $\mathbf{f}_=|_x(x) = x$. Thus, a function can generate more than one way in LCTRG. The problem is that these two situations are described with the same topological relations, making it impossible to distinguish one situation from the other. Therefore, if applying a function, it generates more than one way in an LCTRG. That has prevented to decide whether this is the action that is part of a solution. This problem occurs with all nodes of the LCTRG unless the starting node is *Equal*, *Contained* or *Contains*.

Appendix 2. Topology

This appendix includes the topology definitions and theorems used in our research. Proofs are omitted but can be consulted in a general text on topology.

Definition A.1 *Given a set, X, $\mathcal{T}$ is a topology defined over X if $\mathcal{T}$ is a collection of subsets of X such that*:

(1) $\emptyset, X \in \mathcal{T}$,

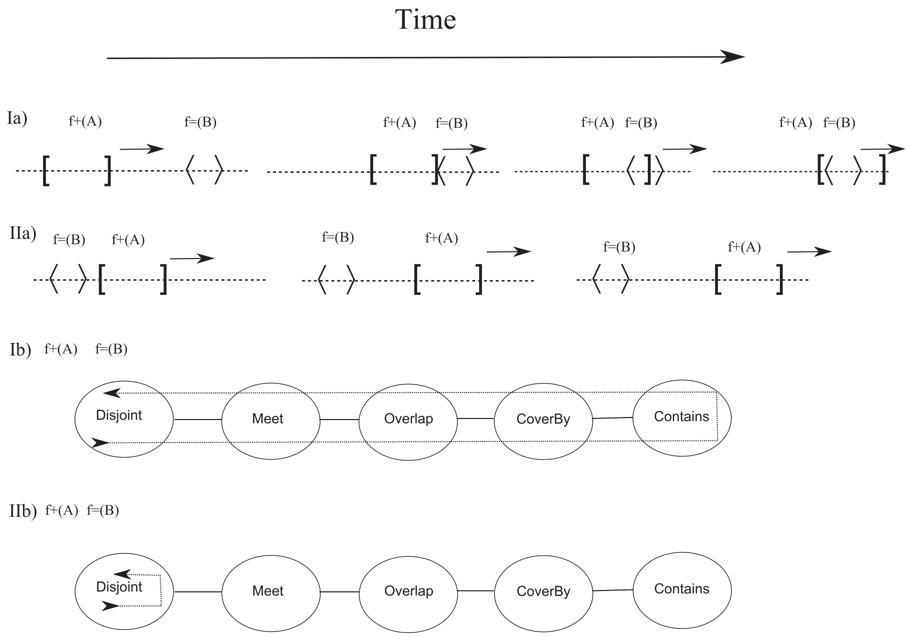

Figure A2. With two objects, two different spatial situations can be created but they cannot be seen differently using topological relations, and if they are not seen differently the agent cannot make a decision to achieve its target. The target is moving to the other side of an object in both situations. While in one situation the target is already achieved and the agent must do the null action, in the other situation forward action must be done. The LCTRG cannot discriminate for which of the situations the function forward is the solution to obtain the desired spatial relation between the two objects. It is due to the same relation that contains states which are target and others which are not target.

(2) $\forall i \in I, U_i \in \mathcal{T} \Rightarrow (\bigcup_i U_i) \in \mathcal{T}$,
(3) $U_1 \in \mathcal{T}, U_2 \in \mathcal{T} \Rightarrow (U_1 \cap U_2) \in \mathcal{T}$.

Each element of $\mathcal{T}$ is called an open element.

Definition A.2 *The pair $(X, \mathcal{T})$ is called ($(X, \mathcal{T})$ is often abbreviated as X if $\mathcal{T}$ is understood) a topological space.*

Definition A.3 *Given a set X, $\mathcal{B}$ is a base if it is a collection of subsets that satisfies*:

(1) $\forall x \in X, \exists B \in \mathcal{B}$,
(2) $\forall x \in B_1 \cap B_2$, *where* $B_1, B_2 \in \mathcal{B}, \exists B_3 \in \mathcal{B} : x \in B_3 \subset B_1 \cap B_2$.

Definition A.4 *Given a base $\mathcal{B}$ over X, the topology generated by $\mathcal{B}$, is the one that, $\mathcal{U}$ is open if and only if for each $x \in \mathcal{U}$ there exists $B \in \mathcal{B}$ such that $x \in B \subset \mathcal{U}$.*

Proposition A.5 *The topology generated by a base is really a topology.*

Somehow, a base contains the bricks needed to build any open element.

Lemma A.6 *Given a base, $\mathcal{B}$, each of its elements is open in the topology that it generates, and each open element in that topology may be written as the union (probably infinite) of elements of $\mathcal{B}$.*

Example The collection of all open intervals $\mathcal{B}_1 = \{(a, b) : a, b \in \mathbb{R}\}$ is a topology in $\mathbb{R}$ and is called *usual topology*.

As open intervals $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ generate a topology (the usual one), an idea to generate topologies is to change the meaning of '$<$'.

Definition A.7 *'$\leq$' is a linear order relation in a set X if for each $a, b, c \in X$ it holds that*

(1) $a \leq a$,
(2) $a \leq b, b \leq a \Rightarrow a = b$,
(3) $a \leq b, b \leq c \Rightarrow a \leq c$

and, moreover, given a, b always one of the relations $a < b$, $b < a$ or $a = b$ holds.

Here, $a < b$ has been used as abbreviation of $a < b, a \neq b$. Intervals are denoted as in $\mathbb{R}$.

$$(a,b) = \{x : a < x < b\}, \quad [a,b] = \{x : a \leq x \leq b\}.$$

Example If in set $X = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$ the order relation $\clubsuit < \diamondsuit < \heartsuit < \spadesuit$ is defined, then

$$(\clubsuit, \heartsuit) = \{\diamondsuit\} \quad [\diamondsuit, \spadesuit) = \{\diamondsuit, \heartsuit\} \quad [\diamondsuit, \clubsuit] = \emptyset.$$

DEFINITION A.8 *Let X be a set with a linear order. It is called topology of the order, $\mathcal{T}_<$, to the one generated by the base $\mathcal{B}_{()}$ whom elements are open intervals (a, b) with $a < b$ and $a, b \in X$ or by the base $\mathcal{B}_{[)}$ whom elements are intervals as $[m, b)$ and/or $\mathcal{B}_{(]}$ whom elements are $(a, M]$ if X has a minimum element, m, and/or maximum element, M.*

Example In the previous example, $m = \clubsuit$ and $M = \spadesuit$, and the topology of the order coincides with the discrete, as each element of X is open; in fact, it is an element of the base:

$$\{\clubsuit\} = [\clubsuit, \diamondsuit) \quad \{\diamondsuit\} = (\clubsuit, \heartsuit) \quad \{\heartsuit\} = (\diamondsuit, \spadesuit) \quad \{\spadesuit\} = (\heartsuit, \spadesuit]$$

Given a topology X and another over Y, there is a canonical manner of creating a topology over the Cartesian product $X \times Y$.

DEFINITION A.9 *Let $\mathcal{B}_X$ and $\mathcal{B}_Y$ be bases of topologies over X and Y, respectively. Product topology is the topology over $X \times Y$ generated by*

$$\mathcal{B}_X \times \mathcal{B}_Y = \{(B_X, B_Y) : B_X \in \mathcal{B}_X, B_Y \in \mathcal{B}_Y\}.$$

As the product topology is defined in $X \times Y$, it can be defined in $X_1 \times X_2 \times \cdots \times X_n$.
In topology, the following lemma is known.

LEMMA A.10 *If $\mathcal{B}_X$ and $\mathcal{B}'_X$ are bases of a same topology in X, and $\mathcal{B}_Y$ and $\mathcal{B}'_Y$ are bases of a same topology in Y, then $\mathcal{B}_X \times \mathcal{B}_Y$ generate the same topology than $\mathcal{B}'_X \times \mathcal{B}'_Y$ over $X \times Y$.*

A.2. *Associated sets*

Given a subset A of a topological space X, there are natural sets associated to A; some of them are described in the following definitions.

DEFINITION A.11 *Interior of A, denoted as A°, is the union of all the open elements contained in A. The interior of an empty set is empty. The interior of X is X. If A is open then it holds that $A = A^\circ$.*

DEFINITION A.12 *Closure of A, and denoted as $\bar{A}$, is the intersection of all closed elements that contain A. The closure of the empty set is empty; this is the unique set whose closure is empty. The closure of X is $\bar{X}$. If A is closed, then it holds that $A = \bar{A}$.*

DEFINITION A.13 *Frontier of A, denoted as ∂A, is the intersection of the closure of A with the closure of the complement of A.*

$$\partial A = \bar{A} \cap (\overline{X - A})$$

DEFINITION A.14 *Exterior of A, denoted as A^-, is the set of all elements not contained in A.*

PROPOSITION A.15 *A°, ∂A and A^- are mutually exclusive and hold that $A^\circ \cup \partial A \cup A^- = X$*